

```
> restart;
```

```
Homework 6
```

```
The following procedure compute  $R_{\{nn\}}$  for the integral of  $f(x)$  over  $[a,b]$ 
```

```
> with(Student[Calculus1]);
```

```
[AntiderivativePlot, AntiderivativeTutor, ApproximateInt, ApproximateIntTutor, ArcLength, ArcLengthTutor, Asymptotes, Clear, CriticalPoints, CurveAnalysisTutor, DerivativePlot, DerivativeTutor, DiffTutor, ExtremePoints, FunctionAverage, FunctionAverageTutor, FunctionChart, FunctionPlot, GetMessage, GetNumProblems, GetProblem, Hint, InflectionPoints, IntTutor, Integrand, InversePlot, InverseTutor, LimitTutor, MeanValueTheorem, MeanValueTheoremTutor, NewtonQuotient, NewtonsMethod, NewtonsMethodTutor, PointInterpolation, RiemannSum, RollesTheorem, Roots, Rule, Show, ShowIncomplete, ShowSolution, ShowSteps, Summand, SurfaceOfRevolution, SurfaceOfRevolutionTutor, Tangent, TangentSecantTutor, TangentTutor, TaylorApproximation, TaylorApproximationTutor, Understand, Undo, VolumeOfRevolution, VolumeOfRevolutionTutor, WhatProblem]
```

(1)

```
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```

```
Problem 1a
```

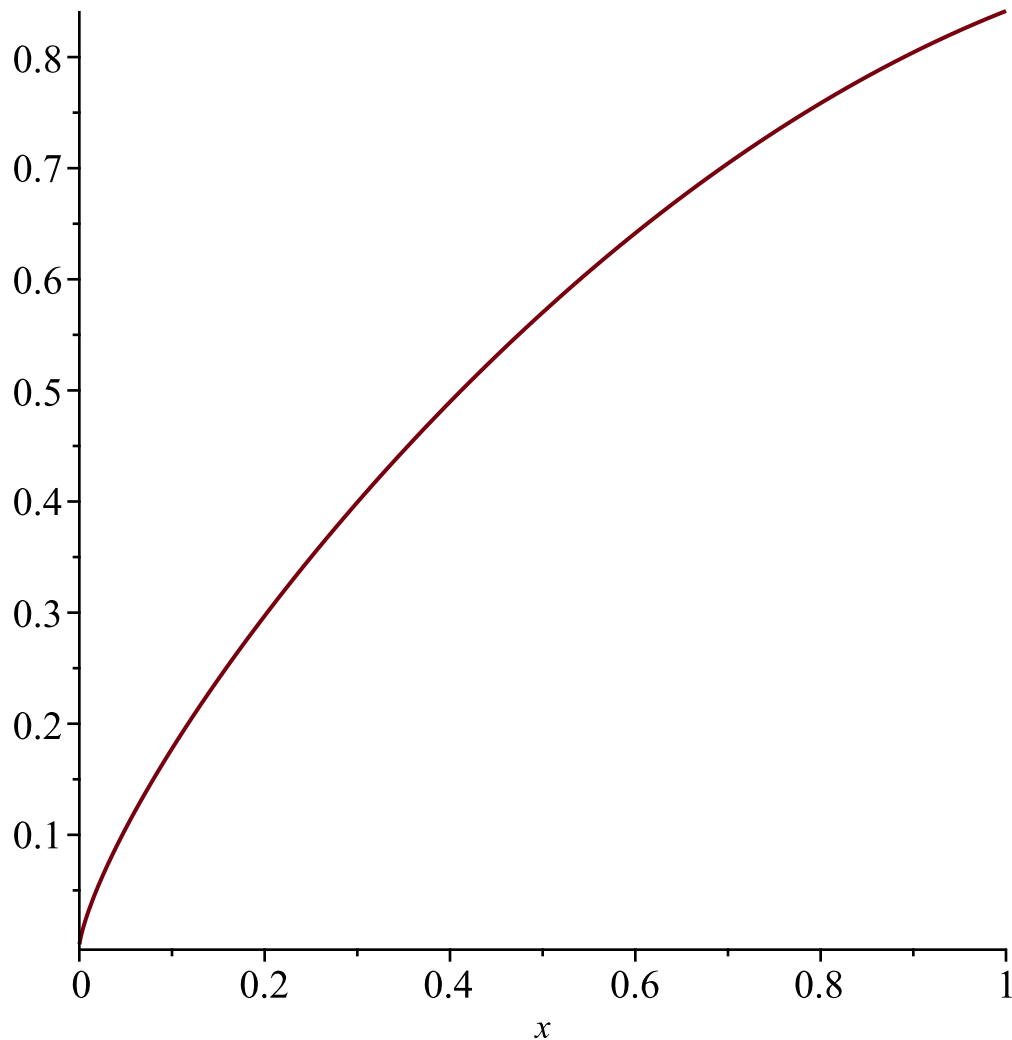
```
Note the limit of  $x^{(-1/4)}\sin(x)$  as  $x \rightarrow 0$  is 0. Because maple perceives there is a singularity, I needed to do the first part of the integration without using the built-in package.
```

```
> f:= proc (x)
  local u;
  if (x<>0) then u:=x(-1/4)*sin(x) else u:=0; fi;
  return (u);
end proc;
```

```
f:= proc(x)
  local u;
  if x<>0 then u := sin(x)/x(1/4) else u := 0 end if; return u
end proc
```

(2)

```
> plot(f(x), x=0..1);
```



```
> A:=(f(0)+4*f(0.125)+f(0.25))*0.25/6+ApproximateInt(f(x),x=0.25.
.1,method=simpson,partition=3);
```

$A := 0.5276650601$

(3)

```
> trueValue1a:=evalf(int(f(x),x=0..1));
```

$trueValue1a := 0.2437477480$

(4)

```
> abs(A-trueValue1a);
```

$2.107 \cdot 10^{-7}$

(5)

Problem 3b

```
> f:= t -> t^2/(1+t^4);
```

$$f := t \rightarrow \frac{t^2}{1+t^4}$$

(6)

```
> trueValue3b:=evalf(int(f(t),t=0..1));
```

$trueValue3b := 0.2437477480$

(7)

```
> A:= ApproximateInt(f(x),x=0.0..1.0,method=simpson,partition=4);
```

$A := 0.2437566796$

(8)

```
> abs(A-trueValue3b);
```

0.0000089316

(9)

Problem 4a

```
> g:= x -> 1/(1+x^4);
```

$$g := x \rightarrow \frac{1}{1+x^4} \quad (10)$$

```
> trueValue4a0to1:=evalf(int(g(t),t=0..1));
```

$$\text{trueValue4a0to1} := 0.8669729872 \quad (11)$$

```
> B:= ApproximateInt(g(x),x=0.0..1.0,method=simpson,partition=9);
```

$$B := 0.8669733048 \quad (12)$$

```
> abs(trueValue4a0to1-B);
```

$$3.176 \cdot 10^{-7} \quad (13)$$

We did the integral from 1 to infinity in Problem 3b)

```
> f:= t -> t^2/(1+t^4);
```

```
trueValue411toInfinity:=evalf(int(f(t),t=0..1));
```

$$f := t \rightarrow \frac{t^2}{1+t^4}$$

$$\text{trueValue411toInfinity} := 0.2437477480 \quad (14)$$

```
> A:= ApproximateInt(f(x),x=0.0..1.0,method=simpson,partition=9);
```

$$A := 0.2437480707 \quad (15)$$

```
> abs(A-trueValue3b);
```

$$3.227 \cdot 10^{-7} \quad (16)$$

Let's compare

```
> X:=A+B;
```

$$X := 1.110721376 \quad (17)$$

```
> trueValue := int(g(x),x=0..infinity);
```

$$\text{trueValue} := \frac{1}{4} \sqrt{2} \pi \quad (18)$$

```
> evalf(abs(X-trueValue));
```

$$6.42 \cdot 10^{-7} \quad (19)$$

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Problem 1a

since $\text{diff}(y \cdot \cos(t), y)$ is $\cos(t)$, which is bounded in absolute value by 1, there is a unique solution, which is easily found to be $e^{\sin(t)}$

```
> f:= t -> exp(sin(t));
```

$$f := t \rightarrow e^{\sin(t)} \quad (20)$$

```
> diff(f(t),t)-f(t)*cos(t);
```

$$0 \quad (21)$$

```
> f(0);
```

$$1 \quad (22)$$

Problem 2a

Since $\text{diff}(\exp(t-y), y) = -\exp(t-y)$ and $\exp(-y)$ goes to infinity as y goes to $-\infty$, we see Theorem 5.4 does not apply. It is easy to solve though,

giving

```
> f:= t -> ln(exp(t)+exp(1)-1);
```

$$f := t \rightarrow \ln(e^t + e - 1) \quad (23)$$

```
> simplify(diff(f(t),t)-exp(t-f(t)));
```

$$0 \quad (24)$$

```
> f(0);
```

1

(25)

Problem 3a

Since $\text{diff}(t^2*y+1,y)=t^2$ is bounded for t in $[0,1]$, the Lipschitz condition is satisfied, so theorem 5.6 applies to show the problem is well-posed.

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Problem 1a and 3a

```
> f := (t,y) -> t*exp(3*t)-2*y;
```

$$f := (t, y) \rightarrow t e^{3t} - 2y$$

(26)

```
> h:=0.5;
```

```
w0:=0;
```

```
w1 := w0+h*f(0,w0);
```

```
w2:= w1+h*f(h,w1);
```

$h := 0.5$

$w0 := 0$

$w1 := 0.$

$w2 := 1.120422268$

(27)

```
> trueSolution := t-> t*exp(3*t)/5-exp(3*t)/25+exp(-2*t)/25;
```

$$\text{trueSolution} := t \rightarrow \frac{1}{5} t e^{3t} - \frac{1}{25} e^{3t} + \frac{1}{25} e^{-2t}$$

(28)

```
> for j from 0 to 2 do
```

```
abs(w[j]-trueSolution(h*j));
```

```
od;
```

0.

0.2836165218

2.098677050

(29)

Problem 5a

```
> f := (t,y) -> y/t - (y/t)^2;
```

$$f := (t, y) \rightarrow \frac{y}{t} - \frac{y^2}{t^2}$$

(30)

```
> h:=0.1;
```

```
w[0]:= 1.0;
```

```
t:=1.0;
```

```
for j from 1 to 10 do
```

```
w[j] := w[j-1] + h* f(t,w[j-1]);
```

```
t:= t+h;
```

```
print(t,w[j]);
```

```
od:
```

$h := 0.1$

$w_0 := 1.0$

$t := 1.0$

1.1, 1.0

1.2, 1.008264463

1.3, 1.021689472

1.4, 1.038514735

1.5, 1.057668193

```

1.6, 1.078461094
1.7, 1.100432165
1.8, 1.123262052
1.9, 1.146723597
2.0, 1.170651570

```

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Problem 7a

```
> trueSolution := t -> t / (1 + ln(t));
```

$$\text{trueSolution} := t \rightarrow \frac{t}{1 + \ln(t)}$$

(32)

```
> t:=1.0;
for j from 1 to 10 do
t:= t+h;
print(t, abs(trueSolution(t)-w[j]));
od:
```

```

t := 1.0
1.1, 0.004281728
1.2, 0.006687851
1.3, 0.008124217
1.4, 0.009019184
1.5, 0.009594161
1.6, 0.009971593
1.7, 0.010222887
1.8, 0.010391505
1.9, 0.010504836
2.0, 0.010580648

```

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Problem 13a

```
> trueSolution(1.25);
(w[3]*(1.3-1.25)+w[2]*(1.25-1.2))/0.1;
Error:= abs(trueSolution(1.25)-(w[3]*(1.3-1.25)+w[2]*(1.25-1.2))
/0.1);
```

```

1.021956907
1.014976968
Error := 0.0069799395

```

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```
> trueSolution(1.93);
(w[10]*(2.0-1.93)+w[9]*(1.93-1.9))/0.1;
Error:= abs(trueSolution(1.93)-(w[10]*(2.0-1.93)+w[9]*(1.93-1.9))
/0.1);
```

```

1.164390171
1.163473178
Error := 0.0009169929

```

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