```
> restart;
```

Homework 4

Problem 1a on page 180

```
> (0.4794-0.5646)/(0.5-0.6);
```

$$0.8520000000 \quad (1)$$

```
> (0.5646-0.6442)/(0.6-0.7)
```

$$0.7960000000 \quad (2)$$

Problem 6c on page 180
We have the two formulae DerivEndpoint for interior points and DeriveMidpoint for enpoints

```
> DerivMidpoint := (f0Plus_h,f0Minus_h,h) -> (f0Plus_h -f0Minus_h)/
  (2*h);
  DerivEndpoint := (f0,f0Plus_h,f0Plus_2h,h) -> (-3*f0+4*f0Plus_h-
  f0Plus_2h)/(2*h);
```

$$DerivMidpoint := (f0Plus\_h, f0Minus\_h, h) \rightarrow \frac{1}{2}\,\frac{f0Plus\_h - f0Minus\_h}{h}$$

$$DerivEndpoint := (f0, f0Plus\_h, f0Plus\_2h, h) \rightarrow \frac{1}{2}\,\frac{-3\,f0 + 4\,f0Plus\_h - f0Plus\_2h}{h} \quad (3)$$

```
> x0:=1.1; F0:=1.52918;
  x1:= 1.2; F1:=1.64024;
  x2:=1.3;F2:=1.70470;
  x3:=1.4; F3:=1.71277;
```

$$x0 := 1.1$$
$$F0 := 1.52918$$
$$x1 := 1.2$$
$$F1 := 1.64024$$
$$x2 := 1.3$$
$$F2 := 1.70470$$
$$x3 := 1.4$$
$$F3 := 1.71277 \quad (4)$$

```
> a[0]:=DerivEndpoint(F0,F1,F2,0.1);
  a[1]:=DerivMidpoint(F2,F0,0.1);
  a[2]:=DerivMidpoint(F3,F1,0.1);
  a[3]:=DerivEndpoint(F3,F2,F1,-0.1);
```

$$a_0 := 1.343600000$$
$$a_1 := 0.8776000000$$
$$a_2 := 0.3626500000$$
$$a_3 := -0.2012500000 \quad (5)$$

Problem 8c on page 180
Actual errors

```
> f:= x -> x*sin(x)+x^2*cos(x);
  df:=unapply(diff(f(x),x),x);
  abs(df(x0)-a[0]);
  abs(df(x1)-a[1]);
```

```
abs(df(x2)-a[2]);
abs(df(x3)-a[3]);
```

$$f := x \to x\sin(x) + x^2\cos(x)$$
$$df := x \to \sin(x) + 3\,x\cos(x) - x^2\sin(x)$$
$$0.033886346$$
$$0.0167907180$$
$$0.0157402840$$
$$0.0309197410 \quad (6)$$

Estimated Error using a rather sloppy maximum of the absolute value of the 3rd derviative

```
> d3f := unapply(diff(f(x),x$3),x);
  #plot(d3f(x),x=1.1..1.4);
  Max := 9 + 7*1.4 - 1.1^2*sin(1.1);
  Error_Endpoint := 0.1^2/3*Max;
  Error_Midpoint := 0.1^2/6*Max;
```

$$d3f := x \to -9\sin(x) - 7\,x\cos(x) + x^2\sin(x)$$
$$Max := 17.72163909$$
$$Error\_Endpoint := 0.05907213029$$
$$Error\_Midpoint := 0.02953606516 \quad (7)$$

pg 189: Problems 1ab

```
> fa:= x-> ln(x);
  fb:= x -> x+exp(x);
  N1:= (f,x,h) -> (f(x+h)-f(x))/h;
  N2 := (f,x,h) -> 2*N1(f,x,h/2)-N1(f,x,h);
  N3 := (f,x,h) -> (4*N2(f,x,h/2)-N2(f,x,h))/3;
```

$$fa := x \to \ln(x)$$
$$fb := x \to x + e^x$$
$$N1 := (f, x, h) \to \frac{f(x+h) - f(x)}{h}$$
$$N2 := (f, x, h) \to 2\,N1\left(f, x, \frac{1}{2}\,h\right) - N1(f, x, h)$$
$$N3 := (f, x, h) \to \frac{4}{3}\,N2\left(f, x, \frac{1}{2}\,h\right) - \frac{1}{3}\,N2(f, x, h) \quad (8)$$

```
> Answer_a:=N3(fa,1.0,0.4);
  Answer_b:=N3(fb,0.0,0.4);
```

$$Answer\_a := 0.9987827568$$
$$Answer\_b := 2.000384148 \quad (9)$$

Using constant term of interpolation polynomials

```
> with(CurveFitting);
  h:=0.4;
  PolynomialInterpolation([h,h/2,h/4],[N1(fa,1.0,h),N1(fa,1.0,h/2),
  N1(fa,1.0,h/4)],x);
  h:=0.4;
  PolynomialInterpolation([h,h/2,h/4],[N1(fb,0.0,h),N1(fb,0.0,h/2),
```

```
N1(fb,0.0,h/4)],x);
```

$$[ArrayInterpolation, BSpline, BSplineCurve, Interactive, LeastSquares, Lowess, PolynomialInterpolation, RationalInterpolation, Spline, ThieleInterpolation]$$

$$h := 0.4$$

$$0.2093472566\, x^2 - 0.4777443164\, x + 0.9987827570$$

$$h := 0.4$$

$$0.1989789333\, x^2 + 0.4933524150\, x + 2.000384150 \quad \textbf{(10)}$$

Problem 6 of page 189
We are assume we have a polynomial g(u)= M - k1*u-k2*u^2-k3*u^3-k4*u^4 up to O(u^5) where N1(h)=g(h^2).

```
> PolynomialInterpolation([u^2,u^2/4,u^2/16,u^2/64],[2.356194,
  -0.4879837,-0.8815732,-0.9709157],u);
```

$$\frac{0.4969378377}{u^2} + \frac{0.9973546343}{u^3} + \frac{1.862036678}{u} - 1.000135150 \quad \textbf{(11)}$$

So we get the 8th order approximation -1.000135150

Problems 13on page 189 : This follows on paper from the divided difference computation of the Newton form of the interpolation polynomials

page 200: Problems 1a, 2a, 6a, 10a

```
> (0.5^4+1^4)*(1-0.5)/2;
```

$$0.2656250000 \quad \textbf{(12)}$$

```
> f:= x -> cos(x)^2;
```

$$f := x \rightarrow \cos(x)^2 \quad \textbf{(13)}$$

```
> ActualAnswer:= int(f(x),x=-0.25..0.25);
  (f(-0.25)+f(0.25))*(0.25-(-0.25))/2;
  (f(-0.25)+4*f(0.0)+f(0.25))*(0.25-(-0.25))/6;
  f(0)*(0.25-(-0.25));
```

$$ActualAnswer := 0.4897127693$$

$$0.4693956404$$

$$0.4897985467$$

$$0.50 \quad \textbf{(14)}$$

page 208: Problem 1ac

```
> with(Student[Calculus1]):
  Prob1a:=ApproximateInt(x*ln(x),x=1..2,method=trapezoid,partition=
  4);
  evalf(Prob1a);
```

$$Prob1a := \frac{5}{16} \ln\left(\frac{5}{4}\right) + \frac{3}{8} \ln\left(\frac{3}{2}\right) + \frac{7}{16} \ln\left(\frac{7}{4}\right) + \frac{1}{4} \ln(2)$$

$$0.6399004777 \quad \textbf{(15)}$$

```
> Prob1c:=ApproximateInt(2/(x^4+4),x=0..2,method=trapezoid,
  partition=6);
  evalf(Prob1c);
```

$$Prob1c := \frac{49708601}{70178550}$$

$$0.7083161593 \quad \textbf{(16)}$$

page 208: Problem 3ac

```
> Prob3a:=ApproximateInt(x*ln(x),x=1..2,method=simpson,partition=4)
;
evalf(Prob3a);
```

$$Prob3a := \frac{3}{16}\ln\left(\frac{9}{8}\right) + \frac{5}{48}\ln\left(\frac{5}{4}\right) + \frac{11}{48}\ln\left(\frac{11}{8}\right) + \frac{1}{8}\ln\left(\frac{3}{2}\right) + \frac{13}{48}\ln\left(\frac{13}{8}\right) + \frac{7}{48}\ln\left(\frac{7}{4}\right) + \frac{5}{16}\ln\left(\frac{15}{8}\right) + \frac{1}{12}\ln(2)$$

$$0.6362953646 \quad \textbf{(17)}$$

```
> Prob3c:=ApproximateInt(2/(x^4+4),x=0..2,method=simpson,partition=
6);
evalf(Prob3c);
```

$$Prob3c := \frac{2171050711410637}{3058726978715850}$$

$$0.7097889830 \quad \textbf{(18)}$$

page 208: Problem 5ac

```
> Prob5a:=ApproximateInt(x*ln(x),x=1..3,method=midpoint,partition=
4);
evalf(Prob5a);
```

$$Prob5a := \frac{5}{8}\ln\left(\frac{5}{4}\right) + \frac{7}{8}\ln\left(\frac{7}{4}\right) + \frac{9}{8}\ln\left(\frac{9}{4}\right) + \frac{11}{8}\ln\left(\frac{11}{4}\right)$$

$$2.932376281 \quad \textbf{(19)}$$

```
> Prob5c:=ApproximateInt(2/(x^4+4),x=0..2,method=midpoint,
partition=6);
evalf(Prob5c);
```

$$Prob5c := \frac{1653959812928}{2327798309525}$$

$$0.7105253948 \quad \textbf{(20)}$$

page 208: Problem 13a
For the trapezoid rule the theroetical error estimate is
(max absolute value of the 2nd derivative of f)*(b-a)*h^2/12 with h=(b-a)/n
So for 2!*2^3/(0+4)^3*1/(12/n^2) < 10^(-5), and thus we need n to be at least

```
> evalf((2!*10^5/4^3*2^3/12)^(1/2));
```

$$45.64354646 \quad \textbf{(21)}$$

so

```
> n:=46;
h:=(2-0.0)/46;
```

$$n := 46$$

$$h := 0.04347826087 \quad \textbf{(22)}$$

```
> Prob13a:=ApproximateInt(1/(x+4),x=0..2,method=trapezoid,
partition=n);
evalf(Prob13a);
error13a:=evalf(abs(Prob13a-int(1/(x+4),x=0..2)));
```

$$Prob13a := \frac{3569733046115684962701460857213388511946345650627}{8803926207046550164190455310821877943262300464000}$$

$$0.4054705778$$

$$error13a := 0.000005469 \quad \textbf{(23)}$$

It is worth noting how truly awful the theoretical estimates are (because 1/(x+4) is so flat between 0 and 2), e.g., for trapezoid

page 208: Problem 13b
For Simpson the theoretical error estimate is
(max absolute value of 4th derivative of f)/*h^4*(b-a)/(180) with h=(b-a)/n
So 4!2^5/(0+4)^5*1/(180/n^4) < 10^(-5), and thus we need n to be at least

```
> evalf((4!*10^5/4^5*2^5/180)^(1/4));
```

$$4.518010020 \quad (24)$$

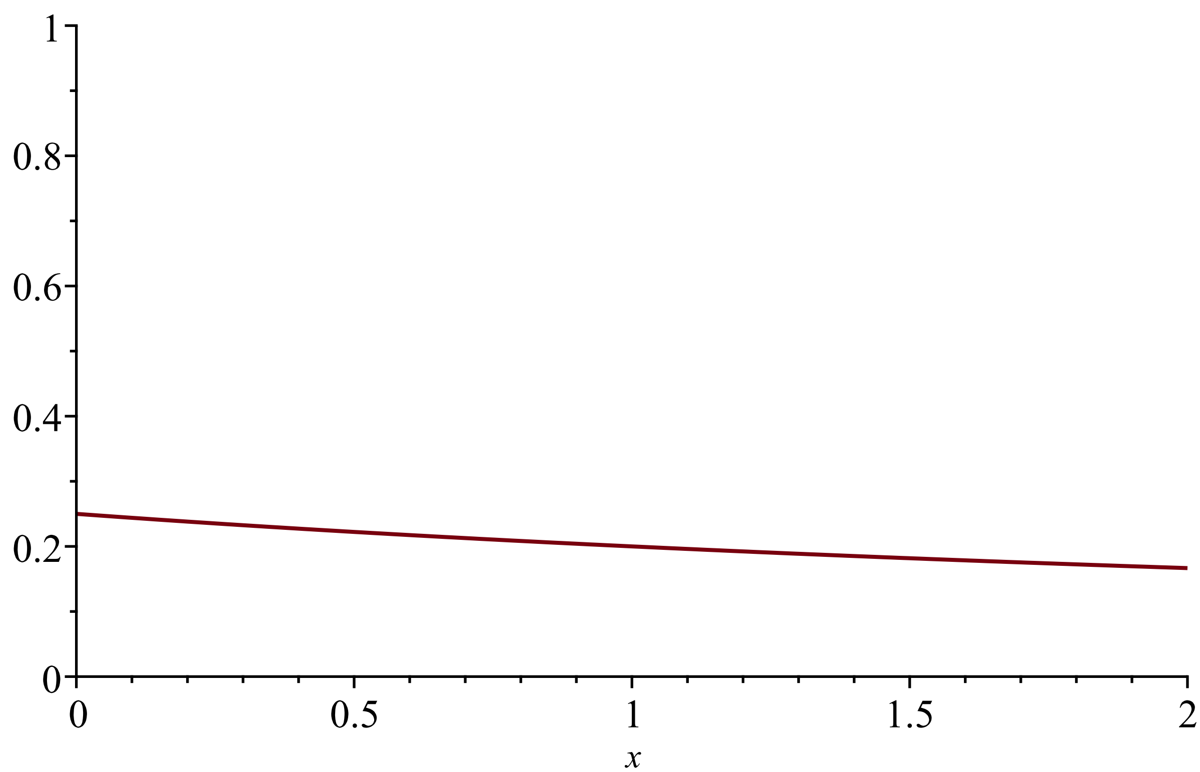
so

```
> A:=ApproximateInt(1/(x+4),x=0..2,method=simpson,partition=1);
  evalf(A);
  errorA:=evalf(abs(A-int(1/(x+4),x=0..2)));
  plot(1/(x+4),x=0..2,view=[0..2,0..1],size=[600,300]);
```

$$A := \frac{73}{180}$$

$$0.4055555556$$

$$errorA := 0.000090447$$




```
> n:=5;
  h:=(2-0.0)/5;
```

$$n := 5$$

$$h := 0.4000000000 \quad (25)$$

```
> Prob13b:=ApproximateInt(1/(x+4),x=0..2,method=simpson,partition=
  n);
  evalf(Prob13b);
  error13b:=evalf(abs(Prob13b-int(1/(x+4),x=0..2)));
```

$$Prob13b := \frac{548199461}{1352025675}$$

$$0.4054652742$$

$$error13b := 1.66\ 10^{-7} \quad \textbf{(26)}$$

It is worth noting how awful the theoretical estimates are (because 1/(x+4) is so flat between 0 and 2), e.g., for simpson

```
> B:=ApproximateInt(1/(x+4),x=0..2,method=simpson,partition=1);
  evalf(B);
  errorB:=evalf(abs(B-int(1/(x+4),x=0..2)));
```

$$B := \frac{73}{180}$$

$$0.4055555556$$

$$errorB := 0.000090447 \quad \textbf{(27)}$$