

```
> restart;
```

The next procedure constructs the i-th Lagrange polynomial for the n points 0, 1/n, ..., (n-1)/n, 1

```
> Lagrange := proc(i,n,p,x)
  local L,h,j,k,Nodes;
  h:=1/n;
  for j from 0 to n do
    Nodes[j]:=j*h;
  od;
  L:=1;
  for k from 0 to n do
    if k<>i then L:= L*(x-Nodes[k])/(Nodes[i]-Nodes[k]); fi;
  od;
  p:=unapply(L,x);
end proc;
```

```
Lagrange := proc(i,n,p,x)
```

```
  local L,h,j,k,Nodes;
```

```
  h := 1/n;
```

```
  for j from 0 to n do Nodes[j] := j*h end do;
```

```
  L := 1;
```

```
  for k from 0 to n do
```

```
    if k<>i then L := L*(x - Nodes[k])/(Nodes[i] - Nodes[k]) end if
```

```
  end do;
```

```
  p := unapply(L,x)
```

```
end proc
```

The weights for the Newton-Cotes formulae for the interval [0,1]

```
> for n from 1 to 12 do
  print("weights for n equal to ",n);
  for i from 0 to n do
    Lagrange(i,n,p,x);
    Int_p:=int(p(x),x=0..1);
    print(Int_p);
  od;
od;
```

```
"weights for n equal to ", 1
```

$$\frac{1}{2}$$

$$\frac{1}{2}$$

```
"weights for n equal to ", 2
```

$$\frac{1}{6}$$

$$\frac{2}{3}$$

$$\frac{1}{6}$$

```
"weights for n equal to ", 3
```

$$\frac{1}{8}$$

(1)

$$\frac{3}{8}$$

$$\frac{3}{8}$$

$$\frac{1}{8}$$

"weights for n equal to ", 4

$$\frac{7}{90}$$

$$\frac{16}{45}$$

$$\frac{2}{15}$$

$$\frac{16}{45}$$

$$\frac{7}{90}$$

"weights for n equal to ", 5

$$\frac{19}{288}$$

$$\frac{25}{96}$$

$$\frac{25}{144}$$

$$\frac{25}{144}$$

$$\frac{25}{96}$$

$$\frac{19}{288}$$

"weights for n equal to ", 6

$$\frac{41}{840}$$

$$\frac{9}{35}$$

$$\frac{9}{280}$$

$$\frac{34}{105}$$

$$\frac{9}{280}$$

$$\frac{9}{35}$$

$$\frac{41}{840}$$

"weights for n equal to ", 7

$$\frac{751}{17280}$$

$$\frac{3577}{17280}$$

$$\frac{49}{640}$$

$$\frac{2989}{17280}$$

$$\frac{2989}{17280}$$

$$\frac{49}{640}$$

$$\frac{3577}{17280}$$

$$\frac{751}{17280}$$

"weights for n equal to ", 8

$$\frac{989}{28350}$$

$$\frac{2944}{14175}$$

$$- \frac{464}{14175}$$

$$\frac{5248}{14175}$$

$$- \frac{454}{2835}$$

$$\frac{5248}{14175}$$

$$- \frac{464}{14175}$$

$$\frac{2944}{14175}$$

$$\frac{989}{28350}$$

"weights for n equal to ", 9

$$\frac{2857}{89600}$$

$$\frac{15741}{89600}$$

$$\begin{array}{r}
 27 \\
 \hline
 2240 \\
 1209 \\
 \hline
 5600 \\
 2889 \\
 \hline
 44800 \\
 2889 \\
 \hline
 44800 \\
 1209 \\
 \hline
 5600 \\
 27 \\
 \hline
 2240 \\
 15741 \\
 \hline
 89600 \\
 2857 \\
 \hline
 89600
 \end{array}$$

"weights for n equal to ", 10

$$\begin{array}{r}
 16067 \\
 \hline
 598752 \\
 26575 \\
 \hline
 149688 \\
 - \quad 16175 \\
 \hline
 199584 \\
 5675 \\
 \hline
 12474 \\
 - \quad 4825 \\
 \hline
 11088 \\
 17807 \\
 \hline
 24948 \\
 - \quad 4825 \\
 \hline
 11088 \\
 5675 \\
 \hline
 12474 \\
 - \quad 16175 \\
 \hline
 199584 \\
 26575 \\
 \hline
 149688 \\
 16067 \\
 \hline
 598752
 \end{array}$$

"weights for n equal to ", 11

$$\begin{array}{r}
 434293 \\
 \hline
 17418240 \\
 4495513 \\
 \hline
 29030400
 \end{array}$$

$$\begin{array}{r}
 \frac{3237113}{87091200} \\
 \frac{560593}{1935360} \\
 \frac{1599257}{14515200} \\
 \frac{2582261}{14515200} \\
 \frac{2582261}{14515200} \\
 \frac{1599257}{14515200} \\
 \frac{560593}{1935360} \\
 \frac{3237113}{87091200} \\
 \frac{4495513}{29030400} \\
 \frac{434293}{17418240}
 \end{array}$$

"weights for n equal to ", 12

$$\begin{array}{r}
 \frac{1364651}{63063000} \\
 \frac{12504}{79625} \\
 \frac{105387}{875875} \\
 \frac{893128}{1576575} \\
 \frac{1144251}{1401400} \\
 \frac{1215504}{875875} \\
 \frac{522602}{375375} \\
 \frac{1215504}{875875} \\
 \frac{1144251}{1401400} \\
 \frac{893128}{1576575} \\
 \frac{105387}{875875}
 \end{array}$$

$$\frac{12504}{79625}$$

$$\frac{1364651}{63063000}$$

(2)

The computation of the sum of the absolute values of the weights

```
> for n from 1 to 25 do
  W:=0;
  for i from 0 to n do
    Lagrange(i,n,p,x);
    Int_p:=int(p(x),x=0..1);
    W:=W+abs(Int_p);
  od;
  print("the sum of absolute values of the weights for n equal to
", n, "is ", evalf(W));
od;
```

"the sum of absolute values of the weights for n equal to ", 1, "is ", 1.
 "the sum of absolute values of the weights for n equal to ", 2, "is ", 1.
 "the sum of absolute values of the weights for n equal to ", 3, "is ", 1.
 "the sum of absolute values of the weights for n equal to ", 4, "is ", 1.
 "the sum of absolute values of the weights for n equal to ", 5, "is ", 1.
 "the sum of absolute values of the weights for n equal to ", 6, "is ", 1.
 "the sum of absolute values of the weights for n equal to ", 7, "is ", 1.
 "the sum of absolute values of the weights for n equal to ", 8, "is ", 1.451216931
 "the sum of absolute values of the weights for n equal to ", 9, "is ", 1.
 "the sum of absolute values of the weights for n equal to ", 10, "is ", 3.064794773
 "the sum of absolute values of the weights for n equal to ", 11, "is ", 1.589389284
 "the sum of absolute values of the weights for n equal to ", 12, "is ", 7.531736644
 "the sum of absolute values of the weights for n equal to ", 13, "is ", 3.247132553
 "the sum of absolute values of the weights for n equal to ", 14, "is ", 20.34354977
 "the sum of absolute values of the weights for n equal to ", 15, "is ", 8.348084926
 "the sum of absolute values of the weights for n equal to ", 16, "is ", 58.45738092
 "the sum of absolute values of the weights for n equal to ", 17, "is ", 22.21673509
 "the sum of absolute values of the weights for n equal to ", 18, "is ", 175.4632224
 "the sum of absolute values of the weights for n equal to ", 19, "is ", 63.24684727
 "the sum of absolute values of the weights for n equal to ", 20, "is ", 544.1771560
 "the sum of absolute values of the weights for n equal to ", 21, "is ", 186.4283864
 "the sum of absolute values of the weights for n equal to ", 22, "is ", 1731.638280
 "the sum of absolute values of the weights for n equal to ", 23, "is ", 567.4264427
 "the sum of absolute values of the weights for n equal to ", 24, "is ", 5626.401824
 "the sum of absolute values of the weights for n equal to ", 25, "is ", 1769.574151

(3)