

Homework 11

(1)

Homogenize $z^3 - 1 + z + z^2$

$$\rightarrow \left(\left(\frac{z}{w} \right)^3 - 1 + \frac{z}{w} + \left(\frac{z}{w} \right)^2 \right) w^3 =$$

$$\boxed{z^3 - w^3 + zw^2 + z^2w}$$

If homogenize $z^3 + z^2w - zw^2$ with respect to $z=0$

$$\rightarrow 1 + w - w^2$$

note homogenizing gives

$$z^2 + wz - w^2$$

what is going on?

ans. $z=0$ is part of $V(z^3 + z^2w - zw^2)$ ~~and~~ and it was lost when we ~~homogenized~~ dehomogenized.

(2)

Find the singular points of
 $V(z^3 - 3zw^2 + 2w^3) = 0$

$$\frac{\partial (z^3 - 3zw^2 + 2w^3)}{\partial z} = 3z^2 - 3w^2$$

$$\frac{\partial (z^3 - 3zw^2 + 2w^3)}{\partial w} = -6zw + 6w^2$$

$$\text{So } z = \pm w$$

$$w=0 \text{ or } w=z$$

$\Rightarrow z=w$ $[1,1]$ singular point.

Find the singular points of $V(z^5 + w^5 + u^5)$

$5z^4 = 5w^4 = 5u^4 = 0$ $[0,0,0]$ not ~~a~~
a point of \mathbb{P}^2 . So no singular points.

$$\begin{aligned} \#e(S^2 - 10 \text{ disks}) &= e(S^2) - 10 e(\text{disk}) \\ &= 2 - 10 = -8 \end{aligned}$$
