Sociology 592 - Research Statistics I
Exam 2 Answer Key
November 5, 2004

1. (10 points each, 30 points total.) You have been asked to serve as a statistical consultant for several proposed projects. For each of the following, your employers want you to tell them:

(i) Which of the cases we have studied their problem falls under (e.g. one sample tests, case I, σ known; nonparametric tests, case II, tests of association). Briefly explain why.
(ii) the null and alternative hypotheses
(iii) whether a Z, T, chi-square, or F test is appropriate; where applicable, also tell what the degrees of freedom for the test are. You DO NOT have to give the formula for the test statistic, nor do you need to specify the acceptance region.

If values for population parameters are not specified (e.g. σ) assume they are unknown; and if two or more unknown σ's are involved, assume they are equal.

a. The Democratic Party believes that President Bush's support will decline over the next year. Three hundred voters will be interviewed and asked their level of support for the President, on a scale that ranges from a low of 0 to a high of 70. One year from now, those same 300 voters will be asked to again rate the President.

2 sample tests, Case IV, Matched Pairs. The same individuals are compared one year apart.

H_0: $\mu_1 = \mu_2$
H_A: $\mu_1 > \mu_2$ (it is believed that support will decline by time 2)

T statistic with 299 d.f.

b. A nationwide study of state-funded Universities has found that 25% of the students at those schools favor the right of gays to get married. Notre Dame wants to see how its students compare. Therefore, 60 Notre Dame students will be asked whether they approve or disapprove of gay marriage.

Single Sample Tests, Case II, binomial parameter p. We are asking whether the proportion of students who favor gay marriage is the same at Notre Dame as it is nationwide.

H_0: $p = .25$
H_A: $p \neq .25$ (nothing has been said about how ND students are expected to differ)

Z statistic.

Alternatively, you could treat this as Nonparametric Tests Case I, sample distribution compared to population distribution. You would use a chi-square statistic with 1 d.f.

c. Doctors are worried about the nationwide shortage of flu vaccine. They have developed a new vaccine that can be quickly manufactured in mass quantities. But, first they must see whether it is more effective, equally effective, or less effective than the old vaccine in preventing the flu. Five hundred randomly selected subjects will receive the new vaccine while another 500 will receive the old. After a month, the doctors will measure how many members of each group have gotten the flu.

Two sample Tests, Case V, Difference Between Two Proportions. We are testing whether the proportion who get the flu is the same in both groups.

H_0: $p_1 = p_2$
H_A: $p_1 \neq p_2$

The researchers have not stated whether they think the vaccine will be more effective or less effective – even if it was less effective it might be better than nothing.

Z Statistic. The sign of the test statistic will indicate whether the new vaccine is more effective or less effective than the old.

Alternatively, you could treat this as nonparametric tests, Case II, tests of association. You'd use a chi-square statistic with 1 d.f. Note that the test statistic would always be positive and would not, in and of itself, tell you which vaccine was more effective. You'd have to look at the 2 X 2 table to see which was more effective.

2. (5 points each, 20 points total). For each of the following, indicate whether the statement is true or false. If you think the statement is false, indicate how the statement could be corrected.

NOTE: These are all pretty easy, but you could waste a great deal of time on some of them or make stupid mistakes if you don't happen to see what the easiest way to approach each problem is.

a. When there are many cells in a table and the expected cell frequencies are very large, e.g. several hundred in each cell, Fisher's Exact Test should be used instead of a chi-square test.

False. Fisher's Exact Test should be used when one or more expected cell frequencies is very small, e.g. less than 5. It might be very time consuming to compute it for a table that had many cells with large expected frequencies.

b. A One-way ANOVA with 8 groups produces a highly significant F value. The researcher wants to determine which groups differ from each other. An LSD (Least Significant Difference) test is desirable because such a test controls for the overall probability of rejecting the hypotheses that some pairs of means are different, when in fact they are equal.

False. The LSD test does NOT control for the overall probability of rejection. Bonferroni, Sidak and Scheffe tests provide such controls.

c. Using a balanced design, a researcher has collected data from 36 respondents on their race (white, black, other), religion (Catholic, NonCatholic) and political conservatism (measured on a continuous scale ranging from 0 to 30). She finds that

$$F_{JK-1,N-JK} = \frac{SS\ Cells/(JK-1)}{SS\ Error/(N-JK)} = \frac{MS\ Cells}{MS\ Error} = 58$$

She should therefore conclude that the political conservatism of individuals is affected by both their race and their religion.

False. This tells you that race or religion or possibly both affect political conservatism. But, it need not be the case that both have effects, e.g. race could affect political conservatism while religion does not. You would need to do additional tests to determine which specific effects are statistically significant.

d. A researcher is interested in the relationship between gender and attitudes toward abortion. Specifically, she believes that men are more likely to be pro-life than women are. She interviews 50 men and 50 women, and asks them whether they are pro-life or pro-choice. She obtains the following:

```
. tab  gender abortion [fw=freq], chi2

           |   Attitudes toward
           |       Abortion
    gender |  pro-life  pro-choic |     Total
-----------+----------------------+----------
      Male |        20         30 |        50
    Female |        35         15 |        50
-----------+----------------------+----------
     Total |        55         45 |       100

          Pearson chi2(1) =   9.0909   Pr = 0.003
```

If she is using the .01 level of significance, she should conclude that the data support her position.

False. The researcher believes that men are more likely to be pro-life. But, in reality, only 40% of the men are pro-life, compared to 70% of the women. The chi-square statistic does not tell you which group is more likely to be pro-life, it just tells you that they are not equally likely to be pro-life. Instead of treating this as Nonparametric Tests Case II Tests of Association, the researcher should treat it as 2 Sample Tests, Case V, Difference Between Proportions. The z-statistic from that test would tell you the direction of the differences. (The z would equal -3.015.)

Answer two of the following three questions. (25 points each; you will get up to 10 points extra credit if you answer all three correctly.)

3. Once again, the Republican Presidential candidate has carried the state of Indiana. Democrats are trying to find out whether support for President Bush was equally strong in all parts of the state or whether his support varied by geographic region. A survey of 100 Indiana voters produces the following results.

```
. tab  candidate region [fw=freq]

   Who did |
  you vote |       Region of the State
      for? |     North    Central      South |     Total
-----------+---------------------------------+----------
      Bush |        15         20         25 |        60
     Kerry |        20         15          5 |        40
-----------+---------------------------------+----------
     Total |        35         35         30 |       100
```

Using our five-step hypothesis testing procedure and the .05 level of significance, determine whether or not support for Bush varied by region. If there is a significant relationship, use the information you have been given to explain exactly what you think that relationship is, i.e. how did support for Bush differ across the state?

This falls under Nonparametric Tests, Case II, Tests of Association.

Step 1.

H_0: Support for Bush did not vary by region
H_A: Support for Bush did differ by region

or, equivalently,

H_0: $P(A_i \cap B_j) = P(A_i)P(B_j)$ (Model of independence)
H_A: $P(A_i \cap B_j) <> P(A_i)P(B_j)$ for some i, j

Step 2. An appropriate test statistic is

$\chi^2_v = \Sigma\Sigma (O_{ij} - E_{ij})^2/E_{ij}$, $v = rc-1-(r-1)-(c-1) = (r-1)(c-1) = 2$

Step 3. For $\alpha = .05$ and $v = 2$, accept H_0 if $\chi^2_2 \leq 5.99$

Step 4. The computed value of the test statistic is:

Candidate/Region	Observed	Expected	$(O_{ij} - E_{ij})^2/E_{ij}$
Bush North	15	.60 * .35 * 100 = 21	$-6^2/21 = 1.714$
Kerry North	20	.40 * .35 * 100 = 14	$6^2/14 = 2.571$
Bush Central	20	.60 * .35 * 100 = 21	$-1^2/21 = 0.048$
Kerry Central	15	.40 * .35 * 100 = 14	$1^2/14 = 0.071$
Bush South	25	.60 * .30 * 100 = 18	$7^2/18 = 2.722$
Kerry South	5	.40 * .30 * 100 = 12	$-7^2/12 = 4.083$

So, $\chi^2_v = \Sigma\Sigma (O_{ij} - E_{ij})^2/E_{ij} = 11.21$.

Step 5. Reject H_0, the computed test statistic falls outside the acceptance region.

Bush's support does vary by region. Looking at the table, it appears that, as you go from North to South, Bush's support increases.

Here is the more detailed Stata output for this problem.

```
. tab  candidate region [fw=freq], chi2 cchi2 expected

+--------------------+
| Key                |
|--------------------|
|     frequency      |
| expected frequency |
| chi2 contribution  |
+--------------------+

   Who did |
  you vote |      Region of the State
      for? |     North    Central      South |     Total
-----------+---------------------------------+----------
      Bush |        15         20         25 |        60
           |      21.0       21.0       18.0 |      60.0
           |       1.7        0.0        2.7 |       4.5
-----------+---------------------------------+----------
     Kerry |        20         15          5 |        40
           |      14.0       14.0       12.0 |      40.0
           |       2.6        0.1        4.1 |       6.7
-----------+---------------------------------+----------
     Total |        35         35         30 |       100
           |      35.0       35.0       30.0 |     100.0
           |       4.3        0.1        6.8 |      11.2

          Pearson chi2(2) =  11.2103   Pr = 0.004
```

4. A major university wants to strengthen its undergraduate statistics course. In the past, this class has been taught using a lecture format with little if any student discussion. Under this format, students have gotten a mean score of 70 on a standardized exam with a population standard deviation of $\sigma = 14$. Sixteen randomly selected students are now in a new statistics course that emphasizes class participation. Instructors believe that this format will produce higher test scores, although they are confident that σ will continue to equal 14. Results show that students in the new course receive an average score of 76 on the exam.

a. Construct the 95% confidence interval. Based on the confidence interval, should you reject the null?

This falls under confidence intervals, Case I, σ known. The critical value for z is 1.96. The 95% c. i. is

$$\bar{x} \pm (z_{\alpha/2} * \sigma/\sqrt{N}), i.e.,$$
$$76 - (1.96 * 14/4) \le \mu \le 76 + (1.96 * 14/4), i.e.,$$
$$69.14 \le \mu \le 82.86$$

Note that, even though 70 falls within the c.i., you do <u>not</u> reject the null. Confidence intervals should not be used to test 1-tailed alternative hypotheses.

b. Using our 5-step hypothesis testing procedure and the .05 level of significance, test whether the instructors' belief is supported by the data.

This falls under single sample tests, case I, σ known.

Step 1. The null and alternative hypotheses are:

H_0: E(X) = 70
H_A: E(X) > 70

Alternative is one-tailed because instructors expect higher scores with the new format.

Step 2. The appropriate test statistic is

$$z = \frac{\bar{x} - \mu_0}{\sqrt{\frac{\sigma^2}{N}}} = \frac{\bar{x} - 70}{\sqrt{\frac{14^2}{16}}} = \frac{\bar{x} - 70}{3.5}$$

Step 3. Accept H_0 if z < 1.645, otherwise reject.

Step 4. The computed value of the test statistic is

$$z = \frac{\bar{x} - 70}{3.5} = \frac{76 - 70}{3.5} = 1.714$$

Step 5. Reject the null. Test scores are significantly higher under the new format.

Here is the Stata output for this problem.

```
. ztesti 76 14 70 16, level(95)

                                                  Number of obs =       16

------------------------------------------------------------------------------
Variable |      Mean    Std. Err.       z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
       x |        76         3.5   21.7143   0.0000       69.14013    82.85987
------------------------------------------------------------------------------

                            Ho: mean(x) = 70

      Ha: mean < 70          Ha: mean ~= 70             Ha: mean > 70
        z =   1.7143             z =   1.7143             z =   1.7143
    P < z =   0.9568       P > |z| =   0.0865         P > z =   0.0432
```

5. Most couples want two children. For those who have two children, a demographer believes that their desire for a third (measured on an interval-level scale) may be related to their social status (high, medium, or low) and their religion (Catholic, Protestant, Jewish, or Other). To test her ideas, she collects random samples of 10 individuals (all of whom currently have two children) for each possible combination of social status and religion. She finds that SS Rows= 120, SS Columns = 144, and SS Interaction = 240. When she tests the hypothesis that there are no differences in desires across social status (i.e. when she tests the hypothesis the row effects are all 0) she gets an F of 5.0.

a) Complete the following Anova table. You do NOT need to indicate whether the F values are statistically significant or not.

Source	SS	D.F.	M. S.	F
A + B (or Main Effects)				
A (Social Status)				
B (Religion)				
AB (or 2-way interaction)				
A + B + AB (or explained)				
Error (or residual)				
Total				

Note that J = 3 (high, medium, low), K = 4 (Catholic, Protestant, Jewish, Other). Since 10 cases are chosen for each possible combination of religion and social class, N = 120. Once we fill in the numbers we know the rest of the calculations are pretty easy:

Source	SS	D.F.	Mean Square	F
A + B (or Main Effects)	SS Main = 264	J + K - 2 = 5	$\frac{\text{SS Main}}{(J + K - 2)} = 52.8$	$\frac{\text{MS Main}}{\text{MS Error}} = 4.4$
A (Social Status)	SS Rows = **120**	J - 1 = 2	$\frac{\text{SS Rows}}{(J - 1)} = 60$	$\frac{\text{MS Rows}}{\text{MS Error}} = \mathbf{5}$
B (Religion)	SS Columns = **144**	K - 1 = 3	$\frac{\text{SS Columns}}{(K - 1)} = 48$	$\frac{\text{MS Columns}}{\text{MS Error}} = 4$
AB (or 2-way interaction)	SS Intraction = **240**	(J - 1) * (K - 1) = 6	$\frac{\text{SS Intrction}}{(J - 1)(K - 1)} = 40$	$\frac{\text{MS Intrction}}{\text{MS Error}} = 3.33$
A + B + AB (or explained)	SS Cells = 504	(J * K) - 1 = 11	$\frac{\text{SS Cells}}{(J * K) - 1} = 45.82$	$\frac{\text{MS Cells}}{\text{MS Error}} = 3.82$
Error (or residual)	SS Error = 1296	N - (J * K) = 108	$\frac{\text{SS Error}}{(N - J * K)} = 12$	
Total	SS Total = 1,800	N - 1 = 119	$\frac{\text{SS Total}}{(N - 1)} = 15.13$	

b) Explain what significant interaction terms might mean. Be specific; don’t just talk about interaction terms in general, rather, talk about what interactions involving the variables in this analysis might be due to.

One possibility is that the effect of social status differs by religion. For example, for Catholics, there may be no differences in desires by social status, whereas for other religions there are, i.e. social status has no effect on Catholics but it does affect people from other religions. Or, higher social status might lead to greater desires in one religion and to lower desires in the others. Such relationships might be due to the differing values religions place on children. You’d of course have to look at the data more to see what was really going on.