```
> restart:
> Digits:=15;
```

$$Digits := 15 \tag{1}$$

```
> Euler := proc( f,a,b,alpha, N )
  local h:
  local xx:
  local j:
  h:=evalf((b-a)/N):
  xx:=alpha:
  for j from 0 to N-1 do
  xx:= xx+f(a+j*h,xx)*h:
  od:
  xx;
  end;
```

$$Euler := \mathbf{proc}(f, a, b, \alpha, N) \tag{2}$$

local $h, xx, j;$

$h := evalf((b-a)/N);\ xx := \alpha;$ **for** j **from** 0 **to** $N-1$ **do** $xx := xx + f(a + j*h, xx)*h$ **end do**; xx

end proc

```
> ModEuler:=  proc( f,a,b,alpha, N )
  local h:
  local xx:
  local j:
  h:=evalf((b-a)/N):
  xx:=alpha:
  for j from 0 to N-1 do
  xx:= xx+(f(a+j*h,xx)+f(a+(j+1)*h,xx+f(a+j*h,xx)*h))*h/2:
  od:
  xx;
  end;
```

$$ModEuler := \mathbf{proc}(f, a, b, \alpha, N) \tag{3}$$

local $h, xx, j;$

$h := evalf((b-a)/N);$

$xx := \alpha;$

for j **from** 0 **to** $N-1$ **do**

$xx := xx + 1/2*(f(a + j*h, xx) + f(a + (j+1)*h, xx + f(a + j*h, xx)*h))*h$

end do;

xx

end proc

```
> p:= x-> x^3-x+7;
```

$$p := x \to x^3 - x + 7 \tag{4}$$

```
> d:=degree(p(x));
```

$$d := 3 \tag{5}$$

```
> N:=100;
```

$$N := 100 \tag{6}$$

```
> for j from 0 to d-1 do
  alpha:= evalf(exp(2*j*Pi*I/d)):
  RandGamma:= 0.196743+0.7981302*I:#very provisional random gamma
  H:= unapply((1-t)*p(x)+t*RandGamma*(x^d-1),t,x):
  f:= unapply(-diff(H(t,x),t)/diff(H(t,x),x),t,x):
  print(Euler(f,1.0,0.0,alpha,N)):
  od:
```

$$1.04396243517603 + 1.51230306990488\ I$$

$$-2.09506087659580 - 0.00283983822444743\,I$$
$$1.05119019867424 - 1.50945304631392\,I \qquad (7)$$

```
> fsolve(p(x),x,complex);
```

$$-2.08674533988267,\ 1.04337266994133 - 1.50528388855442\,I,\ 1.04337266994133 + 1.50528388855442\,I \qquad (8)$$

```
> for j from 0 to d-1 do
  alpha:= evalf(exp(2*j*Pi*I/d)):
  RandGamma:= 0.196743+0.7981302*I:#very provisional random gamma
  H:= unapply((1-t)*p(x)+t*RandGamma*(x^d-1),t,x):
  f:= unapply(-diff(H(t,x),t)/diff(H(t,x),x),t,x):
  print(ModEuler(f,1.0,0.0,alpha,N)):
  od:
```

$$1.04335027725293 + 1.50525806443268\,I$$
$$-2.08670350183645 - 0.00000308317550635\,I$$
$$1.04335432066964 - 1.50525495255674\,I \qquad (9)$$

```
> restart:
> Digits:=15;
```

$$Digits := 15 \tag{10}$$

```
> Euler := proc( f,a,b,alpha, N )
  local h:
  local xx:
  local j:
  h:=evalf((b-a)/N):
  xx:=alpha:
  for j from 0 to N-1 do
  xx:= xx+f(a+j*h,xx)*h:
  od:
  xx;
  end;
```

$$Euler := \mathbf{proc}(f, a, b, \alpha, N) \tag{11}$$

$$\mathbf{local}\ h, xx, j;$$

$$h := evalf((b-a)/N);\ xx := \alpha;\ \mathbf{for}\ j\ \mathbf{from}\ 0\ \mathbf{to}\ N-1\ \mathbf{do}\ xx := xx + f(a + j*h, xx)*h\ \mathbf{end\ do};\ xx$$

$$\mathbf{end\ proc}$$

```
> p:= x-> x^3-x+7;
```

$$p := x \to x^3 - x + 7 \tag{12}$$

```
> d:=degree(p(x));
```

$$d := 3 \tag{13}$$

```
> N:=10;
```

$$N := 10 \tag{14}$$

Euler as predictor + Newton as corrector

```
> a:=1.0:
  b:=0:
  for j from 0 to d-1 do
  alpha:= evalf(exp(2*j*Pi*I/d)):
  RandGamma:= 0.196743+0.7981302*I:#very provisional random gamma
  H:= unapply((1-t)*p(x)+t*RandGamma*(x^d-1),t,x):
  f:= unapply(-diff(H(t,x),t)/diff(H(t,x),x),t,x):
  dHx:= unapply(diff(H(t,x),x),t,x):
  h:=evalf((b-a)/N):
  xx:=alpha:
  for k from 0 to N-1 do
  xx:= xx+f(a+k*h,xx)*h:
  xx:= xx- H(a+(k+1)*h,xx)/dHx(a+(k+1)*h,xx):
  od:
  print(xx);
  od:
```

$$1.04335988840111 + 1.50530141371428\ I$$
$$-2.08675263277533 - 0.00003447783316228\ I$$
$$1.04339173031879 - 1.50526795966888\ I \tag{15}$$