ACMS 40485   Applied Complex Analysis (3-0-3) Syllabus

Prerequisites: Applied Mathematical Methods II (ACMS 20750) or Mathematical Methods II (PHYS 20452); Applied Linear Algebra (ACMS 20620) or Linear Algebra (MATH 20610) ; Numerical Analysis (ACMS 40390) or Numerical PDE (PHYS 50051).

Rationale for prerequisites:

- Applied Methods I & II, which are required entry-level ACMS classes: covers complex numbers, complex power series, Fourier series, the Laplace transform and the Fourier integral, orthogonal polynomials, some special functions, complex variable up to and including Laurent series, integration using residues, and the complex contour integral inversion of the Laplace transform. This background allows this course to start with well over half the material in Math 40480 (Complex Variables) assumed.
- Applied Linear Algebra: covers basic matrix algebra, which is needed for applications to differential equations, e.g., the matrix Riemann-Hilbert problem.
- Numerical analysis: covers polynomial interpolation and solution of differential equations: needed for several applications. Interpolation theory may be approached using the Cauchy integral. This leads to a deeper understanding of the theory, e.g., the estimates behind Runge's famous example showing how badly interpolation at equally space points is. Other applications include the Fast Fourier transform and use of conformal mapping to numerically solve differential equations. Numerical ODE methods (marching methods such as the order 4 Runge-Kutta algorithm or rkf45 quickly and efficiently numerically compute solutions of functions satisfying polynomial relations with coefficients solutions of ODEs with rational coefficients.

Textbook: "Complex Variables: Introduction and Applications" (Cambridge Texts in Applied Mathematics) by M.J. Ablowitz and A.S. Fokas.

Complex analysis is a core part of applied and computational mathematics. As time permits, we will cover the following.

1. The course will start with a review (about three classes) of complex variables with an emphasis on contour integration and the residue calculus.
2. The course will cover infinite product expansions (Weierstrass) and infinite partial fraction expansions (Mittag-Leffler) of functions.
3. Basic special functions will be covered, e.g., Bessel functions, elliptic functions, and if time permits, theta functions.
4. Interpolation by polynomials in terms of the Cauchy integral and applications (including Runge's example). Fast Fourier transform via complex interpolation theory.

5. Numerical computation of analytic continuations of a very large class of functions, including those satisfying polynomial relations with coefficients solutions of ODEs with rational coefficients, e.g., Bessel functions, roots of rational functions.
6. Conformal mapping including the Schwarz-Christophel transformation for uniformization of polygons, and applications to the numerical solution of PDEs in the plane involving the Laplacian.
7. Asymptotic integration methods will be covered (including the stationary phase method and method of steepest descent). Applications of these methods to differential equations will include:
    a. Schrodinger's equation;
    b. to Burgher's equation (weak nonlinear and diffusion: weak shock waves in compressible fluid dynamics);
    c. Korteweg-deVries (KDV) equation; and
    d. the WKB method and the Mellin transform method.
8. Scalar and Matrix Riemann-Hilbert problems; Plemelj formulae; applications to:
    a. the Fourier transform;
    b. the Radon transform (basis of computerized tomography, i.e., the CT scan);
    c. singular integral equations;
    d. planar wing theory;
    e. the Wiener-Hopf method; and
    f. reconstruction of the potential of the time-independent Schrodinger equation from scattering data.