```
> restart:
  with(LinearAlgebra):
  Digits := 30;
  a := 2.0;b:=4.0;
  w := t -> ln(2+sin(t));
  IP := proc( f,g ) evalf(int(f(t)*g(t)*w(t),t=a..b)) end;
```

$$Digits := 30$$

$$a := 2.0$$

$$b := 4.0$$

$$w := t \rightarrow \ln(2 + \sin(t))$$

$$IP := \mathbf{proc}(f, g)\ \ evalf(int(f(t) * g(t) * w(t), t = a..b))\ \ \mathbf{end\ proc} \tag{1}$$

Note the inner product IP above

```
> n:=4;
```

$$n := 4 \tag{2}$$

The following routines work up to n=5.
Does anyone see why there is a difficulty for n>=6?

```
> p0 := unapply(1,t):
> j:='j':k:='k':
> for j from 0 to n-1 do;
> vv := t*p||j(t):
  g:= unapply(vv,t):
> for k from 0 to j do;
  if (k >= j-2) then
  u:=IP(p||k,p||k);
  vv := vv-IP(g,p||k)/u*p||k(t):
  vv:=expand(vv):
  fi:
> od:
  p||(j+1):=unapply(expand(vv),t):
> od:
```

```
> for j from 0 to n do p||j(t); od;
```

$$1$$

$$t - 2.795313489653151808761329765 44$$

$$t^2 - 5.795744776325429926972171852 63\, t + 8.110771076439471618378426630 81$$

$$t^3 - 8.805367558025701520178324090 12\, t^2 + 25.284431326408943957394024078 6\, t - 23.657456953999138640057476778 8$$

$$t^4 - 11.809787425414533218265628169 2\, t^3 + 51.479726353980967492973037262 0\, t^2 - 98.117197979935867781770412996 8\, t + 68.970353905986748956278046896 0 \tag{3}$$

Lagrange basis functions for points indexed by i from 1 to N

```
> L :=  proc( N::integer, i::integer, v::Vector, x)
          description "Lagrange function";
  local j,T;
  T:=1:
  for j from 1 to N do
  if (j<>i) then T:= T*(x-v[j])/(v[i]-v[j])
  fi;
  od;
  T;
  end proc;
```

$$L := \mathbf{proc}(N::integer, i::integer, v::Vector, x) \tag{4}$$

$$\mathbf{local}\ j, T;$$

$$\mathbf{description}\ \text{"Lagrange function"};$$

$$T := 1;\ \mathbf{for}\ j\ \mathbf{to}\ N\ \mathbf{do}\ \mathbf{if}\ j <> i\ \mathbf{then}\ T := T * (x - v[j]) / (v[i] - v[j])\ \mathbf{end\ if}\ \mathbf{end\ do};\ T$$

$$\mathbf{end\ proc}$$

Computation of nodes x and weights

```
> for N from 1 to n do
  if (N=n) then print(n): fi:
  tempX:= Vector(N):
  x := Vector(N):
  y:=Vector(N):
  weights:=Vector(N):
  tempX:= fsolve(p||N(t),t):
  if (N=n) then
  print('nodes'):  fi:
  for j from 1 to N do
  x[j] :=tempX[j]:
  if (N=n) then print(x[j]): fi:
  od:
  if (N=n) then print('weights'): fi:
  for j from 1 to N do
  weights[j]:=int(L(N,j,x,t)*w(t),t=a..b):
  if (N=n) then print(weights[j]): fi:
  od:
  od:
```

$$4$$

nodes

$$2.12727598126564351109771428852$$
$$2.60696420385785998726448593007$$
$$3.25636956006789405707653299728$$
$$3.81917768022313566282689495332$$

weights

$$0.33378494875355245681468943931 6$$
$$0.55693611840198708819123269436 4$$
$$0.41259061298729438758158487991 2$$
$$0.13444359143631083751403831621 7$$

(5)

```
> d:='d':
  N:= 'N':
  for d from 0 to 2*n-1
  do
  ww:=0:
  for N from 1 to n do
  ww:= ww+x[N]^d*weights[N]:
  od:
  print(d,evalf(int(t^d*w(t),t=a..b)-ww)): od:
```

$$0, 0.$$
$$1, 0.$$
$$2, 0.$$
$$3, 0.$$
$$4, -2.\ 10^{-27}$$
$$5, -2.9\ 10^{-26}$$
$$6, -2.7\ 10^{-25}$$
$$7, -1.93\ 10^{-24}$$

(6)