```
> with(LinearAlgebra);
```

(1)

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]

```
> A:=Matrix(4,datatype = complex(sfloat));
  B:=RandomMatrix(4, generator = 0 .. 1.0);
```

$$A := \begin{bmatrix} 0. + 0.\,I & 0. + 0.\,I & 0. + 0.\,I & 0. + 0.\,I \\ 0. + 0.\,I & 0. + 0.\,I & 0. + 0.\,I & 0. + 0.\,I \\ 0. + 0.\,I & 0. + 0.\,I & 0. + 0.\,I & 0. + 0.\,I \\ 0. + 0.\,I & 0. + 0.\,I & 0. + 0.\,I & 0. + 0.\,I \end{bmatrix}$$

$$B := \begin{bmatrix} 0.507858284661118153 & 0.117417650855805911 & 0.262211747780845439 & 0.111119223440598769 \\ 0.424166759713807240 & 0.221746734017240099 & 0.594896074008614328 & 0.438869973126103208 \\ 0.318778301925882301 & 0.711215780433682943 & 0.408719846112552077 & 0.979748378356085192 \\ 0.296675873218326891 & 0.602843089382082975 & 0.258064695912066933 & 0.904880968679892872 \end{bmatrix}$$

(2)

```
> for j from 1 to 4 do
  for k from 1 to 4 do
  A(j,k):= B(j,k) + 0.*I;
  od:
  od:
> A;
```

(3)

[[0.507858284661118153 + 0. I, 0.117417650855805911 + 0. I, 0.262211747780845439 + 0. I, 0.111119223440598769 + 0. I],
[0.424166759713807240 + 0. I, 0.221746734017240099 + 0. I, 0.594896074008614328 + 0. I, 0.438869973126103208 + 0. I],
[0.318778301925882301 + 0. I, 0.711215780433682943 + 0. I, 0.408719846112552077 + 0. I, 0.979748378356085192 + 0. I],
[0.296675873218326891 + 0. I, 0.602843089382082975 + 0. I, 0.258064695912066933 + 0. I, 0.904880968679892872 + 0. I]]

We did the complex form of the Schur Lemma in class.
The real form differs slightly (due to the fact that eigenvalues of a

real matrix may be complex).

```
> SchurForm(A);
```

$$\left[\left[1.82491377021677 - 3.28320558267607\ 10^{-18}\,I,\ 0.367295500855236 + 1.38777878078145\ 10^{-17}\,I,\ 0.0413085072736971 - 5.89805981832114\ 10^{-17}\,I,\ 0.547348045752282 + 6.93889390390723\ 10^{-17}\,I\right],\left[0. + 0.\,I,\ 0.342250057750036 - 7.07841101058017\ 10^{-18}\,I,\ -0.180052284242930 - 1.52655665885959\ 10^{-16}\,I,\ -0.331629585829796 - 2.42861286636753\ 10^{-17}\,I\right],\left[0. + 0.\,I,\ 0. + 0.\,I,\ -0.222536246858923 + 5.05723412908172\ 10^{-18}\,I,\ -0.338188686096925 - 1.83880688453542\ 10^{-16}\,I\right],\left[0. + 0.\,I,\ 0. + 0.\,I,\ 0. + 0.\,I,\ 0.0985782523629265 + 6.46076705521729\ 10^{-19}\,I\right]\right] \quad (4)$$

```
> T,Z := SchurForm(A, output=['T','Z']):
> T;
  Z;
```

$$\left[\left[1.82491377021677 - 3.28320558267607\ 10^{-18}\,I,\ 0.367295500855236 + 1.38777878078145\ 10^{-17}\,I,\ 0.0413085072736971 - 5.89805981832114\ 10^{-17}\,I,\ 0.547348045752282 + 6.93889390390723\ 10^{-17}\,I\right],\left[0. + 0.\,I,\ 0.342250057750036 - 7.07841101058017\ 10^{-18}\,I,\ -0.180052284242930 - 1.52655665885959\ 10^{-16}\,I,\ -0.331629585829796 - 2.42861286636753\ 10^{-17}\,I\right],\left[0. + 0.\,I,\ 0. + 0.\,I,\ -0.222536246858923 + 5.05723412908172\ 10^{-18}\,I,\ -0.338188686096925 - 1.83880688453542\ 10^{-16}\,I\right],\left[0. + 0.\,I,\ 0. + 0.\,I,\ 0. + 0.\,I,\ 0.0985782523629265 + 6.46076705521729\ 10^{-19}\,I\right]\right]$$

$$\left[\left[0.219036761512636 + 6.93889390390723\ 10^{-18}\,I,\ 0.945299512555574 + 0.\,I,\ -0.216964094969587 - 1.38777878078145\ 10^{-16}\,I,\ -0.106575373151025 - 2.42861286636753\ 10^{-17}\,I\right],\left[0.455161838581246 - 1.04083408558608\ 10^{-17}\,I,\ 0.102682443569910 - 3.64291929955129\ 10^{-17}\,I,\ 0.883146249743396 + 0.\,I,\ 0.0483396115626368 + 1.95156391047391\ 10^{-17}\,I\right],\left[0.661431241889163 + 0.\,I,\ -0.287618457568129 + 9.71445146547012\ 10^{-17}\,I,\ -0.272597803089244 + 1.38777878078145\ 10^{-17}\,I,\ -0.636769010607463 - 2.08166817117217\ 10^{-17}\,I\right],\left[0.554399954958834 - 2.08166817117217\ 10^{-17}\,I,\ -0.114633198554512 + 1.38777878078145\ 10^{-17}\,I,\ -0.314117367888374 + 0.\,I,\ 0.762122167976819 + 0.\,I\right]\right] \quad (5)$$

```
> Z.HermitianTranspose(Z);
```

$$\left[\left[1.00000000000000 + 0.\,I,\ 1.91686944095437\ 10^{-16} - 8.17806307158922\ 10^{-17}\,I,\ -4.44089209850063\ 10^{-16} - 3.31534262854856\ 10^{-17}\,I,\ -1.11022302462516\ 10^{-16} + 2.03714197194047\ 10^{-17}\,I\right],\left[1.91686944095437\ 10^{-16} + 8.17806307158922\ 10^{-17}\,I,\ 0.999999999999999 + 0.\,I,\ 6.90419943438769\ 10^{-16} - 3.00585298159696\ 10^{-17}\,I,\ 7.00828284294630\ 10^{-16} + 2.13288663537235\ 10^{-17}\,I\right],\left[-4.44089209850063\ 10^{-16} + 3.31534262854856\ 10^{-17}\,I,\ 6.90419943438769\ 10^{-16} + 3.00585298159696\ 10^{-17}\,I,\ 1.00000000000000 + 0.\,I,\ 1.11022302462516\ 10^{-16} - 1.35997836510819\ 10^{-17}\,I\right],\left[-1.11022302462516\ 10^{-16} - 2.03714197194047\ 10^{-17}\,I,\ 7.00828284294630\ 10^{-16} - 2.13288663537235\ 10^{-17}\,I,\ 1.11022302462516\ 10^{-16} + 1.35997836510819\ 10^{-17}\,I,\ 1.00000000000000 + 0.\,I\right]\right] \quad (6)$$

```
> Z . T . HermitianTranspose(Z)-A;
```

$$\left[\left[6.66133814775094\ 10^{-16} - 1.02105458339470\ 10^{-18}\,I,\ 3.19189119579733\ 10^{-16}\right. \quad (7)$$

$-8.16553339621839\ 10^{-17}\,I,\ 2.77555756156289\ 10^{-16}-4.87201985616224\ 10^{-17}\,I,$
$4.30211422042248\ 10^{-16}+1.10681389586876\ 10^{-16}\,I],$
$[4.44089209850063\ 10^{-16}-2.92673840427805\ 10^{-17}\,I,\ 1.05471187339390\ 10^{-15}$
$-2.71057035821028\ 10^{-17}\,I,\ -1.11022302462516\ 10^{-16}+5.92302854945067\ 10^{-17}\,I,$
$1.16573417585641\ 10^{-15}-8.08593876211424\ 10^{-17}\,I],$
$[3.88578058618805\ 10^{-16}+6.13202030272540\ 10^{-17}\,I,\ -1.11022302462516\ 10^{-16}$
$-8.92861956011960\ 10^{-20}\,I,\ 2.22044604925031\ 10^{-16}-5.59365773356113\ 10^{-17}\,I,$
$6.66133814775094\ 10^{-16}+7.06771895835776\ 10^{-17}\,I],$
$[8.88178419700125\ 10^{-16}+1.34742867162108\ 10^{-17}\,I,\ 8.88178419700125\ 10^{-16}$
$-2.49831496091022\ 10^{-17}\,I,\ 9.99200722162641\ 10^{-16}-8.22893875379093\ 10^{-17}\,I,$
$1.77635683940025\ 10^{-15}+6.60392747240074\ 10^{-17}\,I]]$

```
> L:=SingularValues(A, output = 'list');
```

$$L := [1.94402988081538,\ 0.607037331707257,\ 0.221738830185565,\ 0.0523609676521310] \quad (8)$$

```
> U, Vt := SingularValues(A, output = ['U', 'Vt']);
```

$U, Vt := [[-0.209580807817669+0.\,I,\ 0.663556240128605+0.\,I,\ -0.706058784300273+0.\,I,$ (9)
$-0.131339233640613+0.\,I],$
$[-0.404701142530598+0.\,I,\ 0.606705455105708+0.\,I,\ 0.651351998873087+0.\,I,\ 0.209442234383227$
$+0.\,I],$
$[-0.671486281758066+0.\,I,\ -0.279992900649665+0.\,I,\ 0.0632601445794775+0.\,I,$
$-0.683160525136148+0.\,I],$
$[-0.584293627930040+0.\,I,\ -0.336460372877923+0.\,I,\ -0.270591427773017+0.\,I,$
$0.687150386057493+0.\,I]],\ [[-0.342329717948642+0.\,I,\ -0.485670857091093+0.\,I,$
$-0.370850780141831+0.\,I,\ -0.713725354495552+0.\,I],$
$[0.667605538186052-0.\,I,\ -0.312204871077792-0.\,I,\ 0.549639566283822-0.\,I,$
$-0.393353925916526-0.\,I],$
$[-0.642232678875812-0.\,I,\ -0.255260409316324-0.\,I,\ 0.714243062255971-0.\,I,$
$0.110617167017028-0.\,I],$
$[0.157004445085207+0.\,I,\ -0.775560481512154+0.\,I,\ -0.224104649801693+0.\,I,$
$0.568887202947458+0.\,I]]$

```
> DiagL:=DiagonalMatrix(L);
```

$$DiagL := \begin{bmatrix} 1.94402988081538 & 0 & 0 & 0 \\ 0 & 0.607037331707257 & 0 & 0 \\ 0 & 0 & 0.221738830185565 & 0 \\ 0 & 0 & 0 & 0.0523609676521310 \end{bmatrix} \quad (10)$$

```
> U . DiagL . Vt-A;
```

$[[-6.66133814775094\ 10^{-16}+0.\,I,\ -4.16333634234434\ 10^{-17}+0.\,I,\ 1.38777878078145\ 10^{-15}+0.\,I,$ (11)
$1.38777878078145\ 10^{-17}+0.\,I],$
$[-7.21644966006352\ 10^{-16}+0.\,I,\ -3.33066907387547\ 10^{-16}+0.\,I,\ 9.99200722162641\ 10^{-16}+0.\,I,$
$5.55111512312578\ 10^{-17}+0.\,I],$
$[2.77555756156289\ 10^{-16}+0.\,I,\ -1.11022302462516\ 10^{-16}+0.\,I,\ -8.32667268468867\ 10^{-16}+0.\,I,$
$-5.55111512312578\ 10^{-16}+0.\,I],$
$[5.55111512312578\ 10^{-16}+0.\,I,\ 1.11022302462516\ 10^{-16}+0.\,I,\ -4.99600361081320\ 10^{-16}+0.\,I,$

$$-1.11022302462516\ 10^{-16} + 0.\ I]]$$

```
> Q1, R1 := QRDecomposition(A):
> Q1;
```

$$[[-0.641128071454881 + 0.\ I,\ -0.509006578661873 + 0.\ I,\ 0.540011723689480 + 0.\ I,\ -0.195587415621242 + 0.\ I],\ [-0.535474609441573 - 0.\ I,\ -0.237334072297770 + 0.\ I,\ -0.781177134373007 + 0.\ I,\ 0.216105912697292 + 0.\ I],\ [-0.402430607333263 - 0.\ I,\ 0.648491533829728 + 0.\ I,\ -0.0978570681966960 + 0.\ I,\ -0.638695804774460 + 0.\ I],\ [-0.374528163049618 - 0.\ I,\ 0.513851701910832 + 0.\ I,\ 0.297613200672688 + 0.\ I,\ 0.712117593035995 + 0.\ I]]$$ **(12)**

```
> R1;
```

$$[[-0.792132348079315 + 0.\ I,\ -0.706016211179127 + 0.\ I,\ -0.747796927463772 + 0.\ I,\ -1.03942952279523 + 0.\ I],\ [0.\ I,\ 0.658594947661843 + 0.\ I,\ 0.123001730648398 + 0.\ I,\ 0.939613940751917 + 0.\ I],\ [0.\ I,\ 0.\ I,\ -0.286314458176401 + 0.\ I,\ -0.109400287143582 + 0.\ I],\ [0.\ I,\ 0.\ I,\ 0.\ I,\ 0.0917293527690591 + 0.\ I]]$$ **(13)**

The QR algorithm does not work for every matrix instantly (there are steps to reduce to a form on which the QR-algorithm works). So if you choose a different matrix A, the simple-minded version below will fail.

```
> for j from 1 to 100 do
  B:= R1 . Q1:
  Q1, R1 := QRDecomposition(B);
  od:
> B;
```

$$[[1.82491377021677 + 0.\ I,\ 0.367295500855237 + 0.\ I,\ 0.0413085072736971 + 0.\ I,\ 0.547348045752283 + 0.\ I],\ [-7.92886755229958\ 10^{-73} + 0.\ I,\ 0.342250057750036 + 0.\ I,\ -0.180052284242930 + 0.\ I,\ -0.331629585829797 + 0.\ I],\ [-7.39424623620348\ 10^{-92} + 0.\ I,\ 9.93365025499439\ 10^{-20} + 0.\ I,\ -0.222536246858923 + 0.\ I,\ -0.338188686096926 + 0.\ I],\ [1.57497659095152\ 10^{-127} + 0.\ I,\ -1.89203674678207\ 10^{-55} + 0.\ I,\ 6.90253427293958\ 10^{-37} + 0.\ I,\ 0.0985782523629267 + 0.\ I]]$$ **(14)**

```
> Eigenvalues(A);
```

$$\begin{bmatrix} 1.82491377021677 - 3.28172327501838\ 10^{-18}\ I \\ 0.342250057750036 + 2.00961724006300\ 10^{-18}\ I \\ -0.222536246858923 + 4.75145523766048\ 10^{-18}\ I \\ 0.0985782523629265 + 6.46076705521729\ 10^{-19}\ I \end{bmatrix}$$ **(15)**