Homework 1

pg 11: 1a, 1d, and 2a follow by continuity and the intermediate value theorem (the plots are not needed, but helps understanding)

```
> p:= x -> x*cos(x)-2*x^2+3*x-1;
  a:=0.2: b:=0.3:
  print(a,p(a));
  print(b,p(b));

  a:=1.2: b:=1.3:
  print(a,p(a));
  print(b,p(b));

  plot(p(x),x=0.2..1.3);
```

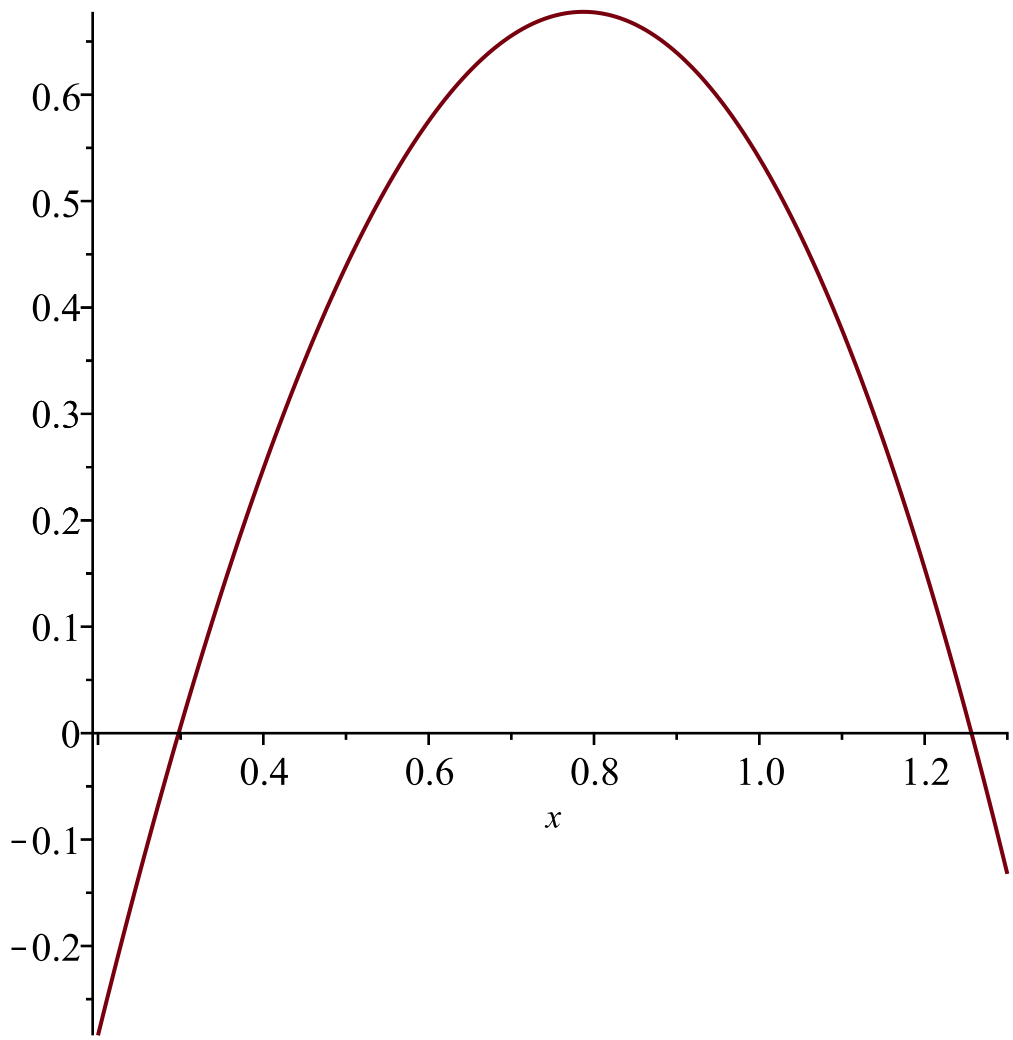
$$p := x \to x\cos(x) - 2x^2 + 3x - 1$$

0.2, -0.2839866844

0.3, 0.006600947

1.2, 0.154829305

1.3, -0.132251523

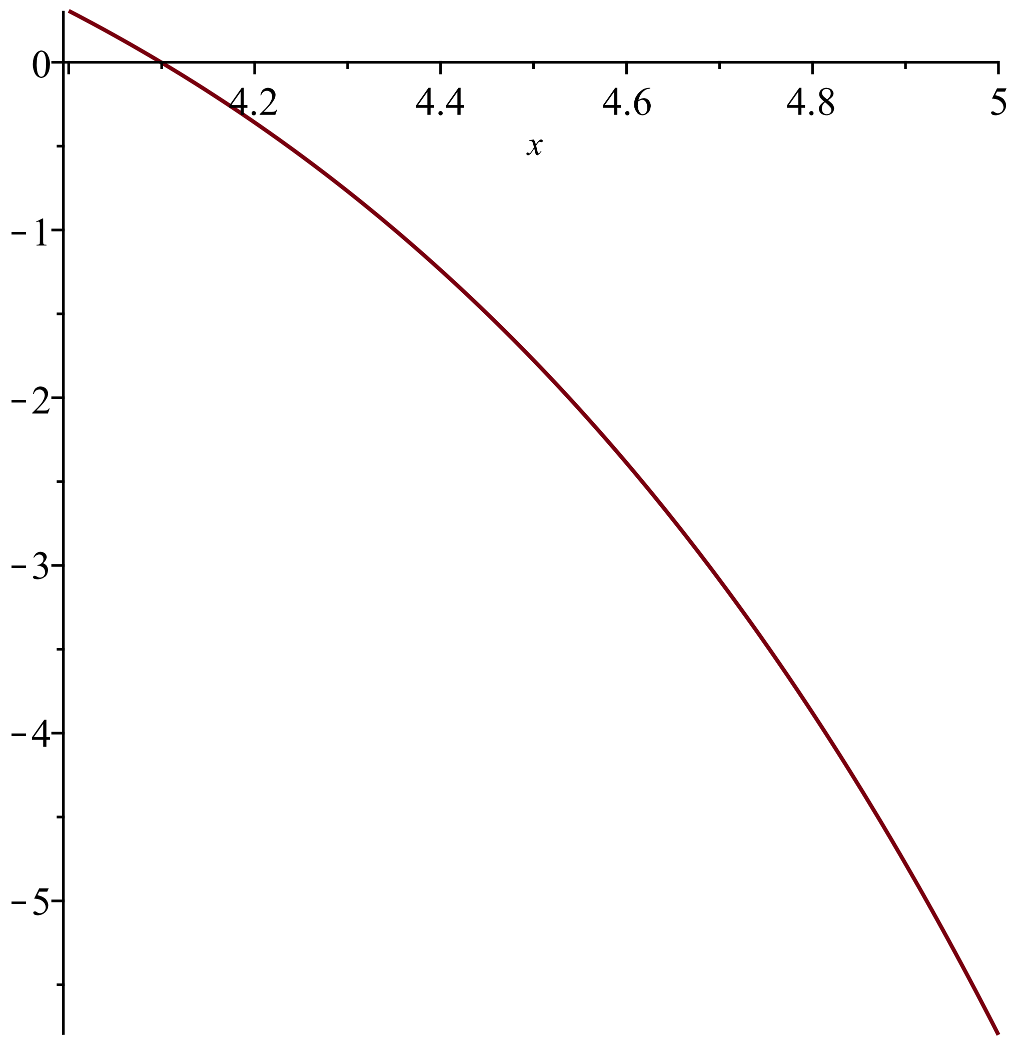


```
> p:= x -> x - ln(x)^x;
  a := 4.0: b := 5.0:
  print(a,p(a));
  print(b,p(b));
  plot(p(x),x=a..b);
```

$$p := x \to x - \ln(x)^x$$

$$4.0,\ 0.306638424$$

$$5.0,\ -5.79869156$$

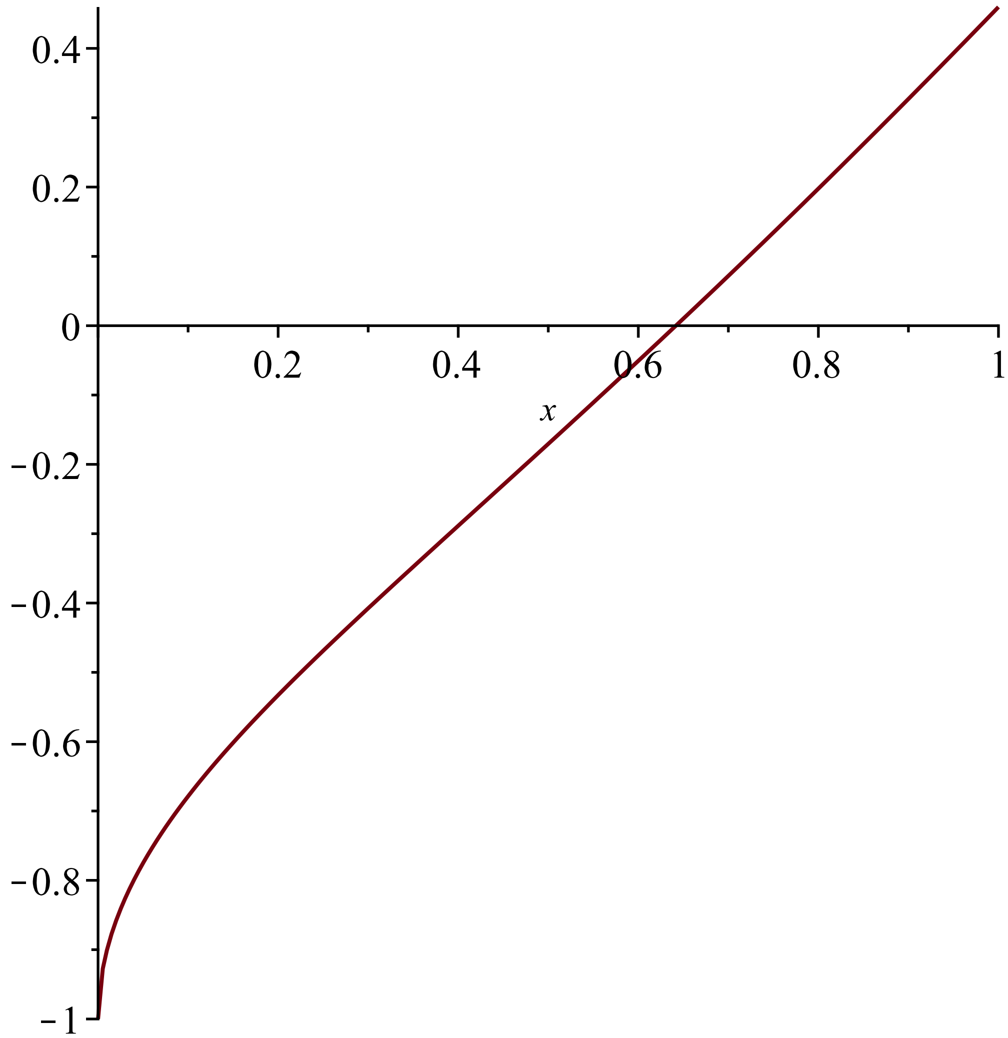



```
> p:= x -> x^(1/2)-cos(x);
  a := 0.0: b := 1.0:
  print(a,p(a));
  print(b,p(b));
  plot(p(x),x=a..b);
```

$$p := x \to \sqrt{x} - \cos(x)$$

$$0.,\ -1.$$

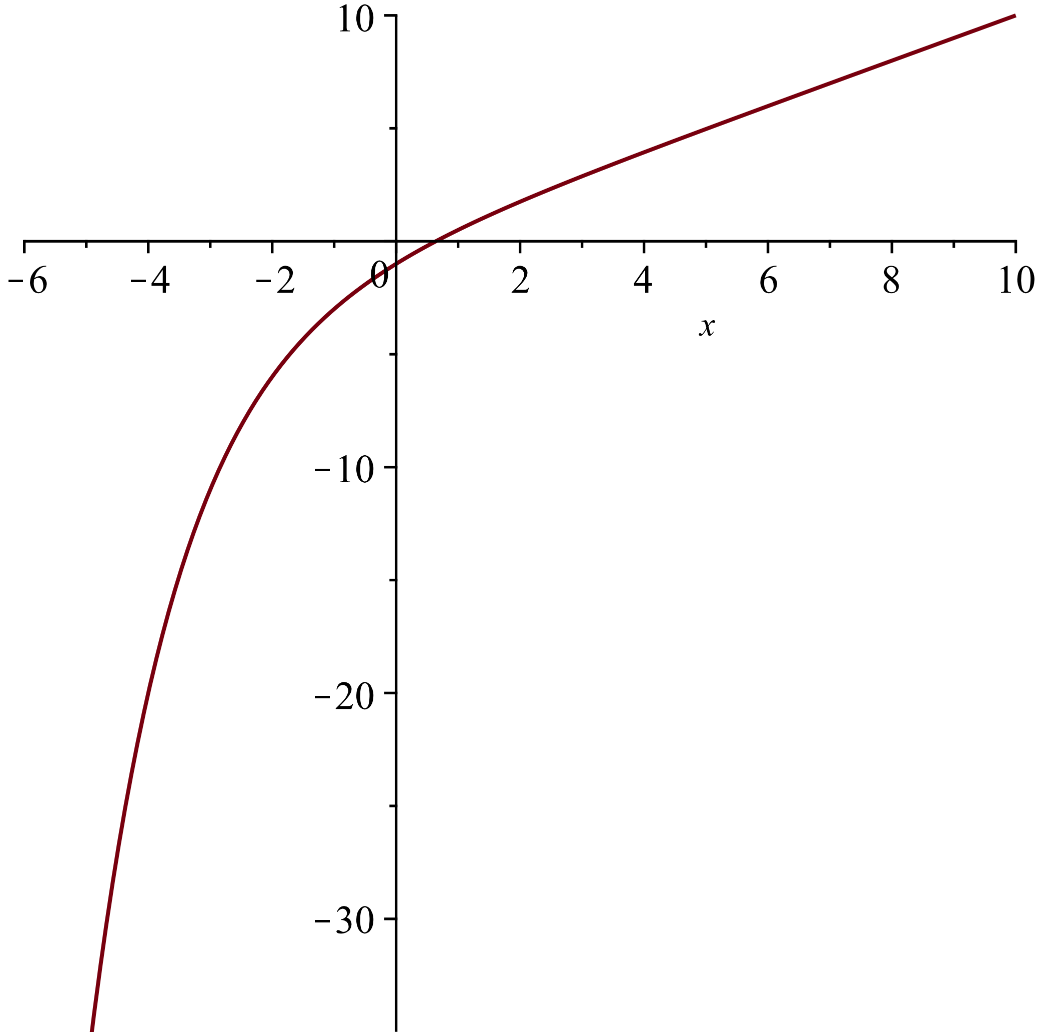
$$1.0,\ 0.4596976941$$



For pg 11: 3a, we could try evaluating at different points to find changes of sign. Instead, I plot first and then evaluate to confirm.

```
> p:= x-> x- 2^(-x);
  plot(p(x),x=-10..10);
  p(0.0); p(1.0);
```

$$p := x \rightarrow x - 2^{-x}$$



$$-1.$$
$$0.5000000000 \quad (1)$$

pg 12: 5a Note the max happens at one of the endpoints or a critical point.

```
> f:= x -> (2-exp(x)+2*x)/3;
  diff(f(x),x);
```

$$f := x \rightarrow \frac{2}{3} - \frac{1}{3}\,e^{x} + \frac{2}{3}\,x$$
$$-\frac{1}{3}\,e^{x} + \frac{2}{3} \quad (2)$$

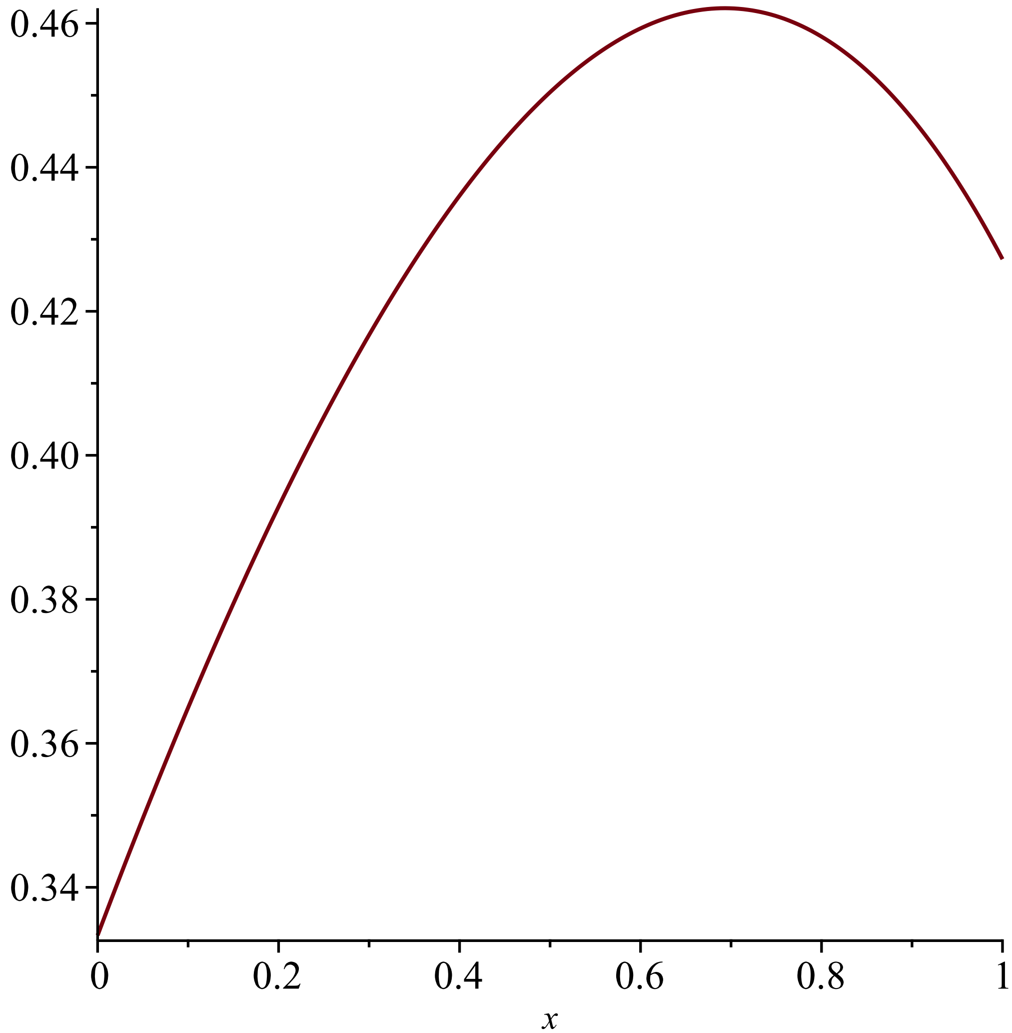
So we need to compare f(0), f(1), and f(ln(2))

```
> f(0.0);
  f(1.0);
  f(ln(2.0));
```

$$0.3333333334$$
$$0.4272393907$$
$$0.4620981204 \quad (3)$$

So the max occurs at ln(2). As the plot makes clear

```
> plot(f(x),x=0..1);
```



pg 12: 7a Note f(0) = f(1) = 0, so we are done by Rolle's Theorem

pg 12: 13c The 3rd Taylor polynomial of f(x) at 1 is f(1)+f'(1)*(x-1)+f''(1)*(x-1)^2/2!+f'''(1)(x-1)^3/3!

```
> f := x -> (x-1)*ln(x);
   f(1)+subs(x=1,diff(f(x),x))*(x-1)+subs(x=1,diff(f(x),x$2))*(x-1)
  ^2/2!+subs(x=1,diff(f(x),x$3))*(x-1)^3/3!;
```

$$f := x \to (x-1)\ln(x)$$

$$\ln(1)\,(x-1) + (x-1)^2 - \frac{1}{2}\,(x-1)^3 \quad (4)$$

Note ln(1) is 0. Maple is leaving it unevaluated. We can force the evaluation, e.g.,

```
> evalf(f(1)+subs(x=1,diff(f(x),x))*(x-1)+subs(x=1,diff(f(x),x$2))*
  (x-1)^2/2!+subs(x=1,diff(f(x),x$3))*(x-1)^3/3!);
```

$$(x-1.)^2 - 0.5000000000\,(x-1.)^3 \quad (5)$$

Integrating this from 0.5 to 1.5 gives

```
> subs(x=1.5,(x-1)^3/3-0.5*(x-1)^4/4)-subs(x=0.5,(x-1)^3/3-0.5*
  (x-1)^4/4);
```

$$0.08333333334 \quad (6)$$

We can also do it using numerical integration. To turn it into a function, there is the useful command   unapply

```
> P_3 := unapply(evalf(f(1)+subs(x=1,diff(f(x),x))*(x-1)+subs(x=1,
  diff(f(x),x$2))*(x-1)^2/2!+subs(x=1,diff(f(x),x$3))*(x-1)^3/3!),
  x);
```

$$P\_3 := x \to (x-1.)^2 - 0.5000000000\,(x-1.)^3 \quad \textbf{(7)}$$

```
> int(P_3(x),x=0.5..1.5);
```

$$0.08333333333 \quad \textbf{(8)}$$

The actual value to several decimal places is

```
> int(P_3(x),x=0.5..1.5);
  int(f(x),x=0.5..1.5);
```

$$0.08802039175 \quad \textbf{(9)}$$

There is the taylor comand you may use later in the course

```
> taylor(f(x),x=1,4);
```

$$(x-1)^2 - \frac{1}{2}(x-1)^3 + O\left((x-1)^4\right) \quad \textbf{(10)}$$

Getting rid of the big O to convert it to a polynomial is done by the convert command (after which you can use the unapply command).
This takes a while to get used to, but it is like in modern computer languages, where every variable has a type.

```
> convert(taylor(f(x),x=1,4),polynom);
  P_3:= unapply(%,x); #the pound sign let's you add a comment and
  the % symbol refers to the last output expression.
```

$$(x-1)^2 - \frac{1}{2}(x-1)^3$$

$$P\_3 := x \to (x-1)^2 - \frac{1}{2}(x-1)^3 \quad \textbf{(11)}$$

pg 25: 5cd   exact first

```
> a:=(1/3-3/11)+3/20;
  evalf((1/3-3/11)+3/20);
  b:=(1/3+3/11)-3/20;
  evalf((1/3+3/11)-3/20);
```

$$a := \frac{139}{660}$$

$$0.2106060606$$

$$b := \frac{301}{660}$$

$$0.4560606061 \quad \textbf{(12)}$$

3 digit chopping gives 0.210 and 0.456 respectively
3 digit rounding gives 0.211 and 0.456 respectively
The relative errors are

```
> evalf(abs(a-0.210)/a);
  evalf(abs(b-0.456)/b);
  evalf(abs(a-0.211)/a);
  evalf(abs(b-0.456)/b);
```

$$0.002877697813$$

$$0.0001328904518$$
$$0.001870503626$$
$$0.0001328904518 \quad \textbf{(13)}$$

We can do 3 digit rounding directly in Maple using evalf[3](number) or globally using Digits := 3;

```
> evalf[3](a);
```

$$0.211 \quad \textbf{(14)}$$

Note Maple's default (and the default in many software products, e.g., C) is round to even.
(Do you understand why this is the default and not the book's rounding up?)

```
> evalf[3](0.211+0.0005);
  evalf[3](0.212+0.0005);
```

$$0.212$$
$$0.212 \quad \textbf{(15)}$$

For pg 36: 6ad, 7cd we need to compute the taylor series. Using the taylor command

```
> taylor(sin(x),x);
```

$$x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \mathrm{O}(x^7) \quad \textbf{(16)}$$

so with x=1/n and n -> infinity, we have order 1/n.

note for 6d, we have ln(n+1)-ln(n)=ln(1+1/n), so we get order 1/n from

```
> taylor (ln(1+x),x);
```

$$x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 + \mathrm{O}(x^6) \quad \textbf{(17)}$$

For 7c, we get order h^2 from

```
> taylor((sin(h)-h*cos(h))/h,h);
```

$$\frac{1}{3}h^2 - \frac{1}{30}h^4 + \mathrm{O}(h^6) \quad \textbf{(18)}$$

and from 7d, we get order h from

```
> taylor((1-exp(h))/h,h);
```

$$-1 - \frac{1}{2}h - \frac{1}{6}h^2 - \frac{1}{24}h^3 - \frac{1}{120}h^4 + \mathrm{O}(h^5) \quad \textbf{(19)}$$