Sociology 592 - Advanced Statistics I
Exam 1
February 12, 1991

Where appropriate, show your work - partial credit may be given. (On the other hand, don't waste a lot of time on excess verbiage.) Do not spend too much time on any one problem. It is legitimate (and probably essential) to refer to results that have previously been proven in class or homework, without re-proving them - for example, you wouldn't need to prove that $P(-1.96 \le Z \le 1.96) = .95$, since we have already shown that in class. Likewise, you are free to refer to anything that was demonstrated in the homework or handouts.

1. (5 pts. each, 20 pts. total). Indicate whether the following statements are true or false. If you think the statement is false, indicate how the statement could be corrected. For false statements, do not just say that you could substitute "not equals" for equals. For example, the statement $P(Z \le 0) = .7$ is false. To make it correct, don't just say $P(Z \le 0) <> .7$, instead say $P(Z \le 0) = .5$ or $P(Z \le .525) = .7$.
    a. $P(-1 \le Z \le 1) = 2F(1)$.
    b. The null and alternative hypotheses are:
        $H_0: \quad \mu = 100 \qquad\qquad H_A: \quad \mu > 100$
        In reality, $\mu = 100$. The researcher rejects the null hypothesis. The researcher has committed a Type I error.
    c. For a binomially distributed variable X, if $N = 17$ and $p = .3$, then $P(5) = .2081$.
    d. If A and B are independent events, then $P(A \mid B) = P(B \mid A)$.

2. (10 pts. each, 30 pts. total) Answer <u>three</u> of the following. The answers to most of these are fairly straightforward, so do not spend a great deal of time on any one problem. NOTE: I will give up to 5 pts. extra credit for each additional problem you do correctly.
    a. In a recent election, Murphy defeated Jones, 60% to 40%. Of those who voted, 30% were Protestant, 30% were Catholic, and 40% belonged to other Religions. Twenty-four percent of the voters were Protestant supporters of Murphy, while another 15% were Catholics who voted for Jones. Complete the following table:

	Murphy (M)	Jones (J)	Σ
Protestant (P)	$P(M \cap P)$	$P(J \cap P)$	$P(P)$
Catholic (C)	$P(M \cap C)$	$P(J \cap C)$	$P(C)$
Other (O)	$P(M \cap O)$	$P(J \cap O)$	$P(O)$
Σ	$P(M)$	$P(J)$	

   b. Recent advances in genetic engineering have made it possible for a women to be 60% successful in choosing the sex of her child. If 11 pregnant women all want girls, what is the probability that a majority of them will be successful?

   c. A gambler is willing to bet that, if he rolls 2 dice 24 times, he will get a 12 (i.e. double-sixes) at least once. If you take him up on his bet, who is more likely to win? (HINT: the probability of getting double-sixes on one roll of a pair of dice is 1/36. Because p is so small, the normal approximation works very poorly).

   d. On a recent 5-question statistics exam, 15% of the students got 1 question right, 20% got 2 questions right, 30% got 3 questions right, 20% got 4 questions right, and the remaining students got perfect scores. Find the mean and variance of the exam scores.

   e. Prove that the following two formulas for the population covariance of X and Y are mathematically equivalent:

$$E[(X - \mu_X)(Y - \mu_Y)] = E(XY) - \mu_X\mu_Y$$

3.   (25 points) A political scientist is studying black-white differentials in voting participation. She gathers data from voting precinct #17, which conveniently has 1000 blacks and 1000 whites that are eligible to vote. Her study reveals that 80% of blacks have low incomes and the remainder have high incomes. For whites, 40% have low incomes and the rest have high incomes. Among blacks, 30% of those with low incomes regularly vote in elections, compared to 80% of the high-income blacks. For whites, 20% of those with low incomes regularly vote, compared to 60% of the high-income whites.
      a.   Finish filling in the numbers for the following table. Remember that, as is already noted in the table, there are a total of 1000 blacks and 1000 whites.

	Black			White		
Vote/Income	Low	High	Σ	Low	High	Σ
Votes						
Does not vote						
Σ			1000			1000

      b.   What percentage of the voters in this precinct are black?
      c.   As these figures show, blacks are generally poorer than whites, and poor people tend to vote less than do wealthier people. Suppose that blacks had the same income distribution as whites. Suppose further that it continued to be the case that blacks maintained their income-specific voting rates, i.e. 30% of the poor blacks and 80% of the rich blacks voted. What percentage of blacks would then vote?

4.   (25 points) The military is concerned about the reliability of the weapons it is using in Operation Desert Storm. The manufacturer of a particular missile has guaranteed that no more than 20% of its missiles will be defective. Random testing of 100 missiles reveals that 30 are defective. Since the military would hate to unjustly accuse any of its contractors of faulty workmanship, test the manufacturer's claim at the .01 level of significance. Be sure to indicate:
      (a)   The null and alternative hypotheses - and whether a one-tailed or two-tailed test is called for.
      (b)   The appropriate test statistic
      (c)   The critical region
      (d)   The computed value of the test statistic
      (e)   Your decision - should the null hypothesis be rejected or not be rejected? Why?
      NOTE: You will receive partial credit if you can at least tell me, if the manufacturer is correct, what is the probability that a random sample of 100 missiles would contain 30 or more defects?