```
> restart:
```

Problem 1

```
> tru := 100+0.495+0.495;
```

$$tru := 100.990 \quad (1)$$

```
> Digits:=3;
```

$$Digits := 3 \quad (2)$$

```
> a:= evalf(evalf(100+0.495)+0.495);
  b:=  evalf(100+evalf(0.495+0.495));
```

$$a := 100.$$

$$b := 101. \quad (3)$$

```
> Digits:=7;
```

$$Digits := 7 \quad (4)$$

```
> abs(a-tru);
  err_a := abs(a-tru)/tru;
```

$$0.990$$

$$err\_a := 0.009802951 \quad (5)$$

```
> abs(b-tru);
  err_b := abs(b-tru)/tru;
```

$$0.010$$

$$err\_b := 0.00009901970 \quad (6)$$

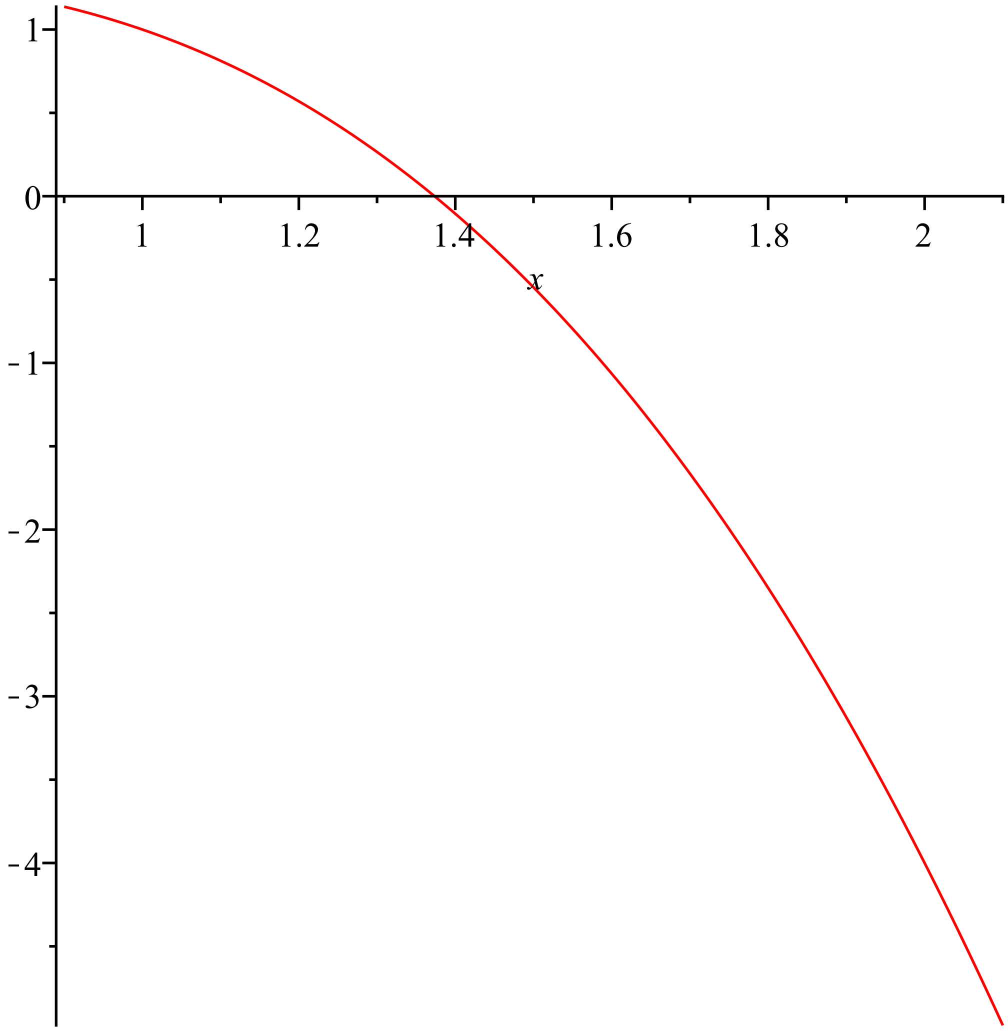
```
> restart:
```

Problem 2

```
> f:= x-> 2^x-x^3;
```

$$f := x \rightarrow 2^x - x^3 \quad (7)$$

```
> plot(f(x),x=0.9..2.1);
```



bisection applies because f(1) > 0 and f(2) < 0

```
> a:=1.0;
  b:=2.0;
  for j from 1 to 10 do
  c:= (a+b)/2:
  print(j,c):
  if (f(c)*f(a) < 0) then
   b:=c: else
  a:=c:
  end if:
  end do:
```

$a := 1.0$

$b := 2.0$

1, 1.500000000

2, 1.250000000

3, 1.375000000

4, 1.312500000

5, 1.343750000

6, 1.359375000

7, 1.367187500

8, 1.371093750

9, 1.373046875

10, 1.374023438 **(8)**

Secant

```
> N:= 7;#number of times we iterate (tried a few until stabilized
  v:=1.0;
  w:=2.0;
> for j from 1 to N do
  J:=(f(w)-f(v))/(w-v);
  x:= evalf(w - f(w)/J):
  s||j:=x;#sets s_j equal to x
  print(x):
  v:=w;
  w:=x;
  od:
```

$N := 7$

$v := 1.0$

$w := 2.0$

1.200000000

1.299688733

1.387296865

1.372505659

1.373455182

1.373467130

1.373467120 **(9)**

Newton-Raphson

```
> N:=5;#number of times we iterate
```

$N := 5$ **(10)**

```
> Newton := proc(f,v)
  local J,x;
  J:=unapply(diff(f(x),x),x);
  v - f(v)/J(v);
  end proc;
> x:=2:
> for j from 1 to N do
> x:= evalf(Newton(f,x)):
  n||j:=x: #sets n_j = x
> print(x):
  od:
```

$Newton := \mathbf{proc}(f, v)\ \ \mathbf{local}\ J, x;\ J := unapply(diff(f(x), x), x);\ v - f(v)/J(v)\ \ \mathbf{end\ proc}$

1.566508972

1.400321218

1.374097673

1.373467479

1.373467120 **(11)**

Problem 3

```
> restart:
> with(CurveFitting):
  with(LinearAlgebra):
> f:= x-> ln(x);
```

$$f := x \rightarrow \ln(x) \quad \textbf{(12)}$$

```
> ln(evalf(exp(1)));#just check ln(x) is the natural log
```

$$0.9999999998 \quad \textbf{(13)}$$

f[1] = 0 the rest are given by

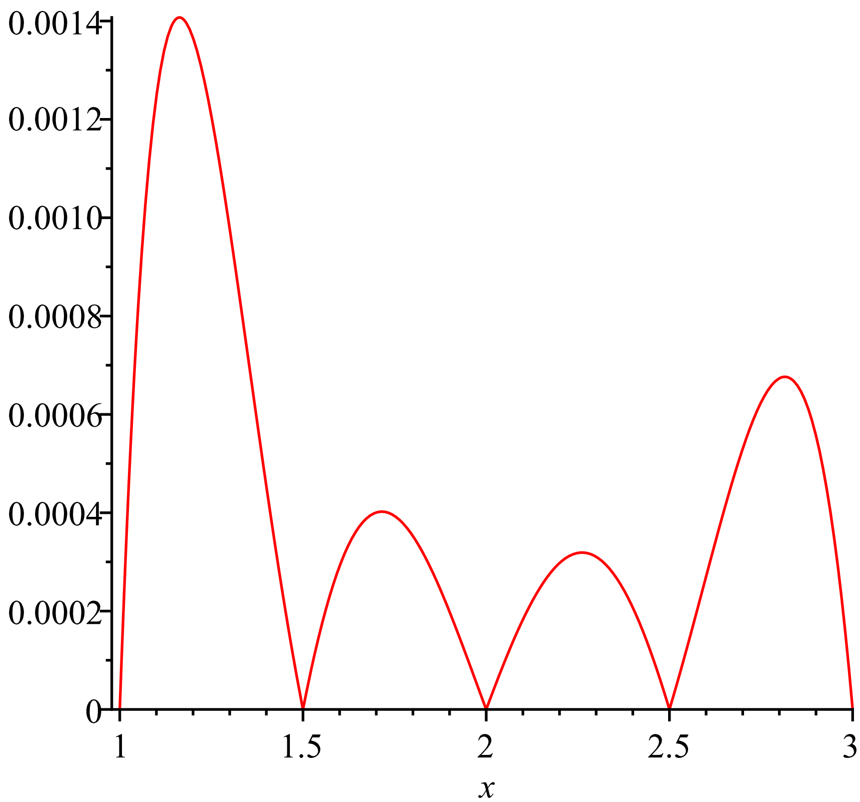
```
> for j from 1 to 4 do
  t:= Vector(j+1):
  y:=Vector(j+1):
  for k from 0 to j do
  t[k+1]:= 1.0+0.5*k:
  y[k+1]:= f(t[k+1]):
  od:
  a||j:=coeff(PolynomialInterpolation(t,y,x),x^j):
  print(a||j):
  od:
```

$$0.8109302162$$
$$-0.2355660712$$
$$0.07099268585$$
$$-0.01968532467 \quad \textbf{(14)}$$

```
> p4:= x -> 0+a||1*(x-1)+a||2*(x-1)*(x-1.5)+a||3*(x-1)*(x-1.5)*(x-2.0)+a||4*
  (x-1)*(x-1.5)*(x-2.0)*(x-2.5):
> p4(1.2);
  RelativeError:=abs(p4(1.2)-ln(1.2))/ln(1.2);
```

$$0.1809560207$$
$$RelativeError := 0.007489712813 \quad \textbf{(15)}$$

```
> plot(abs(p4(x)-ln(x)),x=1..3);
```



Problem 4

```
> restart:
  with(LinearAlgebra):
  Digits := 30;
  a := 1.0;b:=3.0;
  w := t -> ln(t);
  IP := proc( f,g ) evalf(int(f(t)*g(t)*w(t),t=a..b)) end;
```

$$Digits := 30$$

$$a := 1.0$$

$$b := 3.0$$

$$w := t \rightarrow \ln(t)$$

$$IP := \mathbf{proc}(f, g)\ \ evalf(int(f(t) * g(t) * w(t), t = a..b))\ \ \mathbf{end\ proc} \qquad (16)$$

Note the inner product IP above

```
> n:=5;
```

$$n := 5 \qquad (17)$$

```
> p0 := unapply(1,t):
> j:='j':k:='k':
> for j from 0 to n-1 do;
> vv := t*p||j(t):
  g:= unapply(vv,t):
> for k from 0 to j do;
  if (k >= j-2) then vv := vv-IP(g,p||k)/IP(p||k,p||k)*p||k(t): fi:
> od:
  p||(j+1):=unapply(expand(vv),t):
> od:
> for j from 0 to n do p||j(t); od;
```

$$1$$

$$t - 2.27170207626787741387082136094$$

$$t^2 - 4.33078395476124522538960620905\ t + 4.43739999795422255887701379205$$

$$t^3 - 6.35728315695600714063796954037\ t^2 + 12.9707118096786237930320751284\ t - 8.44032211211912173640808192465$$

$$t^4 - 8.37230065866675388219097818328\ t^3 + 25.5348164386704048002040158703\ t^2 - 33.5114542115802242357842146432\ t + 15.9160972517194271142177495097$$

$$t^5 - 10.3819654944929746671134112487\ t^4 + 42.1129442522976810523543105864\ t^3 - 83.2551459788001362012342145788\ t^2 + 80.0540585999125994563774862537\ t - 29.8979661798135917298729920718 \qquad (18)$$

Lagrange basis functions for points indexed by i from 1 to N

```
> L :=  proc( N::integer, i::integer, v::Vector, x)
          description "Lagrange function";
  local j,T;
  T:=1:
  for j from 1 to N do
  if (j<>i) then T:= T*(x-v[j])/(v[i]-v[j])
  fi;
  od;
  T;
  end proc;
```

$L := \mathbf{proc}(N::integer, i::integer, v::Vector, x)$ **(19)**

$\quad \mathbf{local}\ j, T;$

$\quad$ **description** "Lagrange function";

$\quad T := 1;\ \mathbf{for}\ j\ \mathbf{to}\ N\ \mathbf{do}\ \mathbf{if}\ j <> i\ \mathbf{then}\ T := T * (x - v[j]) / (v[i] - v[j])\ \mathbf{end\ if}\ \mathbf{end\ do};\ T$

end proc

Computation of nodes x and weights

```
> for N from 1 to n do
tempX:= Vector(N):
x := Vector(N):
y:=Vector(N):
weights:=Vector(N):
tempX:= fsolve(p||N(t),t):
if (N=5) then
print('nodes'):  fi:
for j from 1 to N do
x[j] :=tempX[j]:
if (N=5) then print(x[j]): fi:
od:
if (N=5) then print('weights'): fi:
for j from 1 to N do
weights[j]:=int(L(N,j,x,t)*w(t),t=a..b):
if (N=5) then print(weights[j]): fi:
od:
od:
```

nodes

1.186855442705112309988785220 85

1.5869722348450547291511226919 3

2.1008170108954401667303456114 9

2.5901229997014259220548863974 4

2.9171978063459415391882713270 0

weights

0.05237447431194211217916148191 06

0.22065738505987014824042156164 6

0.39055478716761758025963505320 5

0.40805915201889542598898627012 7

0.22419106744600380751753134387 9 **(20)**

```
> d:='d':
N:= 'N':
for d from 0 to 9
do
ww:=0:
for N from 1 to 5 do
ww:= ww+x[N]^d*weights[N]:
od:
print(d,evalf(int(t^d*w(t),t=a..b)-ww)): od:
```

0, 0.

1, $1.\ 10^{-29}$

2, 0.

3, 0.

4, $1.\ 10^{-28}$

5, $1.\ 10^{-27}$

6, $1.0\ 10^{-26}$

7, $7.1\ 10^{-26}$

8, $4.2\ 10^{-25}$

9, $2.08\ 10^{-24}$ **(21)**

Problem 5

```
> restart:
> with(student);
```

[D, *Diff*, *Doubleint*, *Int*, *Limit*, *Lineint*, *Product*, *Sum*, *Tripleint*, *changevar*, *completesquare*, *distance*, *equate*, *integrand*, *intercept*, *intparts*, *leftbox*, *leftsum*, *makeproc*, *middlebox*, *middlesum*, *midpoint*, *powsubs*, *rightbox*, *rightsum*, *showtangent*, *simpson*, *slope*, *summand*, *trapezoid*] **(22)**

```
> f := x -> sqrt(2-cos(x)^3);
  a:=0;
  b:=1;
  Digits:=40;
  truInt:=evalf(int(f(x),x=a..b));
```

$$f := x \rightarrow \sqrt{2 - \cos(x)^3}$$

$$a := 0$$

$$b := 1$$

$$Digits := 40$$

$$truInt := 1.159024559177677199290961267950013527915 \quad \textbf{(23)}$$

```
> for j from 3 to 5 do
  S||j:=evalf(simpson(f(x),x=a..b,2*2^j));
  RelError||j:= (truInt-S||j)/truInt;
  od;
```

$$S3 := 1.159024546770964334441409896048931417527$$

$$RelError3 := 1.070444346205403904439170665056232788845\ 10^{-8}$$

$$S4 := 1.159024558395648453168756694698770553113$$

$$RelError4 := 6.747300908593779162793502816488257942570\ 10^{-10}$$

$$S5 := 1.159024559128696810113621147904107511718$$

$$RelError5 := 4.226000975517851806848313327213486967694\ 10^{-11} \quad \textbf{(24)}$$

Note the 63.8 -- truely an h^6 method

```
> RelError3/RelError4;
  RelError4/RelError5;
```

$$15.86477853451029392385759144937264014640$$

$$15.96616031960799220071078795909647821259 \quad \textbf{(25)}$$

```
> RichExtrap34:=(16*S4-S3)/15;
  RichExtrap45:=(16*S5-S4)/15;
```

$$RichExtrap34 := 1.159024559170627394417246481275426495486$$

$$RichExtrap45 := 1.159024559177566700576612111451129975625 \quad \textbf{(26)}$$

```
> RichardsonRelError34:= (truInt-RichExtrap34)/truInt;
  RichardsonRelError45:= (truInt-RichExtrap45)/truInt;
```

$$RichardsonRelError34 := 6.082532779733841071859857054074644618737\ 10^{-12}$$

$$RichardsonRelError45 := 9.533768156522485442532338035547470493136\ 10^{-14} \quad \textbf{(27)}$$

```
> RichardsonRelError34/RichardsonRelError45;
```

$$63.79988143064401134186600763903724338024 \quad \textbf{(28)}$$

Problem 6

```
> restart:
> with(CurveFitting);
```

[*ArrayInterpolation*, *BSpline*, *BSplineCurve*, *Interactive*, *LeastSquares*, *PolynomialInterpolation*, *RationalInterpolation*, *Spline*, *ThieleInterpolation*] **(29)**

```
> f:= (t,x) -> max((x-t),0)^2;
```

$$f := (t, x) \rightarrow \max(x - t, 0)^2 \quad \textbf{(30)}$$

```
> f0:= unapply(f(0,x),x);
  f1:= unapply(f(0.5,x),x);
  f2:= unapply(f(0.5,x),x);
  f3:= unapply(f(1.0,x),x);
```

$$f0 := x \rightarrow \max(0, x)^2$$

$$f1 := x \rightarrow \max(0, x - 0.5)^2$$

$$f2 := x \rightarrow \max(0, x - 0.5)^2$$

$$f3 := x \rightarrow \max(0, x - 1.0)^2 \quad \textbf{(31)}$$

Note diff(f(t,x),t)= -2*max((x-t),0)

```
> f01:= unapply((f1(x)-f0(x))/0.5,x);
  f23:= unapply((f3(x)-f2(x))/0.5,x);
  f12:= unapply(-2*max((x-0.5),0),x);
```

$$f01 := x \rightarrow 2.000000000 \max(0, x - 0.5)^2 - 2.000000000 \max(0, x)^2$$

$$f23 := x \rightarrow 2.000000000 \max(0, x - 1.0)^2 - 2.000000000 \max(0, x - 0.5)^2$$

$$f12 := x \rightarrow -2 \max(0, x - 0.5) \quad \textbf{(32)}$$

```
> f012:= unapply((f12(x)-f01(x))/0.5,x);
  f123:= unapply((f23(x)-f12(x))/0.5,x);
  f0123:= unapply(f123(x)-f012(x),x);
```

$$f012 := x \rightarrow -4.000000000 \max(0, x - 0.5) - 4.000000000 \max(0, x - 0.5)^2 + 4.000000000 \max(0, x)^2$$

$$f123 := x \rightarrow 4.000000000 \max(0, x - 1.0)^2 - 4.000000000 \max(0, x - 0.5)^2 + 4.000000000 \max(0, x - 0.5)$$

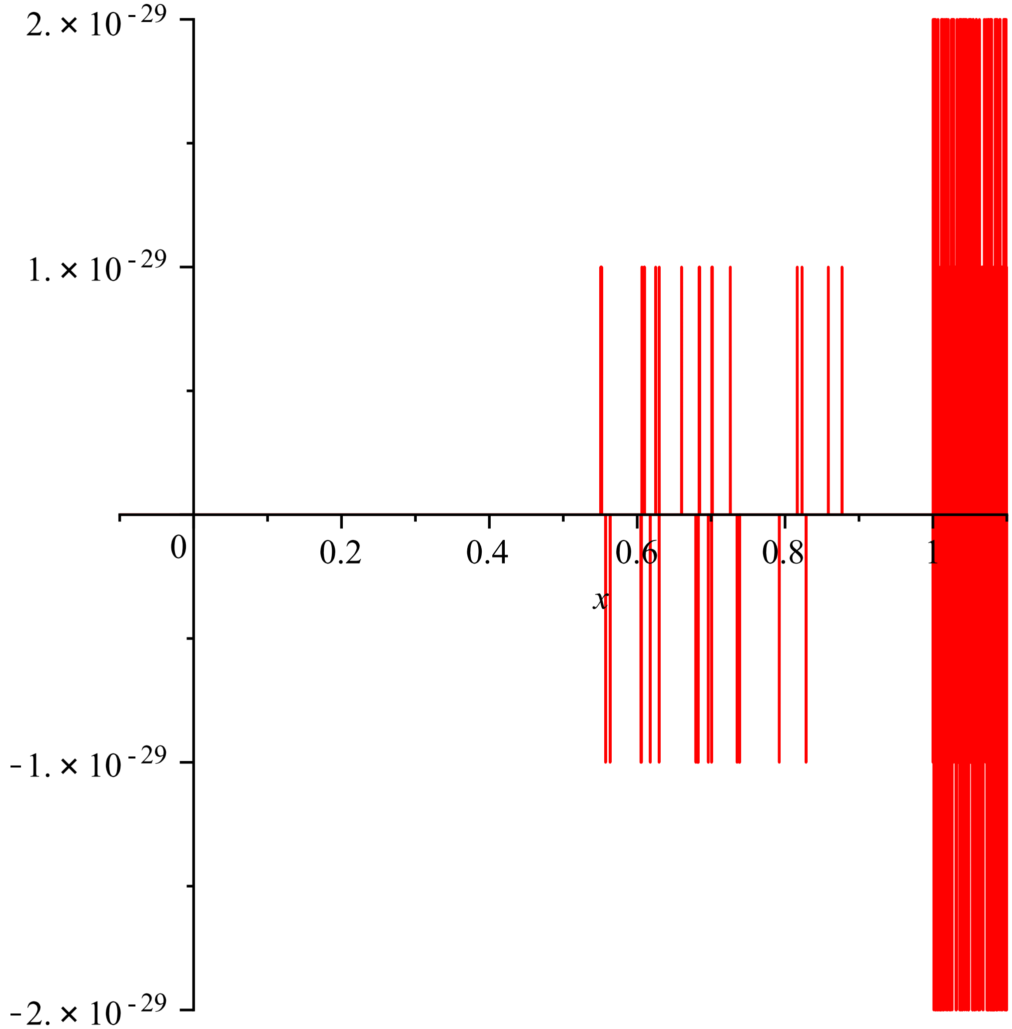
$$f0123 := x \rightarrow 4.000000000 \max(0, x - 1.0)^2 + 8.000000000 \max(0, x - 0.5) - 4.000000000 \max(0, x)^2 \quad \textbf{(33)}$$

So our version of B_{0,3}(x)=(-1)^3*f0123

```
> plot(-f0123(x),x=-0.1..1.1);
> Digits:=30;
  kn:= [0,0.5,0.5,1];
  plot(BSpline(3,x,knots=kn)+f0123(x),x=-0.1..1.1);
```

$$Digits := 30$$

$$kn := [0, 0.5, 0.5, 1]$$



```
> restart:
```

Problem 7

```
> f:= (t,y)-> 3/t*y+t^3*exp(t);
```

$$f := (t, y) \rightarrow \frac{3\,y}{t} + t^3\,e^t \quad (34)$$

```
> g:= t-> t^3*(exp(t)-exp(1.0));
```

$$g := t \rightarrow t^3\,\left(e^t - e^{1.0}\right) \quad (35)$$

```
> diff(g(t),t)-f(t,g(t));
```

$$0 \quad (36)$$

```
> h:=0.05;
  w:=0:
  for j from 1 to 20 do
  w:= w+ h*f(1+(j-1)*h,w):
  od;
```

$h := 0.05$

$w := 0.1359140914$

$w := 0.3207348089$

$w := 0.5643986227$

$w := 0.8781766665$

$w := 1.274806852$

$w := 1.768637479$

$w := 2.375783273$

$w := 3.114294859$

$w := 4.004342744$

$w := 5.068416971$

$w := 6.331543699$

$w := 7.821520036$

$w := 9.569168580$

$w := 11.60861321$

$w := 13.97757779$

$w := 16.71770957$

$w := 19.87492924$

$w := 23.49980960$

$w := 27.64798522$

$w := 32.38059531$ (37)

```
> abs(w-g(2.0))/g(2.0);
```

$$0.1334253855 \quad (38)$$

```
> h:=0.025;
  w:=0:
  for j from 1 to 40 do
  w:= w+ h*f(1+(j-1)*h,w):
  od;
```

$h := 0.025$

$w := 0.06795704570$

$w := 0.1479644368$

$w := 0.2412355345$

$w := 0.3490639463$

$w := 0.4728273853$

$w := 0.6139916955$

$$w := 0.7741150482$$
$$w := 0.9548523169$$
$$w := 1.157959638$$
$$w := 1.385299163$$
$$w := 1.638844015$$
$$w := 1.920683448$$
$$w := 2.233028228$$
$$w := 2.578216236$$
$$w := 2.958718305$$
$$w := 3.377144303$$
$$w := 3.836249466$$
$$w := 4.338940992$$
$$w := 4.888284912$$
$$w := 5.487513239$$
$$w := 6.140031416$$
$$w := 6.849426066$$
$$w := 7.619473059$$
$$w := 8.454145910$$
$$w := 9.357624520$$
$$w := 10.33430427$$
$$w := 11.38880548$$
$$w := 12.52598326$$
$$w := 13.75093776$$
$$w := 15.06902480$$
$$w := 16.48586699$$
$$w := 18.00736523$$
$$w := 19.63971072$$
$$w := 21.38939740$$
$$w := 23.26323494$$
$$w := 25.26836222$$
$$w := 27.41226132$$
$$w := 29.70277212$$
$$w := 32.14810743$$
$$w := 34.75686872 \quad \textbf{(39)}$$

```
> abs(w-g(2.0))/g(2.0);
```

$$0.06983118051 \quad \textbf{(40)}$$