Soc 591 - Feb. 16,1988 - Answers to first exam.

1. a. False. See Harnett, p. 102. Substitute either
   1. $P(A \cup B) = P(A) + P(B)$, or
   2. $P(A \cap B) = 0$

   b. True. The null hyp. has been accepted when it should have been rejected, a type II error.

   c. False. Substitute either
   1. $P(Z \geq 1.5) = 1 - .9332 = .0668$, or
   2. $P(Z \leq 1.5) = .9332$

   d. True. The bigger α is, the smaller the acceptance region, and the more likely it is you will reject H_0.

2. a. You know (from the homework) the expectation and variance of the sum obtained from tossing 2 fair dice are 7 and 35/6. Since tosses are independent, all we have to do is multiply these numbers by 30 - yielding $E(X) = 210$ and $V(X) = 175$. (See the answer key for homework 2 if you need additional details.)

   b. .50, of course. With 11 kids, you can either have more boys than girls, or more girls than boys. Each of these outcomes is equally likely, hence there is a 50% chance there will be more boys than girls.

   If you simply must do this the hard way - you could use the normal approximation to the binomial. Note that $X \sim N(5.5, 2.75)$. Hence, you want to find the probability that $X \geq 5.5$. (Note that you use 5.5 instead of 6, because of the correction for continuity). Now, $P(X \geq 5.5) = P(Z \geq 0) = .50$.

   Real masochists can also solve this problem using the binomial distribution, computing the probability of 6 boys, 7 boys, 8 boys, etc. This yields P(6 boys) = .2259, P(7 boys) = .1611, P(8 boys) = .0806, P(9 boys) = .0269, P(10 boys) = .0054, P(11 boys) = .0005, add these all up and you get .50.

   Question: In a family of 10 children, what is the probability that there will be more boys than girls?

   c. $P(X \geq x) = P(Z \geq z) = .05$. As we have demonstrated numerous times, the appropriate value for z is 1.645. So, x = (1.645 * $10,000) + $25,000 = $41,450.

   d. Note that all 6 agree if all 6 vote to convict or all 6 vote to acquit (i.e. all 6 get heads or all 6 get tails). P(all convict) = $.5^6 = .015625$, P(all acquit) = .015625, hence P(all convict or all acquit) = .03125.

   e. $P(A) > P(B) ==> 1/P(B) > 1/P(A) ==>$
   $P(A \cap B)/P(B) \geq P(A \cap B)/P(A) ==> P(A \mid B) \geq P(B \mid A)$. Note that, when A and B are mutually exclusive events, $P(A \cap B) = 0 = P(A \mid B) = P(B \mid A)$, otherwise $P(A \mid B) > P(B \mid A)$ (since probabilities are always nonnegative).

3. E1 = takes bus, P(E1) = .30 ("30% of all commuters ride the bus to work")
   E2 = doesn't take bus, P(E2) = .70
   A = Income under $10,000
   P(A | E1) = .50 ("Of those who commute to work by bus, 50% have incomes under $10,000...")
   P(A | E2) = .20 ("Of those who commute to work by some other means, 20% have incomes below $10,000...")

a. $P(A \cap E1) = P(E1)P(A \mid E1) = .3 * .5 = .15$
b. $P(A) = \Sigma Ei\ P(A \mid Ei) = (.3 * .5) + (.7 * .2) = .29$
c. $P(E1 \mid A) = P(A \cap E1)/P(A) = .15/.29 = .517$

NOTE: This is very similar to problem 2.26 in Harnett, which you worked in homework 2.

4. a. H_0: p = .25 (or E(X) = 30)
H_A: p < .25 (or E(X) < 30)

A one tailed test is called for - the civil rights group thinks the city discriminates, hence it believes there will be fewer blacks than the mayor claims.

b. The appropriate test statistic is
$Z = (\#\text{ blacks in sample} - Np_0)/\sqrt{(Np_0q_0)} =$
$(\#\text{ blacks in sample} - 30)/\sqrt{22.5}$

c. As we have previously proven, $P(Z \geq -1.645) = 1 - .05$. Hence, we will accept H_0 if $z_c \geq -1.645$, otherwise we will reject H_0. Or, equivalently, accept H_0 if $X \geq 22.2$, reject H_0 otherwise.

d. $z_c = (18 - 30)/\sqrt{22.5} = -2.53$. Or, if you want to be more precise and apply the correction for continuity, $z_c = (18.5 - 30)/\sqrt{22.5} = -2.42$.

e. Reject H_0 - z_c does not lie in the acceptance region.

NOTE: If you just wanted the probability that the sample would contain 18 or fewer blacks, given that the mayor is correct, you would compute z_c as shown in step d. Then, F(-2.53) = 1 - F(2.53) = .0057 (or, with correction for continuity, F(-2.42) = .0078). Hence, there is less than a 1% chance that the mayor could be correct and the sample only contain 18 or fewer supporters.