Homework 10 (Due Thursday, April 16)

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What is the Euler characteristic of the complex plane minus the disc consisting of all points of absolute value strictly less than 1?

What is the Euler characteristic of the complex plane minus the disc consisting of all points of absolute value less than or equal to 1?

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#4 $$\frac{1}{1+t^4} = 1 - t^4 + t^8 - t^{12} \ldots$$

(only converges when $|t| < 1$, but Watson's lemma says only contribution near 0 counts)

$$\int_0^q \frac{e^{-kt}}{1+t^4}\,dt = \sum_{m=0}^{\infty} (-1)^m \frac{(4m)!}{k^{4m+1}}$$

#5 $$\int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-k\sin^4 t}\,dt \sim \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-kt^4}\,dt$$

$$= 2\int_0^{\frac{1}{2}} e^{-kt^4}\,dt$$

let $u = kt^4$, $du = kt^3 4\,dt$

$$dt = \frac{du}{4k\left(\frac{u}{k}\right)^{3/4}} = \frac{du}{4k^{\frac{1}{4}}u^{3/4}}$$

$$= 2\int_0^{\frac{k}{16}} \frac{e^{-u}}{u^{3/4}k^{1/4}}\frac{du}{4}$$

$$\sim \frac{2}{4}\frac{\Gamma\left(\frac{1}{4}\right)}{k^{1/4}} = \frac{\Gamma\left(\frac{1}{4}\right)}{2k^{1/4}}$$

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1) a

$$\int_0^2 [\sin(t)+t]e^{ikt}\,dt$$

$$= \frac{e^{ikt}}{ik}(\sin(t)+t)\Big|_0^2 - \int_0^2 \frac{e^{ikt}}{ik}(\cos(t)+1)\,dt$$

$$= \frac{e^{ik2}(\sin(2)+2)}{ik} + \frac{e^{ikt}}{k^2}(\cos(t)+1)\Big|_0^2$$

$$+ \int_0^2 \frac{e^{ikt}}{k^2}\sin(t)\,dt$$

$$= \frac{e^{ik2}(\sin(2)+2)}{ik} + \frac{e^{ik2}}{k^2}(\cos(2)+1) - \frac{2}{k^2}$$

$$+ \frac{e^{ikt}}{ik^3}\sin(t)\Big|_0^2 - \int \frac{e^{ikt}}{ik^3}\cos(t)\,dt$$

$$\Big| \qquad \Big| \le \frac{\text{constant}}{k^3}$$

1c)

$$\int_0^\infty \frac{e^{ikt}}{1+t^2}\,dt = \frac{e^{ikt}}{ik(1+t^2)}\Big|_0^\infty + \int_0^\infty \frac{2e^{ikt}}{ik(1+t^2)^2}\,t\,dt$$

$$\parallel$$

$$\frac{1}{ik} + \underbrace{\frac{2e^{ikt}\,t}{-k^2(1+t^2)^2}\Big|_0^\infty}_{=0}$$

$$-\int_0^\infty \frac{2e^{ikt}}{k^2}\left(\frac{1}{(1+t^2)^2} - \frac{\cancel{2}4t^2}{(1+t^2)^3}\right)dt$$

$$= \frac{1}{ik} - \int_0^\infty \frac{2e^{ikt}}{k^2}\,\frac{1-3t^2}{(1+t^2)^3}\,dt$$

$$= \frac{1}{ik} - \frac{2e^{ikt}}{k^3 i}\,\frac{1-3t^2}{(1+t^2)^3}\Big|_0^\infty$$

$$+ \int_0^\infty \frac{2e^{ikt}}{ik^3}\left(\frac{-6t}{(1+t^2)^3} - \frac{(6t)(1-3t^2)}{(1+t^3)^4}\right)$$

$$\left|\frac{-2e^{kt}}{k^3 i}\left(\quad\right)\Big|_0^\infty\right| = \frac{2}{k^3} \qquad \text{so } \frac{1}{ik} \text{ is the answer}$$

$$\left|\int_0^\infty \frac{2e^{ikt}}{ik^3}\left(\quad\right)\right| \le \frac{C}{k^3}$$

$$e(\mathbb{C} - \Delta_{1,(0)}) = e(\mathbb{C}) - e(\Delta_{1,(0)})$$
$$= 1 - 1 = 0$$

$$e(\mathbb{C} - \overline{\Delta_{1,(0)}}) = 1 - 1 = 0$$

Basically $e(S^1) = 0$ so
we have the same ~~[illegible]~~ answer.