Sociology 592 - Research Statistics I
Sample Exam 2 Answer Key
November 11, 1994

1. (10 points each, 30 points total, up to 10 points extra credit). You have been asked to serve as a statistical consultant for several proposed projects. For three of the following, your employers want you to tell them:

(i) Which of the cases we have studied their problem falls under (e.g. one sample tests, case I, σ known; nonparametric tests, case II, tests of association). Briefly explain why.
(ii) the null and alternative hypotheses
(iii) whether a Z, T, chi-square, or F test is appropriate; where applicable, also tell what the degrees of freedom for the test are. You DO NOT have to give the formula for the test statistic, nor do you need to specify the acceptance region.

If values for population parameters are not specified (e.g. σ) assume they are unknown; and if two or more unknown σ's are involved, assume they are equal. [NOTE: You must do three of the following, and you'll receive 5 points extra credit for each additional problem you get right.]

a. The Republican party claims that it is equally popular with men and women. Others, however, feel that there is a “gender gap”, with the party having less support among women than among men. Random samples of 100 men and 100 women will be asked whether or not they support the Republican party.

2 sample tests, case V, $p_1 = p_2$. "Popularity" is measured as a dichotomy -- either you support the party or you don't -- hence we want to compare the proportions of men and women supporting the Republicans.

H_0: $p_M = p_W$
H_A: $p_M > p_W$

The alternative hypothesis is one-tailed because some feel the party is more popular with men than with women. A Z statistic is appropriate.

b. It is known that business executives have an average score of 87 on a test of leadership ability. The military now wants to know how its officers compare with the business population. A random sample of 186 military officers will therefore be given this same test.

Single sample tests, case III, sigma unknown. We want to know whether military leaders have a mean of 87. Note that there isn't a sample of business executives, it is just somehow known that business leaders have a mean of 87.

H_0: $E(X) = 87$
H_A: $E(X) <> 87$

The alternative is 2-tailed because nothing has been said about the military having better or worse leaders. A T statistic, with 185 degrees of freedom, is appropriate.

c. Bill Clinton is worried after Tuesday's election results. He needs to find out whether support for his re-election varies across the country. He wants to poll 50 Easterners, 50 Westerners, 50 Northerners, and 50 Southerners. Using a scale that ranges from 1 to 100, respondents will be asked how much they support the Clinton presidency.

One way Anova. The dependent variable, Support, is continuous, and the independent variable, Region, is categorical. Since there are more than 2 regions involved, none of the 2 sample tests will work.

H_0: $E(X_1) = E(X_2) = E(X_3) = E(X_4)$
H_A: The means are not all equal.

An F statistic, with df = 3, 196, is appropriate.

d. A software company advertises that 90 percent of the calls to its technical support lines are answered in 5 minutes or less. Disgruntled customers claim this figure is far too high. To resolve the controversy, the company agrees to let an independent auditor record how long it takes for a random sample of 500 calls to get answered.

Single sample tests, Case II, binomial parameter p. Note that the company does not claim that the average waiting time is 5 minutes, rather it claims that 90% of the calls are answered in 5 minutes; the other 10% might take days! Hence, even though the waiting time is recorded, the proper strategy will be to dichotomize waiting time into <= 5 minutes (success) versus more than 5 minutes (failure).

H_0: $p = .90$
H_A: $p < .90$

The alternative should be one-tailed because the critics think the company does not do as well as it claims. A Z statistic is appropriate.

e. Last Resort Savings and Loan wants to know whether the racial characteristics of an ad and the race of a viewer are in any way related to the perceived effectiveness of the ad. It has therefore developed three new commercials. In one commercial, all the speakers are black, in another commercial half the speakers are black and half are white, and in the third commercial all the speakers are white. Each commercial will be shown to 10 whites and 10 blacks (i.e. 60 people altogether). Respondents will be asked to rate the effectiveness of the ads on a scale ranging from 0 to 100.

Two way Anova. The dependent variable, Effectiveness, is continuous, while the two independent variables, racial characteristics of the ad and race of the viewer, are both categorical. The phrase “in any way related” implies that the lender just wants to know if somehow, someway, somewhere there is a relationship between the dependent and independent variables. Hence, the appropriate null and alternative hypotheses are

H_0: All τ's, λ's, and $(\tau\lambda)$'s = 0
H_A: At least one τ, λ, or $(\tau\lambda)$ does not equal 0

This tests whether there are any effects at all. If the null hypothesis is true, then every cell in the table will have the same true mean. The appropriate <u>test statistic</u> is

$$F_{JK-1,N-JK} = \frac{SS\ Cells/(JK-1)}{SS\ Error/(N-JK)}$$

Since J = 3 (3 types of ads) and K = 2 (2 race categories) the d.f. are 5, 54.

2. (5 points each, 20 points total). For <u>each</u> of the following, indicate whether the statement is true or false. If you think the statement is false, indicate how the statement could be corrected.

NOTE: These are all pretty easy, but you could waste a great deal of time on some of them or make stupid mistakes if you don't happen to see what the easiest way to approach each problem is.

a. A researcher wants to know whether husbands and their wives significantly differ in how much they read. Eighteen husbands and their wives are asked how many books they have read in the past year. The researcher gets a T value of 2.08. If she is using the .05 level of significance, she should reject the null hypothesis.

False. Note that this is a matched pairs problem; there are 18 pairs rather than 36 cases. The appropriate d.f. are 17. Also, the alternative hypothesis is 2-tailed, since nothing has been said about which spouse is expected to read more. For a T with 17 d.f. and a 2-tailed alternative, do not reject the null if the test statistic falls between -2.11 and 2.11. Since 2.08 falls in this range, do not reject.

b. Twelve (12) randomly selected patients have received a drug, while another 12 randomly selected patients have not. A researcher's null and alternative hypotheses are

H_0: $\mu_1 - \mu_2 = 0$
H_A: $\mu_1 - \mu_2 > 0$

The computed value of the test statistic is -2.50. If $\alpha = .05$, you should reject the null hypothesis.

False. Note that the alternative hypothesis is one-tailed, and we are looking for positive values of T. The fact that the T statistic was negative means that things came out exactly the opposite of what the researcher hypothesized, i.e. the first group had a lower mean than the second group. Ergo, do not reject.

c. A professor believes that men are more likely to get an A in her class than women are. A chi-square statistic will tell her whether or not her hypothesis seems to be supported.

False. A chi-square statistic will tell you whether men and women differ in the probability of receiving an A, but if they do differ it won't tell you how. A T-Test should be used instead. That is, this problem falls under 2 sample tests, Case V, test of $p_1 = p_2$. You should not approach it as nonparametric tests, Case II, tests of association.

d. A researcher has collected data from 60 respondents on their race (white or nonwhite), gender (male or female), and political conservatism (measured on a scale ranging from 1 to 30). She computes

$$F_{JK-1,N-JK} = \frac{SS\ Cells/(JK - 1)}{SS\ Error/(N - JK)} = 3$$

If she is using the .05 level of significance, she should conclude that men and women differ in how politically conservative they are.

False. While the F value is significant, it may or may not be the case that men and women differ. It could instead be that whites and nonwhites differ while men and women are the same. The researcher should explicitly test whether the column (gender) effects differ from 0.

3. (25 points) A researcher selected a sample of 25 homes, and careful logs were kept of how many hours per week the TV set was on. The mean number of hours per week turned out to be 44, with a sample standard deviation of 10. Use both confidence intervals and our 5 step procedure to test the hypothesis that families watch an average of 40 hours of TV a week. Use the .05 level of significance.

This falls under Single sample tests, case III, σ unknown.

Step 1. The null and alternative hypotheses are

H_0: $\mu = 40$
H_A: $\mu <> 40$

Note that the alternative hypothesis is two-tailed. Nothing has been said about whether researchers expect the number to be higher than 40, or less. Just because you know that the observed number was 44 doesn't mean you can cheat when specifying the hypothesis!

Step II: The appropriate statistic is

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{N}}} = \frac{\bar{x} - 40}{\frac{10}{\sqrt{25}}} = \frac{\bar{x} - 40}{2}$$

D.F. = N - 1 = 24.

Step III. Accept H_0 if $-2.064 \leq t_{24} \leq 2.064$

Step IV. The computed value of the test statistic is

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{N}}} = \frac{\bar{x} - 40}{\frac{10}{\sqrt{25}}} = \frac{\bar{x} - 40}{2} = \frac{44 - 40}{2} = 2$$

Step V. Do not reject. The computed value of the test statistic falls within the acceptance region.

To solve this problem using confidence intervals: The 95% c.i. is

$$\bar{x} \pm (t_{\alpha/2,v} * s/\sqrt{N}),\ i.e.$$

$$44 - (2.064 * 10/5) \leq \mu \leq 44 + (2.064 * 10/5),\ i.e.$$

$$39.872 \leq \mu \leq 48.128$$

Since 40 falls within the c.i., do not reject H_0.

4. (25 points) A politician wants to know whether his crime bill is equally popular with members of all political parties. Interviews are conducted with 100 democrats, 100 Republicans, and 50 independents. Forty percent of the Democrats, sixty percent of the Republicans, and half of the independents say they support the bill, while the rest say they oppose it. Use our five step procedure and the .01 level of significance to test whether party affiliation is related to support for the bill.

This falls under nonparametric tests, Case II, tests of association. First, let's reconstruct the original crosstabulation. We are told that

40% of the 100 Democrats support the bill, the other 60% do not
60% of the 100 Republicans support the bill, the other 40% do not
50% of the 50 Independents support the bill, the other 50% do not

Hence, we get

Party / Support	**Supports Bill**	**Opposes Bill**	**Total**
Democrat	40	60	**100**
Republican	60	40	**100**
Independent	25	25	**50**
Total	**125**	**125**	**250**

Step 1. The null and alternative hypotheses are

H_0: $p(A_i \cap B_j) = P(A_i) * P(B_j)$ for all i,j
H_A: $p(A_i \cap B_j) <> P(A_i) * P(B_j)$ for some i,j

where A_1 = Democrat, A_2 = Republican, A_3 = Independent,
B_1 = Supports bill, B_2 = Opposes bill. Note that we are testing the model of independence.

Step 2. A chi-square statistic is appropriate.

$\chi^2_v = \Sigma\Sigma\,(O_{ij} - E_{ij})^2/E_{ij}$, where $v = rc-1-(r-1)-(c-1) = (r-1)(c-1) = 2$, $E_{ij} = P(A_i) * P(B_j) * N$

Step 3. For $\alpha = .01$ and $v = 2$, accept H_0 if

$$\chi^2_v \le 9.21$$

Step 4. To compute the test statistic, note that $P(A_1)= P(A_2)= 100/250 = .4$, $P(A_3) = 50/250 = .2$, $P(B_1) = P(B_2) = 125/250 = .5$. Hence,

Party/Support	Observed	Expected	$(O_{ij} - E_{ij})^2/E_{ij}$
Dem Supporter	40	.4 * .5 * 250 = 50	2
Dem Oppose	60	.4 * .5 * 250 = 50	2
Rep Support	60	.4 * .5 * 250 = 50	2
Rep Oppose	40	.4 * .5 * 250 = 50	2
Ind Support	25	.2 * .5 * 250 = 25	0
Ind Oppose	25	.2 * .5 * 250 = 25	0

The value of the test statistic is the sum of the last column, 8.

Step 5. Do not reject H_0. The computed test statistic is less than the critical value. Note that, if we used the .05 level of significance, the critical value would be 5.99, and we would reject.

Here is an SPSS for Windows 6.1 solution to the above problem:

```
-> * Exam 2, November 1994, Problem 4.
-> Data list Free / Party Support Freq.
-> Begin Data.
-> 1 1 40
-> 1 2 60
-> 2 1 60
-> 2 2 40
-> 3 1 25
-> 3 2 25
-> End data.
-> Value labels Party 1 'Democrat' 2 'Republican' 3 'Independent'/
->              Support 1 'Supports' 2 'Opposes'.
-> Weight by freq.
-> CROSSTABS
->   /TABLES=party  BY support
->   /FORMAT= AVALUE NOINDEX BOX LABELS TABLES
->   /STATISTIC=CHISQ
->   /CELLS= COUNT EXPECTED .

PARTY  by  SUPPORT

                    SUPPORT      Page 1 of 1
            Count  |
           Exp Val |Supports Opposes
                   |                  Row
                   |    1.00|    2.00| Total
PARTY      --------+--------+--------+
             1.00  |    40  |    60  |   100
  Democrat         |  50.0  |  50.0  | 40.0%
                   +--------+--------+
             2.00  |    60  |    40  |   100
  Republican       |  50.0  |  50.0  | 40.0%
                   +--------+--------+
             3.00  |    25  |    25  |    50
  Independent      |  25.0  |  25.0  | 20.0%
                   +--------+--------+
            Column     125      125      250
             Total   50.0%    50.0%   100.0%

      Chi-Square                 Value          DF              Significance
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Pearson                          8.00000         2                 .01832
Likelihood Ratio                 8.05421         2                 .01783
Mantel-Haenszel test for         2.84571         1                 .09162
      linear association

Minimum Expected Frequency -   25.000
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