Sociology 591
Advanced Social Statistics
Exam 1
February 11, 1988

Where appropriate, show your work - partial credit may be given. (On the other hand, don't waste a lot of time on excess verbiage.) Do not spend too much time on any one problem. It is legitimate (and probably essential) to refer to results that have previously been proven in class or homework, without re-proving them - for example, you wouldn't need to prove that $P(-1.96 \leq Z \leq 1.96) = .95$, since we have already shown that in class. Likewise, you are free to refer to anything that was demonstrated in the homework or handouts.

1. (5 pts. each, 20 pts. total). Indicate whether the following statements are true or false. If you think the statement is false, indicate how the statement could be corrected. For false statements, do not just say that you could substitute "not equals" for equals. For example, the statement $P(Z \leq 0) = .7$ is false. To make it correct, don't just say $P(Z \leq 0) <> .7$, instead say $P(Z \leq 0) = .5$ or $P(Z \leq .525) = .7$.

   (a) If A and B are mutually exclusive events, then $P(A \cup B) = 0$.

   (b) The null hypothesis is $E(X) = 100$. In reality, $E(X) = 95$. The researcher does not reject the null hypothesis. The researcher has committed a Type II error.

   (c) $P(Z \geq 1.5) = .9332$

   (d) You are more likely to reject the null hypothesis when $\alpha = .05$ than when $\alpha = .01$. (HINT: If in doubt, remember the results from the homework.)

2. (10 pts. each, 30 pts. total) Answer 3 of the following. The answers to most of these problems are (almost) obvious, do not spend a great deal of time on any one problem. NOTE: Be especially careful on problems (a) and (b) - there are hard, time-consuming ways of solving those problems, and then there are other ways... NOTE: I will give up to 5 pts. extra credit for each additional problem you do correctly.

   a. Find the expectation and variance of the sum obtained in tossing 60 fair dice.

   b. In a family of 11 children, what is the probability that there will be more boys than girls?

   c. If Income ~ N($25000, $10000^2), how high does your income have to be for you to be among the wealthiest 5% of the population?

   d. Suppose that in a jury trial there are 6 jurors, and suppose that all 6 jurors daydream during the trial. During deliberations, each juror determines guilt or innocence by flipping a fair coin. What is the probability that all 6 jurors will reach the same decision?

   e. Prove that if $P(A) > P(B)$ then $P(A \mid B) > P(B \mid A)$.

3. (20 pts. total) In a certain suburban city, 30% of all commuters ride the bus to work. Of those who commute to work by bus, 50% have incomes under $10,000, 35% have incomes of $10,000 to $20,000, and 15% have incomes over $20,000. Of those who commute to work by some other means, 20% have incomes below $10,000, 35% have incomes of $10,000 to $20,000, and 45% have incomes over $20,000. (HINT: More information has been given than you will need to answer the following questions.)
   a. What is the probability that a randomly selected commuter will have an income under $10,000 and will take the bus to work?

b. What is the probability that a randomly selected commuter will have an income under $10,000?

c. If a randomly selected commuter has an income of less than $10,000, what is the probability s/he commutes by bus?

4. (30 pts.) The mayor claims that blacks account for 25% of all city employees. A civil rights group disputes this claim, and argues that the city discriminates against blacks. A random sample of 120 city employees has 18 blacks. Test the mayor's claim at the .05 level of significance. Be sure to indicate:

(a) The null and alternative hypotheses - and whether a one-tailed or two-tailed test is called for.
(b) The appropriate test statistic
(c) The critical region
(d) The computed value of the test statistic
(e) Your decision - should the null hypothesis be rejected or not be rejected? Why?

NOTE: You will receive partial credit if you can at least tell me, if the mayor is correct, what is the probability that a random sample of 120 city employees would contain 18 or fewer blacks?