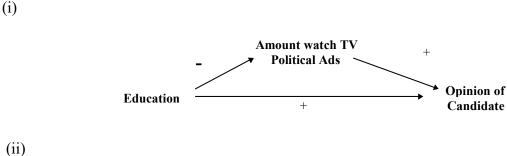
# Soc 63993, Homework #5 Answer Key: Model Mis-Specification/Equality Constraints/Group Comparisons

Richard Williams, University of Notre Dame, <a href="https://academicweb.nd.edu/~rwilliam/">https://academicweb.nd.edu/~rwilliam/</a> Last revised February 15, 2015

1. Model mis-specification. A campaign manager has found that the amount of time spent watching TV political ads is negatively correlated with favorable opinion of her candidate. Two models have been proposed to explain this relationship:





A. Suppose that model (i) is correct. What harm will result from estimating model (ii) and relying on the results? If appropriate, discuss such things as biased coefficients, inflated standard errors, and misguided policy decisions (particularly with regards to the use of TV advertising). Similarly, discuss the harm that will result if Model (ii) is correct and model (i) is mistakenly estimated and relied upon.

If model (ii) is estimated, the data will seem to support it even if Model (i) is correct. Model two predicts a negative effect of TV on opinion. Since the correlation between TV and opinion is negative, the bivariate regression coefficient will indeed be negative.

However, as the "true" model (i) shows, the effect of TV on opinion is actually positive. The negative correlation between TV and opinion arises from the fact that they share a common cause, Education. Better educated people are less likely to watch TV ads and more likely to like the candidate. Hence, those who watch TV ads tend to be disproportionately composed of the lesser-educated who are less likely to like the candidate. However, they would like the candidate even less if they didn't watch the TV ads. Put another way, suppressor effects are present.

From a policy standpoint, this could lead to a grave mistake. The campaign could mistakenly conclude that it should turn away from TV advertising, when that advertising is actually helpful.

Probably less harm is done if model (ii) is correct but model (i) is estimated instead. The expected effect of Education on Opinion is zero, and the expected effect of TV on Opinion is negative. That is, adding extraneous variables to the model does not bias coefficients. Hence, when model (i) is estimated, the campaign will hopefully discover that the data don't support it (whereas in the previous case the data did seem to support the model, even though it was wrong). However, adding extraneous variable does tend to increase standard errors and make estimates less precise. Hence, there is a greater risk that the campaign will conclude that TV does not have

a significant effect (i.e. it is "neutral") when in fact the TV ads are harmful. For that matter, because the estimates are less precise, there may even be a small chance that the estimated effect of TV winds up being positive, as Model (i) suggests.

Model (i) is estimated, yielding the following results. Based on this information, determine what the regression coefficient would be for model (ii). Compute the regression coefficient using both the formula for omitted variable bias and the formula for the slope coefficient in a bivariate regression.

	S11m
•	Suit

Variable	Obs	Mean	Std. Dev.	Min	Max
opinion		79	9.4	0,,102,1	99.79181
educ	200	14	2.7	6.597328	20.61872
tv	200	15	5.6	8872261	34.85936

# . corr

(obs=200)

	opinion	educ	tv
opinion educ	1.0000 0.3500	1.0000	
tv	-0.2200	-0.9000	1.0000

#### . corr, cov (obs=200)

		opinion	educ	tv
opinion		88.36		
educ		8.883	7.29	
tv	1	-11.5808	-13.608	31.36

### . reg opinion educ tv, beta

Source	SS	df	MS	Number of obs =	200
 +-				F(2, 197) =	20.17
Model	2989.21851	2	1494.60926	Prob > F =	0.0000
Residual	14594.421	197	74.0833553	R-squared =	0.1700
 				Adj R-squared =	0.1616
Total	17583.6395	199	88.3599975	Root MSE =	8.6072

opinion	Coef.	Std. Err.	t	P> t	Beta
tv	2.785185 .8392856 27.41812	.2499591	3.36	0.001	.8 .5

We are asked to compute the coefficients for the incorrectly-specified bivariate regression. I'll do this for both TV and Education as the IVs.

Opinion regressed on TV only

(Bonus) Opinion regressed on education only

$$b = \frac{s_{TV,Opinion}}{s_{TV}^2} = \frac{-11.581}{31.360} = -.369$$

$$b = \frac{s_{Educ,Opinion}}{s_{Educ}^2} = \frac{8.883}{7.29} = 1.219$$

$$b_{TV}^* = b_{TV} + b_{Educ} \frac{Cov(TV,EDUC)}{V(TV)}$$

$$= .839286 + 2.785185 \frac{-13.608}{31.360} = -.369$$

$$b_{Educ}^* = b_{Educ} + b_{TV} \frac{Cov(TV,EDUC)}{V(Educ)}$$

$$= 2.785185 + .839286 \frac{-13.608}{7.290} = 1.219$$

To confirm – note that we are given the means, correlations and standard deviations, so we can use the corr2data command to create a pseudo-replication of the data.

- . matrix input means =  $(79\14\15)$ . matrix input sds =  $(9.4\2.7\5.6)$
- . matrix input corr =  $(1,.35,-.22 \setminus .35,1,-.90 \setminus -.22,-.90,1)$
- . corr2data opinion educ tv, n(200) means(means) sds(sds) corr(corr)
- . reg opinion educ tv, beta

200 20.17	Number of obs = $F(2, 197) =$		MS	=	df	SS +	Source
0.0000 0.1700	Prob > F = R-squared =		94.6093 9833575	2 149 7 74.0	2 197	2989.2186   14594.4214	Model Residual
8.6072	Adj R-squared = Root MSE =					17583.64	
Beta			t				opinion
.8 .5		0.000	5.37 3.36	34337 99591	.518	2.785185 8392857	educ tv
		0.012	2.54	77459	10.7	27.41812	cons

### . reg opinion tv, beta

Source	SS	df			Number of obs = F( 1, 198) =	
Model   Residual	851.048099 16732.5919	1 851. 198 84	.048099		Prob > F = R-squared = Adj R-squared =	0.0017 0.0484
opinion				P> t		Beta
tv		.1163682	-3.17	0.002		22

#### . reg opinion educ, beta

Source		df	MS		Number of obs =	
Model Residual	15429.6443 +	1 21 198 77	53.99576		F( 1, 198) = Prob > F = R-squared = Adj R-squared = Root MSE =	0.0000 0.1225
opinion	•	Std. Err	. t	P> t		Beta
educ _cons	1.218518	.2317688	5.26	0.000		.35

C. Based on these results, which model do you think is most plausible? Why?

Model (i) gets a clear edge. All coefficients are in the predicted direction, and all effects are statistically significant.

D. The campaign manager is concerned by the large correlation between educ and tv. Suppose the manager decided to "solve" the problem of multicollinearity by excluding education from the model. What would be the consequence of that decision? Do you think this would be a good idea in this case?

It would be a terrible mistake if you decided to "solve" the problem of multicollinearity by excluding education from the model. As noted above, this serious mis-specification would lead to very erroneous conclusions concerning TV ads. Further, even with this high correlation, effects are statistically significant. Stick with Model (i).

Incidentally, keep in mind that omitted variable bias can cause the magnitude of the coefficients for the remaining variables to be inflated either upwards or downwards. In this case, omitting education would cause the effect of TV to go down so much that the estimated effect actually switches from being positive to negative. This is because there are suppressor effects present in this example: TV and Education both positively affect opinion, but they are negatively correlated with each other.

2. Equality constraints. From the course web page, download gender.dta. This is yet another modified version of our income/education/job experience example. The sample now consists of 225 men and 275 women. Regress income on education and job experience. Test the following hypotheses:

H<sub>0</sub>:  $\beta_{\text{Educ}} = \beta_{\text{Jobexp}}$ H<sub>A</sub>:  $\beta_{\text{Educ}} \neq \beta_{\text{Jobexp}}$ 

Perform a Wald test, an incremental F test, and a likelihood ratio chi-square test. The results should all be identical or nearly identical.

## (i) Wald test:

- . use https://academicweb.nd.edu/~rwilliam/xsoc63993/statafiles/gender.dta, clear
- . \* Unconstrained model
- . reg income educ jobexp

Source	SS	df 	MS		Number of obs F( 2, 497)	
Model   Residual    Total	22352.7545 23157.8824 45510.6369	497 46.5	76.3773 5953368 		Prob > F R-squared Adj R-squared Root MSE	= 0.0000 = 0.4912
income	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
educ   jobexp   _cons	1.309229 .8533107 -1.076636	.0838474 .0670888 1.205717	15.61 12.72 -0.89	0.000 0.000 0.372	1.14449 .7214982 -3.445568	1.473968 .9851233 1.292295

. test educ = jobexp

(1) educ - jobexp = 0 
$$F(1, 497) = 15.63$$
$$Prob > F = 0.0001$$

## To confirm that Stata got it right:

### . vce

$$F_{1,N-K-1} = \left(\frac{(b_{Educ} - b_{Job \exp})}{\sqrt{s^2 b_{Educ} + s^2 b_{Job \exp} - 2s_{b_{Educ}, b_{Job \exp}}}}\right)^2 = \left(\frac{(1.309229 - .8533107)}{\sqrt{.00703 + .004501 - 2* - .000883}}\right)^2$$
$$= \left(\frac{.4559183}{.115312619}\right)^2 = 3.95375897^2 = 15.63$$

## (ii) Incremental F test:

- . \* Unconstrained model
- . reg income educ jobexp

Source	SS	df		MS		Number of obs F(2, 497)		500 239.86
Model   Residual   + Total	22352.7545	2 497	1117 46.5	6.3773 953368  036811		Prob > F R-squared Adj R-squared Root MSE	= = =	0.0000 0.4912 0.4891 6.8261
income	Coef.	 Std.	 Err.	t	P> t	[95% Conf.	In	terval]
educ   jobexp   _cons	1.309229 .8533107 -1.076636	.0838 .0670 1.205	888	15.61 12.72 -0.89	0.000 0.000 0.372	1.14449 .7214982 -3.445568		.473968 9851233 .292295

- . est store unconstrained
- . \* Constrained model
- . gen jobed = educ + jobexp
- . reg income jobed

Source	SS	df		MS		Number of obs F( 1, 498)		500 450.84
Model   Residual	21624.34	1 498	21 47.9	.624.34 9644516		Prob > F R-squared Adj R-squared	=	
	45510.6369					Root MSE		6.9256
income	Coef.			t		[95% Conf.	In	terval]
jobed   _cons	1.037904	.0488	816	21.23 -0.45	0.000 0.653	.9418644 -2.935159	_	.133944 .841978

- . est store constrained
- . \* Use Buis's -ftest- command
- . ftest constrained unconstrained

Assumption: constrained nested in unconstrained

$$F(1, 497) = 15.63$$
  
 $prob > F = 0.0001$ 

If you prefer to do things the hard way – From the unconstrained model, we get

$$SSE_u = 23157.8824$$
,  $R_u^2 = .4912$ ,  $N = 500$ ,  $K = 2$ .

From the constrained model, we get

$$SSE_c = 23886.2969, R^2_c = .4751, J = 1.$$

Using the incremental F test, we get

$$F_{1,N-K-1} = \frac{(SSE_c - SSE_u)*(N - K - 1)}{SSE_u*1} = \frac{(R_u^2 - R_c^2)*(N - K - 1)}{(1 - R_u^2)*1}$$
$$= \frac{(23886.30 - 23157.88)*497}{23157.88} = \frac{(.4912 - .4751)*497}{1 - .49115} = 15.63$$

## (iii) Likelihood ratio chi square test:

#### . 1rtest constrained unconstrained, stats

Likelihood-ratio test LR chi2(1) = 15.48 (Assumption: constrained nested in unconstrained) Prob > chi2 = 0.0001

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
constrained   unconstrai~d		-1837.243 -1837.243	-1676.082 -1668.34	2	3356.165 3342.68	3364.594 3355.324

Note: N=Obs used in calculating BIC; see [R] BIC note

The test statistics are all highly significant. It is very unlikely that the effects of education and job experience are equal.

- 3. Group comparisons. Using the same data as in problem 2, do the following:
- (a) Do T-tests of whether the means of men and women significantly differ on education, job experience, and income. If using Stata, use commands such as

#### . ttest educ, by (female)

Two-sample t test with equal variances

Group	Obs	Mean	Std. Err.	Std. Dev.	. [95% Conf	. Interval]
male female	225 275	11.22222 10.63636	.298438 .1733252	4.47657 2.874273	10.63412 10.29515	11.81033 10.97758
combined	500	10.9	.1650287	3.690154	10.57576	11.22424
diff	 	.5858586	.3310136		0644967	1.236214

Degrees of freedom: 498

Ho: 
$$mean(male) - mean(female) = diff = 0$$

Men have slightly more education than women do. The difference is significant if you use a 1-tailed test.

### . ttest jobexp, by(female)

Two-sample t test with equal variances

Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf	. Interval]
male female	225 275		.3569664 .2249718	5.354497 3.730735	13.40767 11.92074	14.81455 12.80653
combined	500	13.15	.2062525	4.611945	12.74477	13.55523
diff		1.747475	.4075443		.9467565	2.548193

Degrees of freedom: 498

Ho: mean(male) - mean(female) = diff = 0

On average, men have almost 2 more years of job experience than do women. The difference is highly significant.

### . ttest income, by(female)

Two-sample t test with equal variances

Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf	. Interval]
male female	225 275	27.81111 21.63636	.76553 .3865032	11.48295 6.40943	26.30255 20.87547	29.31967 22.39726
combined	500	24.415	.4270917	9.550062	23.57588	25.25412
diff	 	6.174747	.8135835		4.576268	7.773227

Degrees of freedom: 498

Ho: mean(male) - mean(female) = diff = 0

Men make more than \$6,000 a year more than women, and the difference is highly significant.

- (b) Test the following. Use a likelihood ratio chi square test. Performing an incremental F test and/or a Wald test using suest is optional.
  - $H_0$ : Model parameters are the same for both men and women
  - H<sub>A</sub>: Model parameters are not the same for both men and women.
- . \* Constrained model: No Gender differences
- . reg income educ jobexp

Source	SS +	df	MS		Number of obs F( 2, 497)		500 239.86
Model   Residual	22352.7545   23157.8824	2 11	176.3773 .5953368		Prob > F	= =	0.0000
Total	45510.6369	499 91	.2036811		Root MSE	=	6.8261
income	Coef.	Std. Err	. t	P> t	[95% Conf.	In	terval]
educ   jobexp   _cons	1.309229   .8533107   -1.076636	.0838474 .0670888 1.205717	15.61 12.72 -0.89		1.14449 .7214982 -3.445568		.473968 9851233 .292295

Note that the constrained model was the unconstrained model in problem 2. In problem 2, we viewed it as unconstrained because the effects of education and job experience were free to differ. In this problem, we view it as constrained because the coefficients are constrained to be the same for both men and women.

- . \* Unconstrained Effects differ by gender
- . reg income educ jobexp if female == 0

Source	l ss	df		MS		Number of obs F( 2, 222)	=	225 210.87
Model   Residual	19350.4582 10185.7638	2 222		.22912 818188		Prob > F R-squared Adj R-squared	= =	0.0000 0.6551 0.6520
Total	29536.222	224	131.	858134		Root MSE		6.7736
income	Coef.	Std. 1	Err.	t	P> t	[95% Conf.	In	terval]
educ   jobexp   _cons	.8195378   1.384972  9294128	.10708 .08953	246	7.65 15.47 -0.62	0.000 0.000 0.536	.6085108 1.208545 -3.88108	1	.030565 .561398 .022254

- . est store male
- . reg income educ jobexp if female == 1

Source	l SS	df	MS		Number of obs	=	275
+	<del></del>				F( 2, 272)	=	120.03
Model	5276.94296	2 263	38.47148		Prob > F	=	0.0000
Residual	5979.19312	272 21	.9823276		R-squared	=	0.4688
+	+				Adj R-squared	=	0.4649
Total	11256.1361	274 41.	.0807886		Root MSE	=	4.6885
income	Coef.	Std. Err	. t	P> t	[95% Conf.	In	terval]
income	Coef. +	Std. Err	. t	P> t	[95% Conf.	In	terval]
income    educ	Coef.    1.525582	Std. Err.	t  15.19	P> t   0.000	[95% Conf. 		terval]  .723261
	·					1	
educ	1.525582	.1004096	15.19	0.000	1.327903	1 •	.723261

. est store female

Likelihood Ratio Test. Doing a Likelihood Ratio Chi Square Test,

. lrtest (male female) both

Likelihood-ratio test  $\begin{array}{cccc} \textit{LR chi2(3)} & = & 213.10 \\ \textit{Prob} > \textit{chi2} & = & 0.0000 \end{array}$ 

Assumption: (both) nested in (male, female)

[Note that the LR Chi Square / Degrees of Fredom = 213.10/3 = 71.03; compare with the incremental F value calculated below]

This is highly significant, ergo we reject the null and conclude that the coefficients for men and women likely are different. This conclusion is not surprising, since, by looking at the coefficients in the separate male and female models, you can see that the effects appear to be very different. Nonetheless, keep in mind that all we know for sure is that at least one parameter (including possibly the intercept) differs between men and women.

*Incremental F Test.* If we want to be masochistic and do an incremental F test, from the constrained model we get

$$SSE_c = 23158$$
,  $N = 500$ ,  $K = 2$ .

From the regressions for males only and females only we get

$$SSE_{Males} = 10186$$
,  $N_{Males} = 225$ 

$$SSE_{Females} = 5979$$
,  $N_{Females} = 275$ .

Hence, by adding up the figures for men and women, for the unconstrained model we get

$$SSE_{u} = 16165$$
,  $N_{u} = 500$ .

Also, note that J = K + 1 = 3, i.e. the constrained model estimates 2 betas and 1 intercept, while the unconstrained model estimates 4 betas and 2 intercepts.

Hence, for the incremental F, we get

$$F_{K+1,N_1+N_2-2K-2} = \frac{(SSE_c - SSE_u) * (N_1 + N_2 - 2K - 2)}{SSE_u * (K+1)} = \frac{(23158 - 16165) * 494}{16165 * 3} = 71.24$$

[Note too that this is almost identical to LR chi square/3 shown above]

# Suest. If we wanted to do this with a Wald chi-square test and the suest command,

```
. quietly reg income educ jobexp if female == 0
. est store male
. quietly reg income educ jobexp if female == 1
. est store female
. suest male female
```

Simultaneous results for male, female

				Numbe	er of obs =	500
	Coef.	Robust Std. Err.	Z	P> z	[95% Conf.	Interval]
	.8195378 1.384972 9294128				.623384 1.170118 -1.898156	1.015692 1.599825 .0393307
male_lnvar   _cons	3.826069	.0705412	54.24	0.000	3.687811	3.964327
female_mean   educ   jobexp   _cons	1.525582 0049199 5.470545	.0930839 .0400294 1.626955	16.39 -0.12 3.36	0.902	1.343141 0833761 2.281772	1.708023 .0735362 8.659318
female_lnvar   _cons	3.090239	.1010872	30.57	0.000	2.892112	3.288366

### . test [male\_mean = female\_mean], constant coef

- ( 1) [male\_mean]educ [female\_mean]educ = 0
- ( 2) [male\_mean]jobexp [female\_mean]jobexp = 0
  ( 3) [male\_mean]\_cons [female\_mean]\_cons = 0

chi2(3) = 180.32Prob > chi2 = 0.0000

Constrained coefficients

	Coof	Robust Std. Err.	z	DNIZI	[95% Conf.	Intervall
	coer.	sta. EII.	2	E/ 2	[95% CONT.	Incervar
male_mean						
educ	1.579664	.0309782	50.99	0.000	1.518948	1.64038
jobexp	.0646106	.0225819	2.86	0.004	.0203509	.1088703
_cons	3.960761	.3267024	12.12	0.000	3.320436	4.601085
male_lnvar						
_cons	3.813277	.0572104	66.65	0.000	3.701146	3.925407
female_mean						
educ	1.579664	.0309782	50.99	0.000	1.518948	1.64038
jobexp	.0646106	.0225819	2.86	0.004	.0203509	.1088703
_cons	3.960761	.3267024	12.12	0.000	3.320436	4.601085
female_lnvar						
cons	2.949792	.092498	31.89	0.000	2.768499	3.131085

Based on your results, explain whether men make more than women and if so why. [Note: these are hypothetical data, and the results are a little peculiar in some respects!]

We know from the T-Tests that, on average, men make significantly more money than do women. We also know from the T-Tests that men benefit from having higher levels of education and job experience than do women. The regressions add additional insights as to why differences exist. For men, both education and job experience have significant effects, with job experience actually having a larger effect than education does. For women, on the other hand, job experience has virtually no effect whatsoever; only education is important. Education actually appears to have a larger effect on women than it does men! But this is more than offset by the advantages men have from higher levels of job experience and education and the much greater effect job experience has on men than women. Perhaps women are more likely to be in dead-end jobs where additional experience does not help you to get promoted into higher paying positions.

It is true that, under certain conditions, a woman would be expected to make more than a comparable man, e.g. when jobexp = 0. However, no such person exists in the sample (the lowest value of jobexp is 3), and overall, men have the advantage.

These would be <u>extremely</u> interesting and important findings, if it weren't for the fact that I made these data up.

(d) Suppose there were no gender-related compositional differences, i.e. women had the same levels of education and job experience as men did. If education and job experience continued to have the same effects on women that they do now, how much would the gap in income between men and women be affected?

We are asking a "what if" question. The following analysis addresses this.

. tabstat income educ jobexp, by(female) columns(variables)

Summary statistics: mean by categories of: female

female	income	educ	jobexp
male   female	27.81111 21.63636	11.22222	14.11111 12.36364
Total	24.415	10.9	13.15

As we saw before, men make \$6174.75 more than women on average.

. reg income educ jobexp if female == 1

Source	SS	df	MS		Number of obs F( 2, 272)		275 120.03
Model   Residual    Total	5276.94296 5979.19312	2 272 	2638.47148 21.9823276		Prob > F R-squared Adj R-squared Root MSE	= = =	0.0000 0.4688
income		Std. E	rr. t	P> t	[95% Conf.	In	terval]
educ   jobexp   _cons	1.525582	.10040 .07735 1.5897	87 -0.06	0.000 0.949 0.001	1.327903 1572178 2.340821		.723261 1473779 .600269

As the final numbers show, if women had the same levels of education and job experience as men, while the effects of education and job experience on women stayed the same, women would make \$22,521.54 on average, an increase of \$22,521.54 - \$21,636.36 = \$885.18. Hence, of the original difference of \$6174.75, \$885.18/\$6174.75 = 14.34% is due to compositional factors. The rest is due to differences in effects, in particular the fact that women get virtually no benefit from their years of job experience.

Some alternative approaches that will also work:

Using the predict command,

```
. reg income educ jobexp if female == 1
. predict mcompfcoef if !female
(option xb assumed; fitted values)
(275 missing values generated)
. sum mcompfcoef

Variable | Obs Mean Std. Dev. Min Max
mcompfcoef | 225 22.52154 6.82074 8.506949 31.35624
```

Using the adjust command,

Using margins with atmeans,

```
. quietly reg income educ jobexp if female == 1
. margins if female == 0, atmeans noesample
                                         Number of obs =
Adjusted predictions
                                                             225
Model VCE : OLS
Expression : Linear prediction, predict()
at : educ = 11.22222 (mean)
jobexp = 14.11111 (mean)
______
         | Delta-method
| Margin Std. Err.
                                 t P>|t| [95% Conf. Interval]
_cons | 22.52154 .323607 69.60 0.000 21.88445 23.15863
Using margins with precise values,
. sum educ if female == 0, meanonly
. scalar malemeaneduc = r(mean)
. sum jobexp if female == 0, meanonly
. scalar malemeanjobexp = r(mean)
. scalar list
malemeanjobexp = 14.111111
malemeaneduc = 11.222222
. quietly reg income educ jobexp if female == 1
. margins, at (educ = `=malemeaneduc' jobexp = `=malemeanjobexp')
Adjusted predictions
                                         Number of obs =
                                                             275
Model VCE : OLS
Expression : Linear prediction, predict()
         : educ = 11.22222
jobexp = 14.11111
                    Delta-method
         | Margin Std. Err. t P>|t| [95% Conf. Interval]
_____
     _cons | 22.52154 .323607 69.60 0.000 21.88445 23.15863
```