HW10

1 Problem 7.2.4

Verify that the following formula is exact for polynomial of degree ≤ 4:

$\int_0^1 f(x)dx \approx 1/90[7f(0) + 32f(1/4) + 12f(1/2) + 32f(3/4) + 7f(1)]$.

2 Problem 7.2.8

Find the coefficients A_0, A_1 of the formula $\int_0^1 f(x)dx \approx A_0 f(0) + A_1 f(1)$ that is exact for all functions of the form $f(x) = ae^x + b\cos(\pi x/2)$. (Hint: This is similar to the method of undetermined coefficients with polynomials.)

3 Problem 7.2.11

Using the polynomial of degree 1 that interpolates $f(x)$ at x_1, x_2, derive a numerical integration formula for $\int_{x_0}^{x_3} f(x)dx$. Here $x_0 < x_1 < x_2 < x_3$.

4 Problem 7.2.19

The **midpoint rule** over the interval $[x_{i-1}, x_{i+1}]$ is given by $\int_{x_{i-1}}^{x_{i+1}} f(x)dx = (x_{i+1} - x_{i-1})f(x_i)$. Determine the composite midpoint rule over the interval $[a, b]$ with uniform spacing of $h = (b-a)/n$ such that $x_i = a + ih$ for $i = 0, ..., n$, n is even.

5 Problem 7.2.21

Which of the following Newton-Cotes formulas for $n = 2$ and $[a, b] = [0, 1]$ is better?

$\int_0^1 f(x)dx \approx af(0) + bf(1/2) + cf(1)$

$\int_0^1 f(x)dx \approx \alpha f(1/4) + \beta f(1/2) + \gamma f(3/4)$. (Hint: Compare the errors.)

6 Problem 7.3.8.a

Determine appropriate values of A_i and x_i so that the quadrature formula

$$\int_{-1}^{1} x^2 f(x)dx \approx \sum_{i}^{n} A_i f(x_i)$$

will be correct when f is any polynomial of degree 3. Use $n = 1$.

7 Problem 7.3.9

Find a quadrature formula $\int_{-1}^{1} f(x)dx \approx c\sum_{i=0}^{2} f(x_i)$ that is exact for all quadratic polynomials.