Sociology 63993
Exam 2 Answer Key
April 1, 2011

I. True-False. (20 points) Indicate whether the following statements are true or false. If false, briefly explain why.

1. A researcher computes a variable $X_4 = X_2 + X_3$. She then estimates the following two models using OLS regression:

   $Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon$

   $Y = \beta_1 X_1 + \beta_4 X_4 + \varepsilon$

   She can use an incremental F test to determine which of these two models is better.

True. Since $X_4 = X_2 + X_3$, the incremental F is a test of whether or not $\beta_2 = \beta_3$.

2. A researcher runs the following:

```
. webuse nhanes2f, clear
. gen femage = female * age
. reg health female age femage

      Source |       SS       df       MS              Number of obs =   10335
-------------+------------------------------           F(  3, 10331) =  549.60
       Model |  2069.28161     3  689.760537           Prob > F      =  0.0000
    Residual |  12965.7398 10331   1.2550324           R-squared     =  0.1376
-------------+------------------------------           Adj R-squared =  0.1374
       Total |  15035.0214 10334   1.4549082           Root MSE      =  1.1203

------------------------------------------------------------------------------
      health |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |  -.2752255   .0648373    -4.24   0.000    -.4023191   -.1481319
         age |  -.0280887   .0009315   -30.15   0.000    -.0299146   -.0262627
      femage |   .0043295   .0012822     3.38   0.001     .0018162    .0068428
       _cons |    4.78594   .0469616   101.91   0.000     4.693886    4.877994
------------------------------------------------------------------------------
```

These results show that age has a negative effect on the health of males and a positive effect on the health of females.

False. The effect of age is less negative for females (-.0280887 + .0043295 = -.0237592) but it is still negative.

3. A researcher has included several extraneous variables in her model. The larger her sample, the more serious this problem will be.

False. Adding extraneous variables increases standard errors. Larger sample sizes decrease standard errors.

4. A researcher regresses income on education. She does not include any dummy variables or interaction terms involving gender. One implication of this model is that, if it is true, the mean income for men will be the same as the mean income for women.

False. If men and women differ in their mean levels of education, they will also differ in their mean incomes.

5. A researcher is interested in the relationship between bmi (Body Mass Index) and health. She does the following:

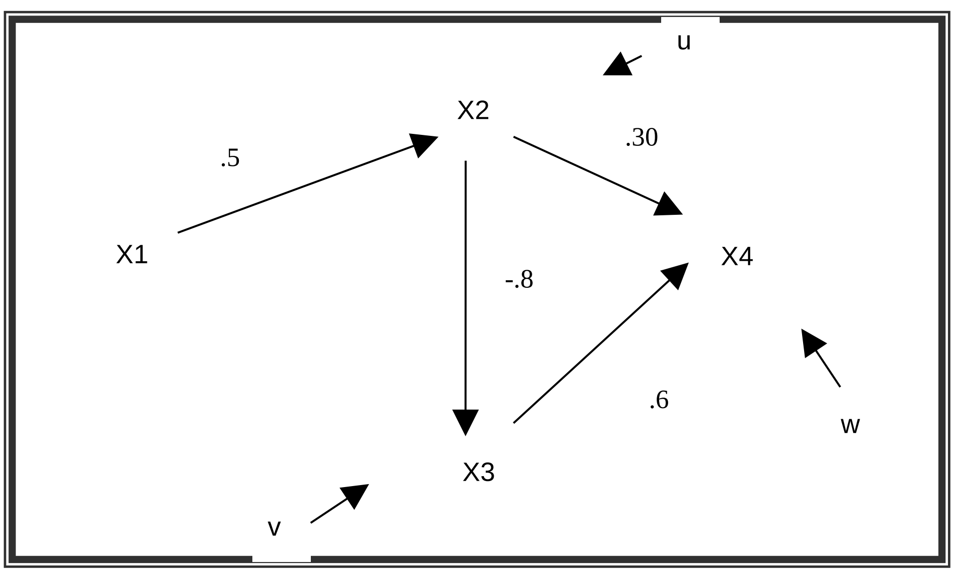
```
. webuse nhanes2f, clear
. gen bmi = weight/ (height/100)^2
. gen bmi2 = bmi * bmi
. reg health bmi bmi2

      Source |       SS       df       MS              Number of obs =   10335
-------------+------------------------------           F(  2, 10332) =  111.24
       Model |  316.928298     2  158.464149           Prob > F      =  0.0000
    Residual |  14718.0931 10332   1.4245154           R-squared     =  0.0211
-------------+------------------------------           Adj R-squared =  0.0209
       Total |  15035.0214 10334   1.4549082           Root MSE      =  1.1935

------------------------------------------------------------------------------
      health |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         bmi |   .0072049   .0152456     0.47   0.637    -.0226794    .0370892
        bmi2 |  -.0007416   .0002646    -2.80   0.005    -.0012601    -.000223
       _cons |   3.731409   .2147848    17.37   0.000      3.31039    4.152429
------------------------------------------------------------------------------
```

Based on these results, she should conclude that bmi is not related to health.

False. The results indicate that there is a curvilinear relationship between bmi and health. Increases in body mass index are good up to point, but after that further increases are harmful. (In other words, it isn't good to be obese.)

II. Path Analysis/Model specification (25 pts).

A sociologist believes that the following model describes the relationship between X1, X2, X3, and X4. All her variables are in standardized form. The estimated value of each path in her model is included in the diagram.



a. (5 pts) Write out the structural equation for each endogenous variable, using both the names for the paths (e.g. β_{42}) and the estimated value of the path coefficient.

$$X_2 = \beta_{21} X_1 + u = .5X_1 + u$$
$$X_3 = \beta_{32} X_2 + v = -.8X_2 + v$$
$$X_4 = \beta_{42} X_2 + \beta_{43} X_3 + w = .3X2 + .6X3 + w$$

b. (10 pts) Part of the correlation matrix is shown below. Determine the complete correlation matrix. (Remember, variables are standardized. You can use either normal equations or Sewell Wright, but you might want to use both as a double-check.)

```
             |       x1       x2       x3       x4
-------------+------------------------------------
          x1 |   1.0000
          x2 |   0.5000   1.0000
          x3 |     ?        ?      1.0000
          x4 |     ?        ?        ?      1.0000
```

Here is the uncensored output:

```
             |       x1       x2       x3       x4
-------------+------------------------------------
          x1 |   1.0000
          x2 |   0.5000   1.0000
          x3 |  -0.4000  -0.8000   1.0000
          x4 |  -0.0900  -0.1800   0.3600   1.0000
```

To confirm that this reproduces the estimated path coefficients:

```
. pathreg (x2 x1) (x3 x2 x1) (x4 x3 x2 x1)

------------------------------------------------------------------------------
          x2 |      Coef.   Std. Err.      t    P>|t|                     Beta
-------------+----------------------------------------------------------------
          x1 |         .5   .0874818     5.72   0.000                       .5
       _cons |   8.90e-09   .0870433     0.00   1.000                        .
------------------------------------------------------------------------------
                 n = 100  R2 = 0.2500  sqrt(1 - R2) = 0.8660

------------------------------------------------------------------------------
          x3 |      Coef.   Std. Err.      t    P>|t|                     Beta
-------------+----------------------------------------------------------------
          x2 |        -.8   .0703452   -11.37   0.000                      -.8
          x1 |   3.08e-09   .0703452     0.00   1.000                 3.08e-09
       _cons |   3.66e-09   .0606154     0.00   1.000                        .
------------------------------------------------------------------------------
                 n = 100  R2 = 0.6400  sqrt(1 - R2) = 0.6000

------------------------------------------------------------------------------
          x4 |      Coef.   Std. Err.      t    P>|t|                     Beta
-------------+----------------------------------------------------------------
          x3 |         .6   .1557167     3.85   0.000                       .6
          x2 |         .3    .164795     1.82   0.072                       .3
          x1 |  -8.07e-09   .1078837    -0.00   1.000                -8.07e-09
       _cons |  -7.66e-09   .0929617    -0.00   1.000                        .
------------------------------------------------------------------------------
                 n = 100  R2 = 0.1620  sqrt(1 - R2) = 0.9154
```

c. (5 pts) Decompose the correlation between X3 and X4 into

- Correlation due to direct effects

  .6

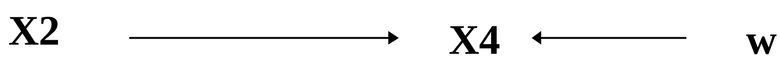
- Correlation due to indirect effects

  0

- Correlation due to common causes

  -.24

d. (5 pts) Suppose the above model is correct, but instead the researcher believed in and estimated the following model:



What conclusions would the researcher likely draw? In particular, what would the researcher conclude about the effect of changes in X2 on X4? Discuss the consequences of this mis-specification, and in what ways, if any, the results would be misleading. Why would she make these mistakes?

In the correctly specified model the direct effect is .3, but in the incorrectly specified model the estimated direct effect is -.18 (the same as the correlation between the variables). This is because the direct effect of X2 on X4 (.3) gets confounded with its indirect effect (X2 affects X3 which in turn affects X4, which adds -.48 to the X2-X4 correlation). How serious a mistake this is depends on the situation. On the one hand, the total effect (direct + indirect) of X2 on X4 really is -.18. So, the predicted change in X4 produced by a change in X2 is correct, even if the model incorrectly explains why that change occurs. But on the other hand, by failing to separate the direct and indirect effects, the researchers may miss the opportunity to make changes in the system, e.g. maybe some sort of change could be made that would make the negative indirect effect of X2 on X4 go away, leaving only the positive direct effect.

III. Group comparisons (25 points). This week, the Supreme Court heard a landmark gender discrimination case against retail giant Wal-Mart. The plaintiffs based their case, in part, on work done by Sociologist William Bielby. Bielby's devastating arguments have put the fear of God into another company making it wonder if it, too, might face such a lawsuit. It has therefore conducted its own study of gender equity within its work force, collecting data from a random sample of 7500 of its employees on the following variables:

Variable	Description
pay	Annual Salary (in thousands of dollars)
qual	A qualifications scale that the company has constructed and believes to be very valid. It takes into account such things as past performance, aptitude test scores, education, and years of experience. The scale ranges from -40 to 40 and has been centered to have a mean of 0 (i.e. 0 means average qualifications; and the higher the score, the more qualified the person is)
female	Coded 1 if female, 0 if male
femqual	female * qual

The results of the analysis are as follows:

```
. ttest pay, by(female)

Two-sample t test with equal variances
------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       0 |    3572     78.1415    .2298254    13.73579     77.6909     78.5921
       1 |    3928    47.23287    .2309217    14.47273    46.78013    47.68561
---------+--------------------------------------------------------------------
combined |    7500    61.95362     .241623    20.92516    61.47997    62.42727
---------+--------------------------------------------------------------------
    diff |            30.90863    .3266069                30.26839    31.54887
------------------------------------------------------------------------------
    diff = mean(0) - mean(1)                                      t =  94.6356
Ho: diff = 0                                     degrees of freedom =     7498

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 1.0000         Pr(|T| > |t|) = 0.0000          Pr(T > t) = 0.0000

. nestreg: reg pay qual female femqual

Block  1: qual

      Source |       SS       df       MS              Number of obs =    7500
-------------+------------------------------           F(  1,  7498) = 5952.86
       Model |  1453171.17     1  1453171.17           Prob > F      =  0.0000
    Residual |   1830359.2  7498  244.112991           R-squared     =  0.4426
-------------+------------------------------           Adj R-squared =  0.4425
       Total |  3283530.37  7499  437.862431           Root MSE      =  15.624

------------------------------------------------------------------------------
         pay |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        qual |   1.438618   .0186459    77.15   0.000     1.402067    1.475169
       _cons |   61.95362   .1804117   343.40   0.000     61.59996    62.30728
------------------------------------------------------------------------------
```

Block 2: female

```
      Source |       SS       df       MS              Number of obs =    7500
-------------+------------------------------           F(  2,  7497) = 5296.84
       Model |  1922795.12     2  961397.559           Prob > F      =  0.0000
    Residual |  1360735.25  7497  181.503969           R-squared     =  0.5856
-------------+------------------------------           Adj R-squared =  0.5855
       Total |  3283530.37  7499  437.862431           Root MSE      =  13.472

------------------------------------------------------------------------------
         pay |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        qual |   .6212315   .0227315    27.33   0.000     .5766715    .6657916
      female |  -22.40072   .4403823   -50.87   0.000    -23.26399   -21.53744
       _cons |   73.68562   .2782026   264.86   0.000     73.14027    74.23098
------------------------------------------------------------------------------
```

Block 3: femqual

```
      Source |       SS       df       MS              Number of obs =    7500
-------------+------------------------------           F(  3,  7496) = 3605.84
       Model |  1939531.43     3  646510.478           Prob > F      =  0.0000
    Residual |  1343998.94  7496  179.295483           R-squared     =  0.5907
-------------+------------------------------           Adj R-squared =  0.5905
       Total |  3283530.37  7499  437.862431           Root MSE      =   13.39

------------------------------------------------------------------------------
         pay |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        qual |   .8329084   .0314714    26.47   0.000     .7712156    .8946012
      female |   -22.3506   .4377256   -51.06   0.000    -23.20866   -21.49253
     femqual |  -.4367667   .0452069    -9.66   0.000    -.5253848   -.3481486
       _cons |   72.16734   .3180414   226.91   0.000     71.54389    72.79079
------------------------------------------------------------------------------


  +-------------------------------------------------------------+
  |       |          Block  Residual                     Change |
  | Block |       F     df        df   Pr > F       R2    in R2 |
  |-------+-----------------------------------------------------|
  |     1 | 5952.86      1      7498   0.0000   0.4426          |
  |     2 | 2587.40      1      7497   0.0000   0.5856   0.1430 |
  |     3 |   93.34      1      7496   0.0000   0.5907   0.0051 |
  +-------------------------------------------------------------+
```

. ttest qual, by(female)

```
Two-sample t test with equal variances
------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       0 |    3572    7.172654    .1191291    7.119893    6.939086    7.406222
       1 |    3928   -6.522586    .1050536    6.584106   -6.728551   -6.316621
---------+--------------------------------------------------------------------
combined |    7500    3.32e-08    .1117328    9.676342   -.2190275    .2190276
---------+--------------------------------------------------------------------
    diff |            13.69524    .1582456                13.38503    14.00545
------------------------------------------------------------------------------
    diff = mean(0) - mean(1)                                      t =  86.5442
Ho: diff = 0                                     degrees of freedom =     7498

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 1.0000         Pr(|T| > |t|) = 0.0000          Pr(T > t) = 0.0000
```

The initial t-test shows that men make substantially more than women. The company then does additional analyses to find out why. It wants your help in answering the following:

a) (15 pts) The researchers estimate a series of models. Which of the models do you think is best, and why? What do these models tell us about how qualifications and gender affect pay?

The third and final model provides the best fit. It says that both the intercepts and the slopes differ by gender. Because qual is centered, we know that the average woman makes $22,000 less than the average man, even after controlling for qualifications. Further, for women, qualifications have an effect that is less than half as large as it is for men (each qualification point is worth, on average, about $833 for men, but only about $396 for women).

b) (10 pts) Suppose the company was sued on the basis that it discriminated against women. What evidence, if any, do you think the company would cite in its defense? What evidence, if any, would its critics cite? Consider both the t-tests and the regression analyses in your answer. If you were the president of the company, would these results make you be worried about a lawsuit?

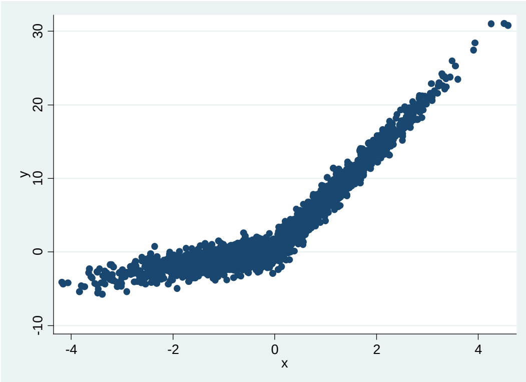
The company would no doubt note that, on average, women are less qualified than men (by about 13.7 points, as the last t-test shows). Critics will no doubt note the evidence raised in point A, namely that a woman with average qualifications earns $22,000 less than a similarly qualified man, and woman are only rewarded half as much for their qualifications as men are. If I were the president, I would be very worried about a lawsuit.

IV. Short answer. Answer *both* of the following questions. (15 points each, 30 points total.) In each of the following problems, a researcher runs through a sequence of commands. Explain why she didn't stop after the first command, i.e. explain what the purpose of each subsequent command was, what it told her, and why she did not run additional commands after the last one. If she had stopped after the first command, what would the consequences have been, i.e. in what ways would her conclusions have been incorrect or misleading?

1.

```
. reg y x

      Source |       SS       df       MS              Number of obs =    2293
-------------+------------------------------           F(  1,  2291) = 9754.77
       Model |  68744.4388     1  68744.4388           Prob > F      =  0.0000
    Residual |  16145.2885  2291  7.04726691           R-squared     =  0.8098
-------------+------------------------------           Adj R-squared =  0.8097
       Total |  84889.7273  2292  37.0374028           Root MSE      =  2.6547

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |    3.94874   .0399807    98.77   0.000     3.870337    4.027142
       _cons |   3.328859   .0554381    60.05   0.000     3.220145    3.437573
------------------------------------------------------------------------------
```

```
. scatter y x
```



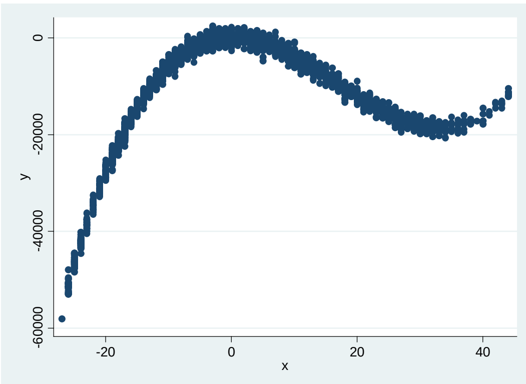

```
. mkspline xlow 0 xhigh = x

. reg y xlow xhigh

      Source |       SS       df       MS              Number of obs =    2293
-------------+------------------------------           F(  2,  2290) =41359.81
       Model |  82602.9569     2  41301.4785           Prob > F      =  0.0000
    Residual |  2286.77032  2290  .998589661           R-squared     =  0.9731
-------------+------------------------------           Adj R-squared =  0.9730
       Total |  84889.7273  2292  37.0374028           Root MSE      =  .99929

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        xlow |    1.02005    .029061    35.10   0.000     .9630619    1.077039
       xhigh |   6.933698   .0294706   235.28   0.000     6.875907     6.99149
       _cons |   .0479089   .0348016     1.38   0.169     -.020337    .1161549
------------------------------------------------------------------------------
```

The scatterplot strongly suggests that the effect of X is not the same across the range of X. In particular, the effect of X becomes much greater once X goes past 0. The mkspline computation and the subsequent regression shows that between -4 and 0, the slope of X is 1, and after that the slope of X is about 7. The R^2 is extremely high and the results are consistent with the scatterplot so the researcher probably thought it was ok to stop at that point. If the researcher had not done the 2nd regression, the researcher would have concluded that the effect of X was about 4 throughout its range, when in reality the effect of X is sometimes much less than that and sometimes much more.

2.

```
. reg y x

      Source |       SS       df       MS              Number of obs =    2293
-------------+------------------------------           F(  1,  2291) =  385.60
       Model |  4.7856e+10     1  4.7856e+10           Prob > F      =  0.0000
    Residual |  2.8433e+11  2291   124108295           R-squared     =  0.1441
-------------+------------------------------           Adj R-squared =  0.1437
       Total |  3.3219e+11  2292   144933916           Root MSE      =   11140

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |   272.3303   13.86839    19.64   0.000     245.1343    299.5262
       _cons |  -12109.01   232.6475   -52.05   0.000    -12565.23   -11652.79
------------------------------------------------------------------------------
```

```
. ovtest

Ramsey RESET test using powers of the fitted values of y
       Ho:  model has no omitted variables
                F(3, 2288) =  94189.71
                  Prob > F =      0.0000
. scatter y x
```




```
. gen x2 = x^2
. gen x3 = x^3
. reg y x x2 x3

      Source |       SS       df       MS              Number of obs =    2293
-------------+------------------------------           F(  3,  2289) =       .
       Model |  3.2990e+11     3  1.0997e+11           Prob > F      =  0.0000
    Residual |  2.2843e+09  2289  997934.435           R-squared     =  0.9931
-------------+------------------------------           Adj R-squared =  0.9931
       Total |  3.3219e+11  2292   144933916           Root MSE      =  998.97

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |   .3380926   2.407808     0.14   0.888    -4.383621    5.059806
          x2 |  -50.07069    .096471  -519.02   0.000    -50.25987   -49.88151
          x3 |   .9974231   .0039199   254.45   0.000     .9897362     1.00511
       _cons |   24.94746   30.93394     0.81   0.420    -35.71402    85.60894
------------------------------------------------------------------------------

. ovtest

Ramsey RESET test using powers of the fitted values of y
       Ho:  model has no omitted variables
                F(3, 2286) =      0.87
                  Prob > F =      0.4561
```

The ovtest command indicated that higher powers of X should be included in the model. The subsequent scatterplot indicated that there were two bends in the data, suggesting that X^2 and X^3 should be added to the model. The final ovtest indicated that no more higher powers were needed so the researcher stopped. If the researcher had not done the follow-up analyses she would have erroneously concluded that the effect of X was linear and positive when in fact the relationship is curvilinear.

Appendix: Stata Code

```
version 11.1

* I-2 - T/F
webuse nhanes2f, clear
gen femage = female * age
reg health female age femage

* I-5 - T/F
webuse nhanes2f, clear
gen bmi = weight/ (height/100)^2
gen bmi2 = bmi * bmi
reg health bmi bmi2

* II - Path Analysis
clear all
matrix input corr = (1,.5,-.4,-.09\.5,1,-.8,-.18\-.4,-.8,1,.36\-.09,-.18,.36,1)
corr2data x1 x2 x3 x4, n(100) corr(corr) double
corr
*** Double-check results
pathreg (x2 x1) (x3 x2 x1) (x4 x3 x2 x1)

* III - Interaction Effects, Group differences
*** Set up data
webuse nhanes2f, clear
set seed 123
sample 7500, count
gen pay = weight - 3*female - .1*female*height
sum height
gen qual = height - r(mean)
gen femqual = female * qual
*** Do analyses
ttest pay, by(female)
nestreg: reg pay qual female femqual
ttest qual, by(female)

* IV-1 - Nonlinear relationships
*** Set up data
use "http://www.indiana.edu/~jslsoc/stata/spex_data/ordwarm2.dta", clear
corr2data e1 e2
gen x = warm + e1
sum x
replace x = x - r(mean)
gen y = x if x <0
replace y = 7*x if x >0
replace y = y + e2
*** Do analyses
reg y x
scatter y x
mkspline xlow 0 xhigh = x
reg y xlow xhigh

* IV-2 - Nonlinear relationships
*** Set up data
use "http://www.indiana.edu/~jslsoc/stata/spex_data/ordwarm2.dta", clear
corr2data e, sd(1000)
sum age
gen x = age - r(mean)
gen y = x - (50 * x^2) + (x^3) + e
*** Do analyses
reg y x
scatter y x
gen x2 = x^2
gen x3 = x^3
reg y x x2 x3
ovtest
```