ACMS 40390: Sample Test II

February 27, 2019

By signing you confirm that you are following the honor code for this test.

Name: ....

To receive credit you must show your work.

Problem Number	Maximum Points	Points attained
1	7	
2	12	
3	7	
4	7	
5	7	
6	7	
7	7	
8	7	
9	7	
10	10	
11	10	
12	12	
TOTAL	100	

Some Useful Results

The following result may be of use. You may assume it in any argument you need to give.

Runge-Kutta Method of Order Four For the ordinary differential equation $y' = f(t, y)$ on $[a, b]$ with initial condition $y(a) = \alpha$ and stepsize h we have

$$\begin{aligned}
w_0 &= \alpha \\
k_1 &= hf\left(t_i, w_i\right) \\
k_2 &= hf\left(t_i + \frac{h}{2}, w_i + \frac{k_1}{2}\right) \\
k_3 &= hf\left(t_i + \frac{h}{2}, w_i + \frac{k_2}{2}\right) \\
k_4 &= hf\left(t_i + h, w_i + k_3\right) \\
w_{i+1} &= w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)
\end{aligned}$$

Letting $f(x) \in C^5[a, b]$ with $a < b$, we use the book's notation $S(f, a, b)$ for the Simpson rule approximation to $\int_a^b f(x)\mathrm{dx}$. We use

$$\mathrm{Err(f, a, b)} := \frac{\left|\mathrm{S(f, a, b)} - \mathrm{S}\left(\mathrm{f, a, \tfrac{a+b}{2}}\right) - \mathrm{S}\left(\mathrm{f, \tfrac{a+b}{2}, b}\right)\right|}{15}$$

as an estimate for the error

$$\left|\int_a^b f(x)\mathrm{dx} - \left(\mathrm{S}\left(\mathrm{f, a, \frac{a+b}{2}}\right) + \mathrm{S}\left(\mathrm{f, \frac{a+b}{2}, b}\right)\right)\right|.$$

Problems

In the following you must show your work.

Problem 1 (7 points total) *We want to approximate*

$$\int_0^1 (1 + 1.06\pi x)\sqrt{\sin(\pi x)}\ \mathrm{d}x$$

with at most an error of 0.005. *We use the Adaptive Simpson's rule and find*

$$\frac{|S(f,0,1) - S(f,0,0.5) - S(f,0.5,1)|}{15} = 0.00508.$$

Should we accept the result or should we subdivide further and if so how? Explain your answer.

Problem 2 (12 points total) *Let* $y' = t^2y^2$ *on* $[0,1]$ *with initial condition* $y(0) = 24$.

1. *What is an explicit local solution near* 0*?*
2. *Is it unique and if so why?*
3. *Is there a continuously differentiable solution on all of* $[0,1]$*? (Either show there is by explicitly constructing the solution or show there is no solution on all of* $[0,1]$*.)*

Problem 3 (7 points total) *What integration method does the Runge-Kutta Method of Order Four applied to solving $y' = f(t)$) on $[1, 2]$ with initial value $y(1) = 0$ and $h = 1.0$ reduce to. (To receive credit you must show this explicitly.)*

Problem 4 (7 points total) *Use the midpoint method to approximate the solution of $y' = t\sin(y)$ on $[1, 1.5]$ with initial value $y(1) = 2$ and $h = 0.5$.*

Problem 5 (7 points total) *Use Taylors method of order three to approximate the solution of* $y' = ty$ *on* $[0, 0.5]$ *with initial value* $y(0) = -1$ *and* $h = 0.5$.

Problem 6 (7 points) *Use the Euler method with* $h = 0.5$ *to solve* $y'' - 3y' + 2y$ *on* $[0, 0.5]$ *and the initial conditions* $y(0) = 2, y'(0) = 5$.

Problem 7 (7 points) *Let*

$$v = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$$

Compute $||v||_1$, $||v||_2$, *and* $||v||_\infty$.

Problem 8 (7 points) *Let*

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Compute $||A||_\infty$.

Problem 9 (7 points) *Let*

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Compute $||A||_1$.

Problem 10 (10 points) *Let*

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Compute $||A||_2$.

Problem 11 (10 points) *Let*

$$v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Use the Gram-Schmidt process on v_1, v_2 *to find a set of orthogonal vectors.*

Problem 12 (12 points) *Let*

$$A = \begin{bmatrix} \frac{5}{2}\pi & \frac{1}{2}\pi \\ \frac{1}{2}\pi & \frac{5}{2}\pi \end{bmatrix}$$

Compute $\cos(A)$ *explicitly.*