# ACMS 690: Numerical Analysis I

Instructor: Andrew Sommese

Class Website: `www.nd.edu/~sommese/ACMS12F690`

This course will give a solid introduction to numerical analysis with careful attention to error estimates and estimation of errors in algorithms. Topics covered will include:

1. divided differences and polynomial interpolation (including generalized Hermite interpolation); two dimensional interpolation; splines; trigonometric interpolation
2. least squares and the basic theory of orthogonal functions
3. numerical integration in one variable, including adaptive methods; Romberg integration; Gauss quadrature and the relations to orthogonal functions and eigenvalues of tridiagonal matrices
4. numerical linear algebra
   (a) direct methods and the analysis of error based on the condition number;
   (b) basic numerical factorizations of matrices, e.g., LU, QR, and singular value decompositions;
   (c) methods to find eigenvalues and eigenvectors such as the QR Method, power method, and inverse power method; and
   (d) iterative methods and Krylov space methods.
5. methods to solve systems of nonlinear equations, e.g., Newton like methods and constrained Newton's methods such as homotopy continuation;
6. numerical solution of ordinary differential equations by marching methods, multistep methods, shooting methods, finite differences, and the finite element method (including such variants as the Galerkin and Rayleigh-Ritz method); and

7. as time permits: solution of simple partial differential equations on the square and disk in $\mathbb{R}^2$ by difference methods; and by the finite element method.

In addition we will point out common general techniques that occur over and over in the course, e.g., Richardson extrapolation shows up in numerical integration (Romberg integration) and in extrapolating high accuracy solutions to differential equations from not so accurate difference scheme solutions.

Some useful books on the material covered are listed below. The books [4, 5] will be main references for the course.

## References


[1] P.J. Davis, Interpolation and approximation. Dover Publications, Inc., New York, 1975.

[2] E. Isaacson and H.B. Keller, *Analysis of Numerical Methods*, Dover Publications, Inc., New York, 1994.

[3] C. Johnson, *Numerical Solution of Partial Differential Equations by the Finite Element Method*, Cambridge University Press, 1987.

[4] J. Stoer and R. Bulirsch, *Introduction to Numerical Analysis*, Springer, 1993.

[5] L.N. Trefethen and D. Bau, *Numerical Linear Algebra*, Society of Industrial and Applied Mathematics, 1997.

[6] E.E. Tyrtyshnikov, *A Brief Introduction to Numerical Analysis*, Birkhäuser, 1997.