Andrew Sommese 2012

```
> restart;
```

```
> with(LinearAlgebra):
  v:= Vector(2);
  w := Vector(2);
```

$$v := \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$w := \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (1)$$

```
> Newton2D := proc(f,g,v)
  local J11, J12, J21, J22, A,B, w;
  with(LinearAlgebra);
  J11:=unapply(diff(f(x,y),x),x,y);
  J12:=unapply(diff(f(x,y),y),x,y);
  J21:=unapply(diff(g(x,y),x),x,y);
  J22:=unapply(diff(g(x,y),y),x,y);
  A := (x,y) -> Matrix([[J11(x,y),J12(x,y)],[J21(x,y),J22(x,y)]]);
  w:= Vector([f(v[1],v[2]),g(v[1],v[2])]);
  B:= MatrixInverse(A(v[1],v[2]));
  VectorAdd(v,-MatrixVectorMultiply(B,w));
  end proc;
```

```
Newton2D := proc(f, g, v)
    local J11, J12, J21, J22, A, B, w;
    with(LinearAlgebra);
    J11 := unapply(diff(f(x, y), x), x, y);
    J12 := unapply(diff(f(x, y), y), x, y);
    J21 := unapply(diff(g(x, y), x), x, y);
    J22 := unapply(diff(g(x, y), y), x, y);
    A := (x, y) → Matrix([[J11(x, y), J12(x, y)], [J21(x, y), J22(x, y)]]);
    w := Vector([f(v[1], v[2]), g(v[1], v[2])]);
    B := LinearAlgebra:-MatrixInverse(A(v[1], v[2]));
    LinearAlgebra:-VectorAdd(v, -LinearAlgebra:-MatrixVectorMultiply(B, w))
end proc
```

(2)

```
> f:= (x,y)->x^2+y^2-1.0;
  g:= (x,y) -> x^2/0.25+y^2/9-1;
```

$$f := (x, y) \to x^2 + y^2 - 1.0$$

$$g := (x, y) \to \frac{x^2}{0.25} + \frac{1}{9} y^2 - 1$$

(3)

```
> v:= Vector([0.1,0.2]);
  for j from 1 to 5 do
  v:= Newton2D(f,g,v);
  od;
```

$$v := \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$$

$$v := \begin{bmatrix} 1.19285714292473 \\ 2.02857142853763 \end{bmatrix}$$

$$v := \begin{bmatrix} 0.692236954690471 \\ 1.20442655934241 \end{bmatrix}$$

$$v := \begin{bmatrix} 0.511214711389569 \\ 0.922460523243180 \end{bmatrix}$$

$$v := \begin{bmatrix} 0.479164525978609 \\ 0.879366621926955 \end{bmatrix}$$

$$v := \begin{bmatrix} 0.478092645310086 \\ 0.878310700378225 \end{bmatrix}$$

(4)

Griewank's Example

```
> f:= (x,y)->29/16 * x^3 -2*x*y;
  g:= (x,y) -> y-x^2;
```

$$(x, y) \to \frac{29}{16}\, x^3 - 2\, x\, y \tag{5}$$
$$(x, y) \to y - x^2$$

```
> v:= Vector([0.1,0.2]);
  for j from 1 to 25 do
  v:= Newton2D(f,g,v);
  od:
  print(v);
```

$$Vector_{column}(\%id = 324379776) \tag{6}$$
$$Vector_{column}(\%id = 326984968)$$

```
> Digits := 20;
```

$$20 \tag{7}$$

```
> v:= Vector([0.01+0.167*I,0.06+0.237*I]);
  for j from 1 to 25 do
  v:= Newton2D(f,g,v);
  od:
  print(v);
```

$$Vector_{column}(\%id = 431849000) \tag{8}$$
$$Vector_{column}(\%id = 436356536)$$