```
> restart;
```

Homework 3

LagrangeInterp gives the Lagrange form of the Interpolation polynomial

```
> Lagrange  := proc(points::list,M,x)
  local n,i,j,L;
  n:=numelems(points);
  for j from 1 to n do
  L:= 1;
  for i from 1 to n do
  if i<>j then L:= L*(x-points[i])/(points[j]-points[i]);
  fi;
  od;
  M[j]:=unapply(L,x);
  od;
  M;
  end proc;
```

```
Lagrange := proc(points::list, M, x)                                        (1)
    local n, i, j, L;
    n := numelems(points);
    for j to n do
        L := 1;
        for i to n do
            if i<>j then L := L* (x − points[i]) / (points[j] − points[i]) end if
        end do;
        M[j] := unapply(L, x)
    end do;
    M
end proc
```

```
> LagrangeInterp  := proc(points,values,f,x)
  local L,j,M;
  L:=0;
  Lagrange(points,M,x);
  for j from 1 to numelems(points)  do
  L:= L+M[j](x)*values[j];
  od;
  f:=unapply(L,x);
  end proc;
```

```
LagrangeInterp := proc(points, values, f, x)                                (2)
    local L, j, M;
    L := 0;
    Lagrange(points, M, x);
    for j to numelems(points) do L := L + M[j](x) * values[j] end do;
    f := unapply(L, x)
end proc
```

pg 112: Problems 1ab

```
> points1 := [0,0.6];
  points2 := [0,0.6,0.9];
  f:= x -> cos(x);
  values1a := [f(0.0),f(0.6)];
  LagrangeInterp(points1,values1a,p_1a,x);
  values2a := [f(0.0),f(0.6),f(0.9)];
  LagrangeInterp(points2,values2a,p_2a,x);
  f:= x -> sqrt(1+x);
  values1b := [f(0.0),f(0.6)];
  LagrangeInterp(points1,values1b,p_1b,x);
  values2b := [f(0.0),f(0.6),f(0.9)];
  LagrangeInterp(points2,values2b,p_2b,x);
```

$$points1 := [0, 0.6]$$

$$points2 := [0, 0.6, 0.9]$$

$$f := x \rightarrow \cos(x)$$

$$values1a := [1., 0.8253356149]$$

$$x \rightarrow -0.291107309\, x + 1.000000000$$

$$values2a := [1., 0.8253356149, 0.6216099683]$$

$$x \rightarrow -1.111111111\, (-1.666666667\, x + 1.000000000)\, (x - 0.9) - 4.585197862\, x\, (x - 0.9) + 2.302259141\, x\, (x - 0.6)$$

$$f := x \rightarrow \sqrt{1 + x}$$

$$values1b := [1., 1.264911064]$$

$$x \rightarrow 0.441518440\, x + 1.000000000$$

$$values2b := [1., 1.264911064, 1.378404875]$$

$$x \rightarrow -1.111111111\, (-1.666666667\, x + 1.000000000)\, (x - 0.9) - 7.027283691\, x\, (x - 0.9) + 5.105203240\, x\, (x - 0.6) \quad \textbf{(3)}$$

```
> p_1a(0.45);
  error_a1 := abs(cos(0.45)-p_1a(0.45));
  p_2a(0.45);
  error_a2 := abs(cos(0.45)-p_2a(0.45));
```

$$0.8690017110$$

$$error\_a1 := 0.0314453914$$

$$0.8981000750$$

$$error\_a2 := 0.0023470274 \quad \textbf{(4)}$$

```
> f:= x -> sqrt(1+x);
  p_1b(0.45);
  error_b1 := abs(f(0.45)-p_1b(0.45));
  p_2b(0.45);
  error_b2 := abs(f(0.45)-p_2b(0.45));
```

$$f := x \rightarrow \sqrt{1 + x}$$

$$1.198683298$$

$$error\_b1 := 0.005476160$$

$$1.203423728$$

$$error\_b2 := 0.000735730 \quad \textbf{(5)}$$

Problems 2bd on page 112

```
> points1 := [1.25,1.6];
  points2 := [1.0,1.25,1.6];
  f:= x -> (x-1)^(1/3);
  values1b := [f(1.25),f(1.6)];
  LagrangeInterp(points1,values1b,p_1b,x);
  values2b := [f(1.0),f(1.25),f(1.6)];
  LagrangeInterp(points2,values2b,p_2b,x);
  f:= x -> exp(2*x)-x;
  values1d := [f(1.25),f(1.6)];
  LagrangeInterp(points1,values1d,p_1d,x);
  values2d := [f(1.0),f(1.25),f(1.6)];
  LagrangeInterp(points2,values2d,p_2d,x);
```

$$points1 := [1.25, 1.6]$$

$$points2 := [1.0, 1.25, 1.6]$$

$$f := x \rightarrow (x-1)^{1/3}$$

$$values1b := [0.6299605249, 0.8434326653]$$

$$x \rightarrow 0.609920401\, x - 0.132439977$$

$$values2b := [0., 0.6299605249, 0.8434326653]$$

$$x \rightarrow -1.799887214\,(4.000000000\, x - 4.000000000)\,(x - 1.6) + 2.409807615\,(1.666666667\, x - 1.666666667)\,(x - 1.25)$$

$$f := x \rightarrow e^{2x} - x$$

$$values1d := [10.93249396, 22.93253020]$$

$$x \rightarrow 34.28581782\, x - 31.92477832$$

$$values2d := [6.389056099, 10.93249396, 22.93253020]$$

$$x \rightarrow -10.64842683\,(-4.000000000\, x + 5.000000000)\,(x - 1.6) - 31.23569703\,(4.000000000\, x - 4.000000000)\,(x - 1.6) + 65.52151485\,(1.666666667\, x - 1.666666667)\,(x - 1.25) \quad (6)$$

```
> f := x -> (x-1)^(1/3);
  p_1b(1.4);
  error_b1 := abs(f(1.4)-p_1b(1.4));
  p_2b(1.4);
  error_b2 := abs(f(1.4)-p_2b(1.4));
```

$$f := x \rightarrow (x-1)^{1/3}$$

$$0.7214485844$$

$$error\_b1 := 0.0153577153$$

$$0.8169446700$$

$$error\_b2 := 0.0801383703 \quad (7)$$

```
> f:= x -> exp(2*x)-x;
  p_1d(1.4);
  error_d1 := abs(f(1.4)-p_1d(1.4));
  p_2d(1.4);
  error_d2 := abs(f(1.4)-p_2d(1.4));
```

$$f := x \rightarrow e^{2x} - x$$

$$16.07536663$$

$$error\_d1 := 1.03071986$$

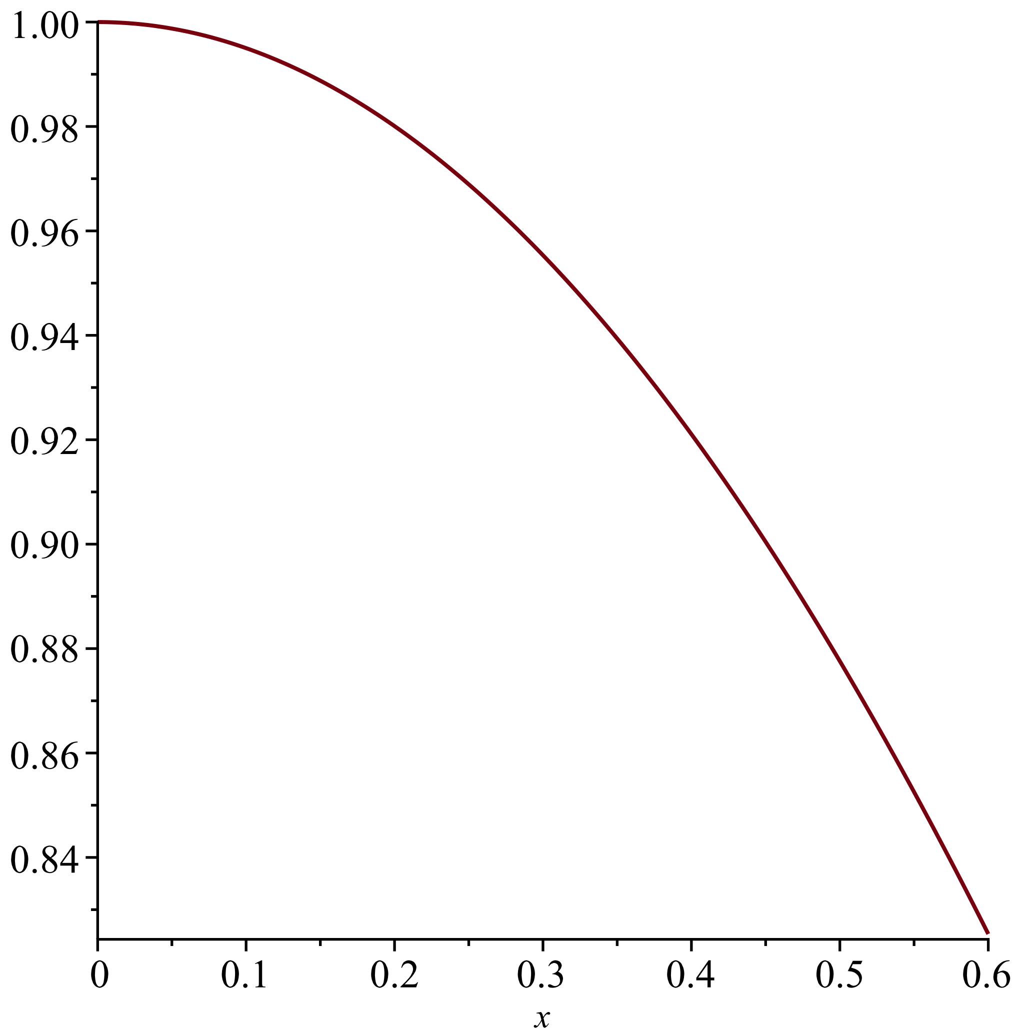
$$15.26976332$$

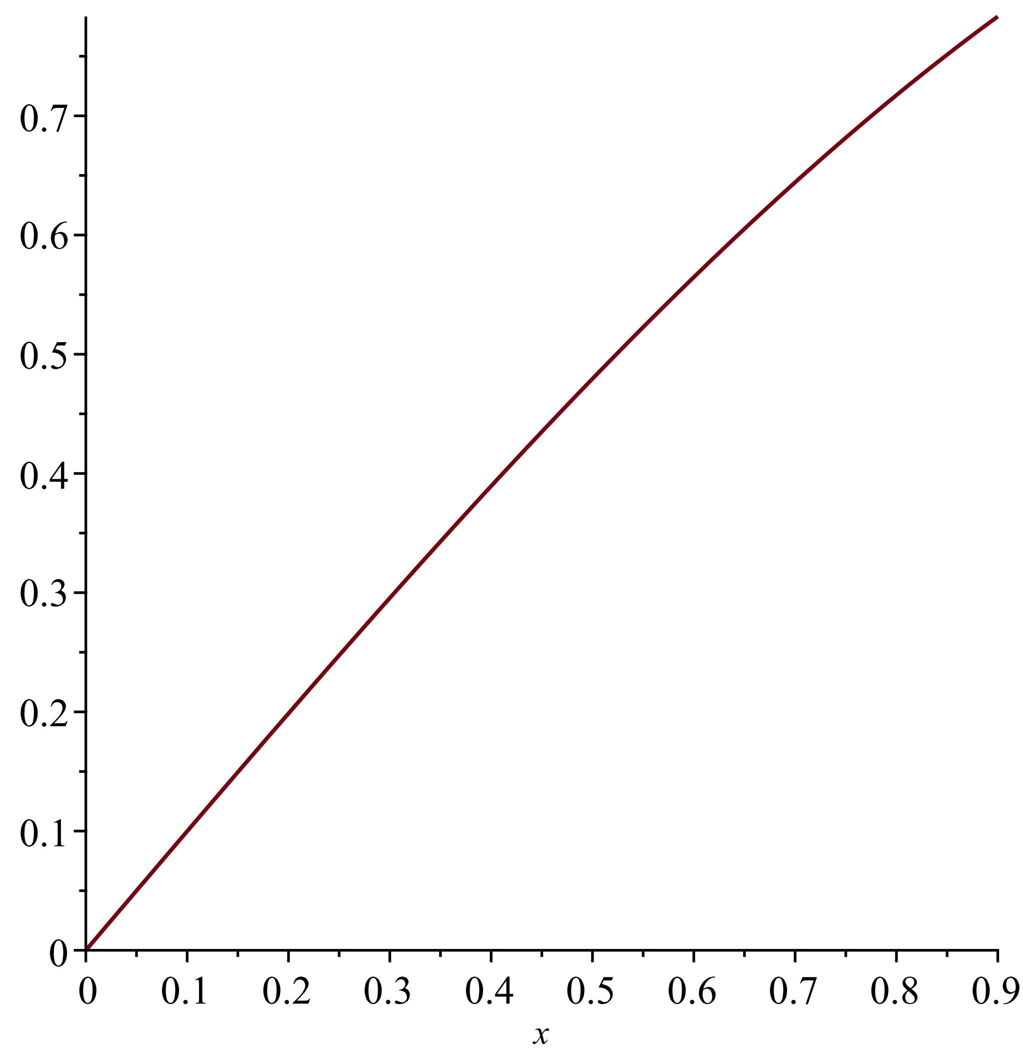
$$error\_d2 := 0.22511655 \quad (8)$$

Problem 3 of page 112
For this problem we need bounds on the absolute values of the appropriate derivatives of the functions in 1a and 1b

For 1a we need the second and third derivatives, i.e., -cos(x) and sin(x). The absolute value of the first has the max 1 at at 0.0
and the second has its max (= sin(0.9)) at 0.6. Graphically

```
> plot(cos(x),x=0.0..0.6);
  plot(sin(x),x=0.0..0.9);
```






```
> ErrorBound_a1 := abs((0.45-0.0)*(0.45-0.6)/2);
  ErrorBound_a2 := abs(sin(0.9)*(0.45-0.0)*(0.45-0.6)*(0.45-0.9)/6)
  ;
```

$$ErrorBound\_a1 := 0.03375000000$$
$$ErrorBound\_a2 := 0.003965592480 \quad \textbf{(9)}$$

The absolute values of the derivatives are decreasin6 with theeir max values of 1/2, 1/4, and 3/8 at 0.0

```
> diff(sqrt(1+x),x);
  diff(sqrt(1+x),x$2);
  diff(sqrt(1+x),x$3);
```

$$\frac{1}{2\sqrt{1+x}}$$
$$-\frac{1}{4\,(1+x)^{3/2}}$$
$$\frac{3}{8\,(1+x)^{5/2}} \quad \textbf{(10)}$$

```
> ErrorBound_a1 := abs(1/4*(0.45-0.0)*(0.45-0.6)/2);
```

```
ErrorBound_a2 := abs(3/8*(0.45-0.0)*(0.45-0.6)*(0.45-0.9)/6);
```

$$ErrorBound\_a1 := 0.008437500000$$
$$ErrorBound\_a2 := 0.001898437500 \quad (11)$$

Page 120 Problem 1a

```
> x[0]:= 8.1; Q[0,0] := 16.94410;
  x[1]:= 8.3; Q[1,0] := 17.56492;
  x[2]:= 8.6; Q[2,0] := 18.50515;
  x[3]:= 8.7; Q[3,0] := 18.82091;
  u:=8.4;
```

$$x_0 := 8.1$$
$$Q_{0,\,0} := 16.94410$$
$$x_1 := 8.3$$
$$Q_{1,\,0} := 17.56492$$
$$x_2 := 8.6$$
$$Q_{2,\,0} := 18.50515$$
$$x_3 := 8.7$$
$$Q_{3,\,0} := 18.82091$$
$$u := 8.4 \quad (12)$$

```
> for i from 1 to 3
  do
  for j from 1 to i
  do
  Q[i,j] := ((u-x[i-j])*Q[i,j-1]-(u-x[i])*Q[i-1,j-1])/(x[i]-x[i-j])
  ;
  od;
  od;
> Q[3,3];
```

$$17.87714250 \quad (13)$$

Problem 3 on page 120: note we show a is more accurate

```
> TrueAnswer := sqrt(3.0);
```

$$TrueAnswer := 1.732050808 \quad (14)$$

```
> f:= z -> 3.0^z;
  x[0] := -2; Q[0,0] := f(x[0]);
  x[1] := -1; Q[1,0] := f(x[1]);
  x[2] :=  0; Q[2,0] := f(x[2]);
  x[3] :=  1; Q[3,0] := f(x[3]);
  x[4] :=  2; Q[4,0] := f(x[4]);
  u:=0.5;
```

$$f := z \rightarrow 3.0^z$$
$$x_0 := -2$$
$$Q_{0,\,0} := 0.1111111111$$
$$x_1 := -1$$
$$Q_{1,\,0} := 0.3333333333$$
$$x_2 := 0$$

$$Q_{2,\,0} := 1.0$$
$$x_3 := 1$$
$$Q_{3,\,0} := 3.0$$
$$x_4 := 2$$
$$Q_{4,\,0} := 9.00$$
$$u := 0.5 \tag{15}$$

```
> for i from 1 to 4
  do
  for j from 1 to i
  do
  Q[i,j] := ((u-x[i-j])*Q[i,j-1]-(u-x[i])*Q[i-1,j-1])/(x[i]-x[i-j])
  ;
  od;
  od;
```

```
> est1:=Q[4,4];
  error1 :=abs(est1-TrueAnswer);
```

$$est1 := 1.708333334$$
$$error1 := 0.023717474 \tag{16}$$

```
> f:= z -> sqrt(z);
  x[0] := 0.0; Q[0,0] := f(x[0]);
  x[1] := 1.0; Q[1,0] := f(x[1]);
  x[2] := 2.0; Q[2,0] := f(x[2]);
  x[3] := 4.0; Q[3,0] := f(x[3]);
  x[4] := 5.0; Q[4,0] := f(x[4]);
  u:= 3.0;
```

$$f := z \rightarrow \sqrt{z}$$
$$x_0 := 0.$$
$$Q_{0,\,0} := 0.$$
$$x_1 := 1.0$$
$$Q_{1,\,0} := 1.0$$
$$x_2 := 2.0$$
$$Q_{2,\,0} := 1.414213562$$
$$x_3 := 4.0$$
$$Q_{3,\,0} := 2.000000000$$
$$x_4 := 5.0$$
$$Q_{4,\,0} := 2.236067977$$
$$u := 3.0 \tag{17}$$

```
> for i from 1 to 4
  do
  for j from 1 to i
  do
  Q[i,j] := ((u-x[i-j])*Q[i,j-1]-(u-x[i])*Q[i-1,j-1])/(x[i]-x[i-j])
  ;
  od;
```

```
od;
> est2:=Q[4,4];
error2 :=abs(est2-TrueAnswer);
```

$$est2 := 1.690606764$$

$$error2 := 0.041444044 \quad \textbf{(18)}$$

page 130 Problem 7

```
> x[0] := -0.1; F[0,0]:= 5.3;
x[1] :=  0.0; F[1,0]:= 2.0;
x[2] :=  0.2; F[2,0]:= 3.19;
x[3] :=  0.3; F[3,0]:= 1.0;
```

$$x_0 := -0.1$$

$$F_{0,0} := 5.3$$

$$x_1 := 0.$$

$$F_{1,0} := 2.0$$

$$x_2 := 0.2$$

$$F_{2,0} := 3.19$$

$$x_3 := 0.3$$

$$F_{3,0} := 1.0 \quad \textbf{(19)}$$

```
> for i from 1 to 3 do
for j from 1 to i do
F[i,j]:= (F[i,j-1]-F[i-1,j-1])/(x[i]-x[i-j]);
od;
od;
> for i from 0 to 3 do print(F[i,i]); od;
p3 := z -> F[0,0]+F[1,1]*(z-x[0])+F[2,2]*(z-x[0])*(z-x[1])+F[3,3]
*(z-x[0])*(z-x[1])*(z-x[2]);
```

$$5.3$$

$$-33.00000000$$

$$129.8333333$$

$$-556.6666665$$

$$p3 := z \rightarrow F_{0,0} + F_{1,1}\left(z - x_0\right) + F_{2,2}\left(z - x_0\right)\left(z - x_1\right) + F_{3,3}\left(z - x_0\right)\left(z - x_1\right)\left(z - x_2\right) \quad \textbf{(20)}$$

Let's confirm this is correct

```
> for i from 0 to 3 do p3(x[i]); od;
```

$$5.3$$

$$2.000000000$$

$$3.189999998$$

$$1.000000002 \quad \textbf{(21)}$$

```
> x[4] := 0.35; F[4,0] := 0.97260;
for i from 1 to 4 do
for j from 1 to i do
F[i,j]:= (F[i,j-1]-F[i-1,j-1])/(x[i]-x[i-j]);
od;
od;
```

$$x_4 := 0.35$$

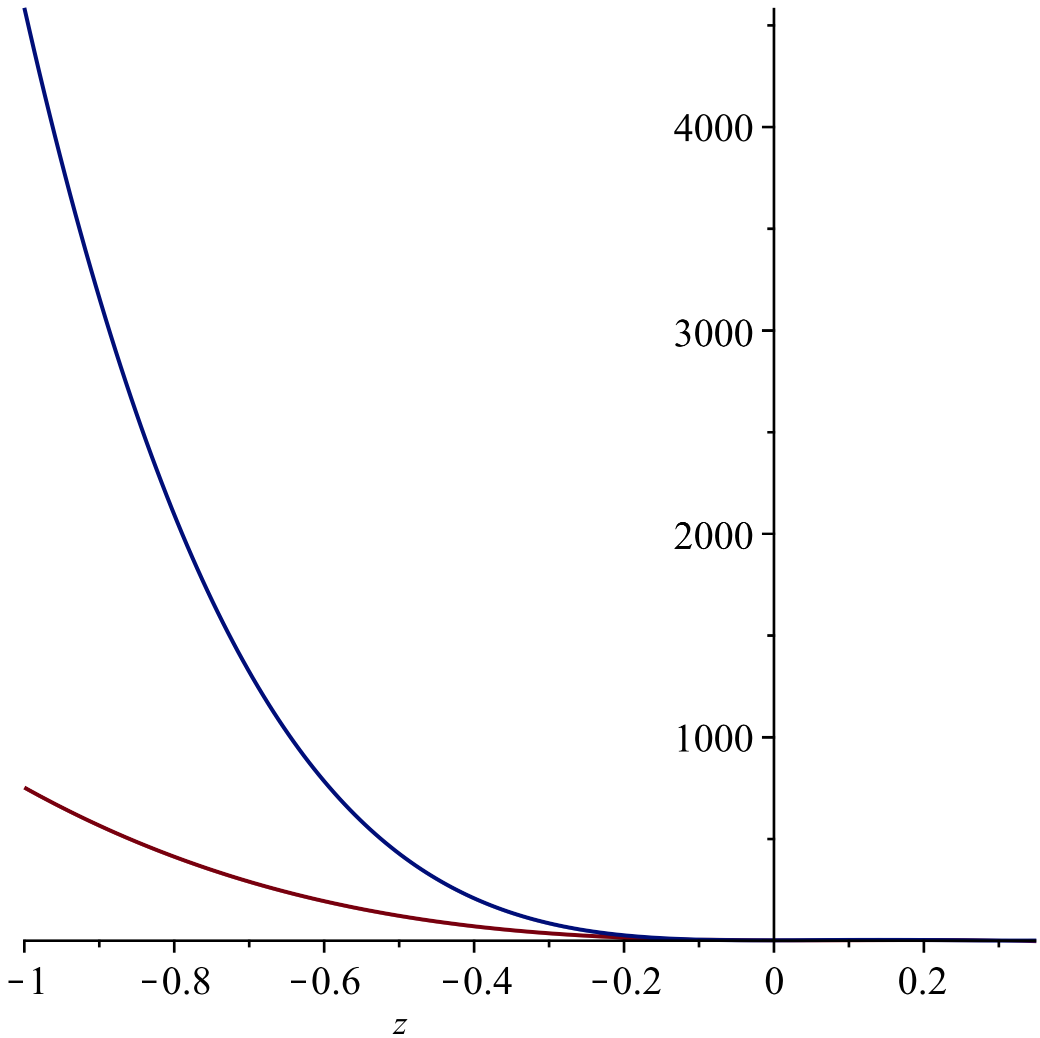
$$F_{4,0} := 0.97260 \quad \textbf{(22)}$$

```
> for i from 0 to 4 do print(F[i,i]); od;
  p4 := z -> F[0,0]+F[1,1]*(z-x[0])+F[2,2]*(z-x[0])*(z-x[1])+F[3,3]
  *(z-x[0])*(z-x[1])*(z-x[2])+F[4,4]*(z-x[0])*(z-x[1])*(z-x[2])*(z-
  x[3]);
```

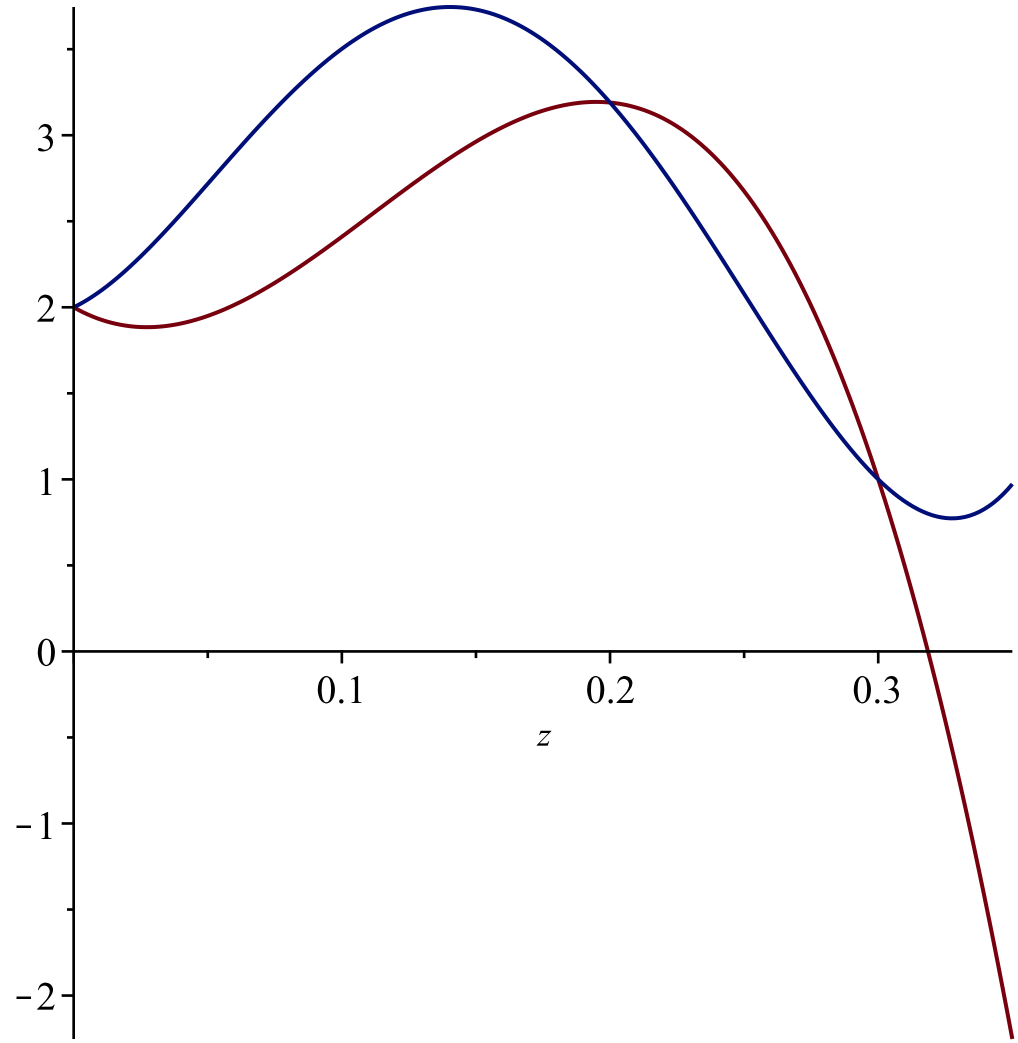
$$5.3$$
$$-33.00000000$$
$$129.8333333$$
$$-556.6666665$$
$$2730.243387$$

$$p4 := z \rightarrow F_{0,0} + F_{1,1}\left(z - x_0\right) + F_{2,2}\left(z - x_0\right)\left(z - x_1\right) + F_{3,3}\left(z - x_0\right)\left(z - x_1\right)\left(z - x_2\right) + F_{4,4}\left(z - x_0\right)\left(z - x_1\right)\left(z - x_2\right)\left(z - x_3\right) \quad \textbf{(23)}$$

```
> plot({p3(z),p4(z)},z=-1..0.35);
```




```
> plot({p3(z),p4(z)},z=0..0.35);
```



Problem 1a on page 139

```
> x[0]:=8.3; f[0]:=17.56492; fprime[0]:=3.116256;
  x[1]:=8.6; f[1]:=18.50515; fprime[1]:=3.151762;
```

$$x_0 := 8.3$$

$$f_0 := 17.56492$$

$$fprime_0 := 3.116256$$

$$x_1 := 8.6$$

$$f_1 := 18.50515$$

$$fprime_1 := 3.151762 \qquad \textbf{(24)}$$

```
> n:=1;
  for i from 0 to n do
  z[2*i] := x[i];
  z[2*i+1]:=x[i];
  Q[2*i,0]:=f[i];
  Q[2*i+1,0]:=f[i];
  Q[2*i+1,1]:=fprime[i];
  if i<>0 then Q[2*i,1] := (Q[2*i,0]-Q[2*i-1,0])/(z[2*i]-z[2*i-1]);
  fi;
  od:
```

```
for i from 2 to 2*n+1 do
for j from 2 to i do
Q[i,j]:= (Q[i,j-1]-Q[i-1,j-1])/(z[i]-z[i-j])
od;
od;
```

$$n := 1 \quad \textbf{(25)}$$

```
> u:='u';
p3:= u -> Q[0,0] + Q[1,1]*(u-z[0]) + Q[2,2]*(u-z[0])*(u-z[1]) + Q
[3,3]*(u-z[0])*(u-z[1])*(u-z[2]);
print(p3(u));
```

$$u := u$$

$$p3 := u \rightarrow Q_{0,\,0} + Q_{1,\,1}\left(u - z_0\right) + Q_{2,\,2}\left(u - z_0\right)\left(u - z_1\right) + Q_{3,\,3}\left(u - z_0\right)\left(u - z_1\right)\left(u - z_2\right)$$

$$-8.3000048 + 3.116256\, u + 0.05948000000\,(u - 8.3)^2 - 0.002022222233\,(u - 8.3)^2\,(u - 8.6) \quad \textbf{(26)}$$

Let's check that p3(u) interpolates correctly

```
> for i from 0 to 1 do
p3(x[i]);
subs(u=x[i],diff(p3(u),u));
od;
```

$$17.56492$$
$$3.116256000$$
$$18.50515000$$
$$3.151762000 \quad \textbf{(27)}$$

Problem 1c on page 139

```
> x[0]:=-0.5;  f[0]:=-0.0247500; fprime[0]:=0.7510000;
x[1]:=-0.25; f[1]:= 0.3339375; fprime[1]:=2.1890000;
x[2]:= 0.00;  f[2]:= 1.1010000; fprime[2]:=4.0020000;
```

$$x_0 := -0.5$$
$$f_0 := -0.0247500$$
$$fprime_0 := 0.7510000$$
$$x_1 := -0.25$$
$$f_1 := 0.3339375$$
$$fprime_1 := 2.1890000$$
$$x_2 := 0.$$
$$f_2 := 1.1010000$$
$$fprime_2 := 4.0020000 \quad \textbf{(28)}$$

```
> n:=2;
for i from 0 to n do
z[2*i] := x[i];
z[2*i+1]:=x[i];
Q[2*i,0]:=f[i];
Q[2*i+1,0]:=f[i];
Q[2*i+1,1]:=fprime[i];
if i<>0 then Q[2*i,1] := (Q[2*i,0]-Q[2*i-1,0])/(z[2*i]-z[2*i-1]);
```

```
fi;
od:
for i from 2 to 2*n+1 do
for j from 2 to i do
Q[i,j]:= (Q[i,j-1]-Q[i-1,j-1])/(z[i]-z[i-j])
od;
od;
```

$$n := 2 \tag{29}$$

```
> u:='u';
p5:= u -> Q[0,0] + Q[1,1]*(u-z[0]) + Q[2,2]*(u-z[0])*(u-z[1]) + Q
[3,3]*(u-z[0])*(u-z[1])*(u-z[2]) + Q[4,4]*(u-z[0])*(u-z[1])*(u-z
[2])*(u-z[3]) + Q[5,5]*(u-z[0])*(u-z[1])*(u-z[2])*(u-z[3])*(u-z
[4]);
print(p5(u));
```

$$u := u$$

$$p5 := u \to Q_{0,0} + Q_{1,1}(u - z_0) + Q_{2,2}(u - z_0)(u - z_1) + Q_{3,3}(u - z_0)(u - z_1)(u - z_2) + Q_{4,4}(u - z_0)(u - z_1)(u - z_2)(u - z_3) + Q_{5,5}(u - z_0)(u - z_1)(u - z_2)(u - z_3)(u - z_4)$$

$$0.35075000 + 0.7510000\,u + 2.735000000\,(u + 0.5)^2 + 1.128000000\,(u + 0.5)^2\,(u + 0.25) - 0.2560000000\,(u + 0.5)^2\,(u + 0.25)^2$$

$$-0.0247500 \tag{30}$$

Let's check that p5(u) interpolates correctly

```
> for i from 0 to 2 do
p5(x[i]);
subs(u=x[i],diff(p5(u),u));
od;
```

$$-0.0247500$$

$$0.751000000$$

$$0.3339375000$$

$$2.189000000$$

$$1.101000000$$

$$4.002000000 \tag{31}$$