HW9

1. Prove that if f is a polynomial of degree at most n and p is the polynomial of degree at most n that interpolates f at a set of $n+1$ distinct nodes, then $p \equiv f$.

2. Problem 6.3.1

Use the extended Newton divided difference method to obtain a Hermite interpolation polynomial that takes these values:

x	0	1	2
$p(x)$	2	-4	44
$p^{'}(x)$	-9	4	

.

3. Given $n+1$ distinct nodes $x_0 = a, x_1, ..., x_n = b$ in the interval $[a,b]$, $y_i = f(x_i)$, $i = 0, 1, ..., n$, and $y_{i_k}^{'} = f^{'}(x_{i_k})$, $k = 0, 1, ..., m$. Here $0 \leq i_k \leq n$, $m \leq n$. Show that $f(x) - P_{m+n+1}(x) = \frac{f^{(m+n+2)}(\xi)}{(m+n+2)!} \prod_{i=0}^{n}(x - x_i) \prod_{k=0}^{m}(x - x_{i_k})$, where $\xi \in (a,b)$ and $P_{m+n+1}(x)$ is a Hermite interpolation polynomial. (Hint: Define the function $g(t) = f(t) - P_{m+n+1}(t) - \frac{f(x)-P_{m+n+1}(x)}{\prod_{i=0}^{n}(x-x_i)\prod_{k=0}^{m}(x-x_{i_k})} \prod_{i=0}^{n}(t - x_i) \prod_{k=0}^{m}(t - x_{i_k})$. Use Rolle's theorem and note that x_i are zeros of $g(t)$ and x_{i_k} are zeros of $g^{'}(t)$.)

4. Problem 6.9.15

Prove that an orthogonal set of nonzero elements of an i. p. s. is necessarily linearly independent.

5. Problem 5.3.5

Let $u_1, u_2, ..., u_n$ be an orthonormal base for a subspcace U in an i. p. s X. Define $P : X \to U$ by the equation $Px = \sum_{i=1}^{n} < x, u_i > u_i$. Prove:

a. P is linear.

b. $P^2 = P$.

c. $Px = x$ if $x \in U$.

d. $||Px||_2 \leq ||x||_2$ for all $x \in X$.

The map P is called the orthogonal projection of X onto U.