NAME:
AME 20231, Thermodynamics
Examination 2
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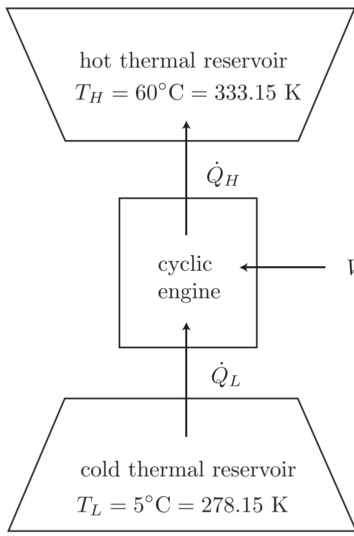
1. (20) A Carnot refrigerator with ammonia NH_3 as a working fluid operates between large thermal reservoirs at 5°C and 60°C. Find the coefficient of performance of the refrigerator. If the refrigerator draws 100 kW of power, find the rate of heat rejected to the high temperature reservoir and the rate of heat drawn from the low temperature reservoir.

Solution

Most people did well on this problem. A few mixed up relative and absolute temperatures. Several tried to get an efficiency when a coefficient of performance was required. We have



$$T_H = 60 + 273.15 = 333.15 \text{ K},$$
$$T_L = 5 + 273.15 = 278.15 \text{ K}.$$

For the refrigerator, we have

$$\beta = \frac{\text{what you want}}{\text{what you pay for}} = \frac{\dot{Q}_L}{\dot{W}}$$

The first law gives us

$$\dot{W} = \dot{Q}_H - \dot{Q}_L.$$

So

$$\beta = \frac{\dot{Q}_L}{\dot{Q}_H - \dot{Q}_L},$$
$$\beta = \frac{1}{\frac{\dot{Q}_H}{\dot{Q}_L} - 1},$$
$$\beta = \frac{1}{\frac{T_H}{T_L} - 1},$$
$$\beta = \frac{1}{\frac{333.15 \text{ K}}{278.15 \text{ K}} - 1},$$
$$\boxed{\beta = 5.05727.}$$

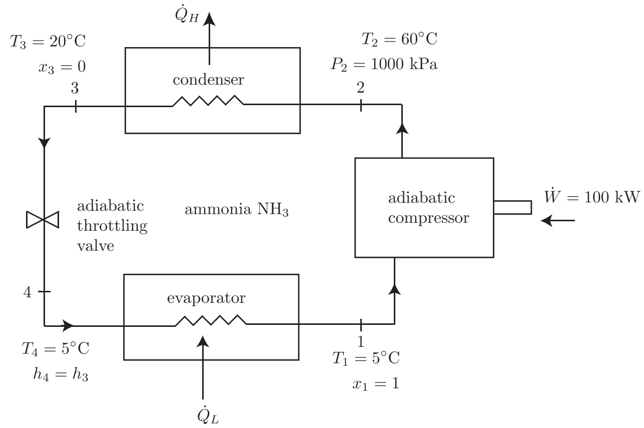
Thus

$$\dot{Q}_L = \beta\dot{W} = 5.05727(100 \text{ kW}) = \boxed{505.727 \text{ kW.}}$$

And

$$\dot{Q}_H = \dot{W} + \dot{Q}_L = \boxed{605.727 \text{ kW.}}$$

2. (40) Consider the refrigeration cycle below for a commercial refrigerator in a grocery warehouse. The working fluid is ammonia, NH_3. Find



(a) the mass flow rate (kg/s),

(b) the heat expulsion rate required from the condenser (kW),

(c) the coefficient of performance,

(d) a correctly oriented sketch, including the vapor dome and appropriate numerical values of P and v, of the cycle on a $P - v$ diagram.

Solution

Performance was mixed on this problem. A large number of people did very good work. Several people incorrectly tried to use a Carnot formula to calculate the coefficient of performance. This was incorrect. The $P - v$ diagrams were mixed. Many people did not put state 4 in the two phase mixture region. Many people mis-labeled and mis located states 1, 2, 3, 4.

The critical temperature and pressure for ammonia is 132.36°C and 11333.2 kPa. We are well below these values.

The tables or problem statement gives us

$$h_1 = 1447.3\ \frac{\text{kJ}}{\text{kg}}, \quad h_2 = 1563.1\ \frac{\text{kJ}}{\text{kg}}, \quad h_3 = 274.3\ \frac{\text{kJ}}{\text{kg}}, \quad h_4 = h_3 = 274.3\ \frac{\text{kJ}}{\text{kg}},$$

$$v_1 = 0.24299\ \frac{\text{m}^3}{\text{kg}}, \quad v_2 = 0.15106\ \frac{\text{m}^3}{\text{kg}}, \quad v_3 = 0.001600\ \frac{\text{m}^3}{\text{kg}}, \quad v_4 = 0.0153096\ \frac{\text{m}^3}{\text{kg}}$$

$$P_1 = 515.9 \text{ kPa}, \quad P_2 = 1000 \text{ kPa}, \quad P_3 = 857.5 \text{ kPa}, \quad P_4 = 515.9 \text{ kPa}.$$

We also have

$$T_1 = 5°\text{C}, \quad T_2 = 60°\text{C}, \quad T_3 = 20°\text{C}, \quad T_4 = 5°\text{C}.$$

For the adiabatic compressor we have

$$_1w_2 = h_1 - h_2 = \left(1447.3\ \frac{\text{kJ}}{\text{kg}}\right) - \left(1563.1\ \frac{\text{kJ}}{\text{kg}}\right) = -115.8\ \frac{\text{kJ}}{\text{kg}}.$$

Thus, the required mass flow rate is

$$\dot{m} = \frac{\dot{W}}{_1w_2} = \frac{-100 \text{ kW}}{-115.8\ \frac{\text{kJ}}{\text{kg}}} = \boxed{0.863558\ \frac{\text{kg}}{\text{s}}.}$$

The heat transferred from the condenser is

$$\dot{Q}_H = \dot{m}(h_3 - h_2) = \left(0.863558\ \frac{\text{kg}}{\text{s}}\right)\left(\left(274.3\ \frac{\text{kJ}}{\text{kg}}\right) - \left(1563.1\ \frac{\text{kJ}}{\text{kg}}\right)\right) = \boxed{-1112.95 \text{ kW}.}$$

After the adiabatic throttling valve, we find that h_4 is such that we have a two phase mixture. Knowing T_4, we conclude $P_4 = P_1 = 515.9$ kPa. We get the quality at state 4 using the tables and find

$$x_4 = \frac{h_4 - h_f}{h_{fg}} = \frac{\left(274.3\ \frac{\text{kJ}}{\text{kg}}\right) - \left(203.58\ \frac{\text{kJ}}{\text{kg}}\right)}{1243.7\ \frac{\text{kJ}}{\text{kg}}} = 0.0568626.$$

The heat transferred to the evaporator is

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = \left(0.863558\ \frac{\text{kg}}{\text{s}}\right)\left(\left(1447.3\ \frac{\text{kJ}}{\text{kg}}\right) - \left(274.3\ \frac{\text{kJ}}{\text{kg}}\right)\right) = 1012.95\ \text{kW}.$$

Note that the net heat in is

$$\dot{Q}_L + \dot{Q}_H = 1012.95\ \text{kW} - 1112.95\ \text{kW} = -100\ \text{kW},$$

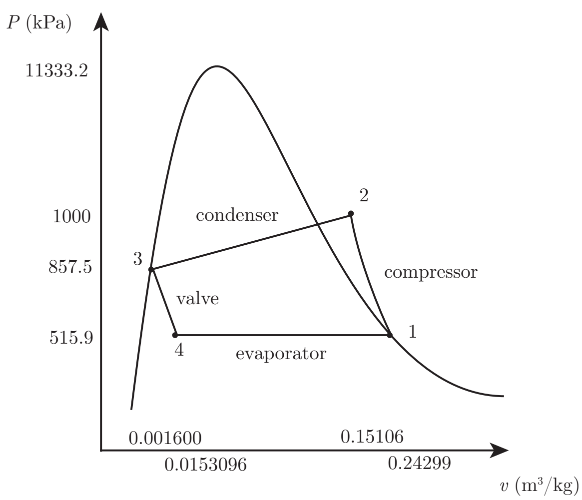
and this is exactly the net work out, $\dot{W} = -100$ kW.

So the coefficient of performance, taking care to account for the sign convention, is

$$\beta = \frac{\text{what you want}}{\text{what you pay for}} = \frac{|\dot{Q}_L|}{|\dot{W}|} = \frac{1012.95\ \text{kW}}{100\ \text{kW}} = \boxed{10.1295.}$$

Curiously, this is better than the Carnot refrigerator operating between the same temperature limits. Consider fixed T_L, which we have in both. The Carnot coefficient of performance of a refrigerator increases as T_H decreases to approach T_L, with $T_H > T_L$. Note that for the actual cycle, we have T_H decreasing from 60°C to 20°C. This enhances the coefficient of performance.

The plot of the cycle in the $P - v$ space is shown. Clearly the condenser is non-ideal as it suffers a



pressure drop due to friction in the pipes.

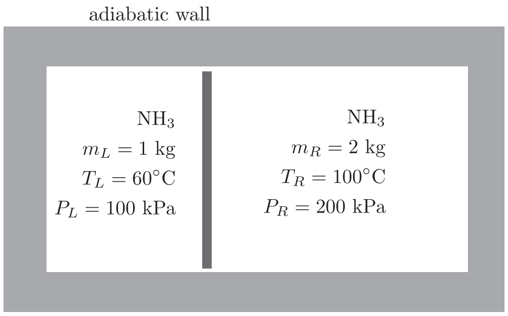
3. (40) An insulated chamber has two compartments, one on the left (L) and one on the right (R). Each is filled with ammonia NH_3 at the conditions indicated below. The partition is removed, and the system comes to thermal and mechanical equilibrium. Find the final temperature and pressure via two different assumptions:

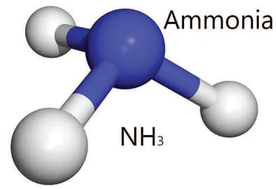
   (a) calorically perfect ideal gas, (use Table A.5),

   (b) non-ideal gas, (use Table B.2.2).

Solution

Performance on this problem was not good in general. On part a, far too many people used Table A.5 to tell us the density of NH_3 was $\rho = 0.694$ kg/m^3. Well, this is true at 25°C, 100 kPa. But our system





is not at this state, so the number is meaningless for this problem. The key to good performance on this problem was respect for mass and energy conservation principles for the mixture. There was no need to calculate any work or heat transfer, which many people tried to do. And for the non-ideal gas portion of the problem, a correct answer required a full commitment to using the tables and not to invoke the ideal gas law.

Let the subscript F denote the final state. Mass conservation gives us

$$m_F = m_L + m_R.$$

Energy conservation gives us for the combined system

$$dE = \delta Q - \delta W.$$

But there is no heat transfer or work for the combined system. So there is no total energy change:

$$dE = 0.$$

In terms of internal, kinetic, and potential energy, this gives

$$d(U + m\text{v}^2/2 + mgz) = 0.$$

Ignoring changes in kinetic and potential energy, we get

$$dU = 0.$$

Thus U is a constant, and using $U = mu$, we must have

$$m_L u_L + m_R u_R = m_F u_F.$$

Employing mass conservation, we then have

$$m_L u_L + m_R u_R = (m_L + m_R) u_F.$$

We could also say

$$m_L(u_L - u_F) + m_R(u_R - u_F) = 0.$$

Now let us consider the various cases

(a) *CPIG:* Here we have $du = c_v\, dT$. This reduces the first law to

$$m_L c_v (T_L - T_F) + m_R c_v (T_R - T_F) = 0.$$

But the c_v is the same for ammonia; thus,

$$m_L (T_L - T_F) + m_R (T_R - T_F) = 0.$$

We solve and get

$$T_F = \frac{m_L T_L + m_R T_R}{m_L + m_R}.$$

Let us convert the temperatures to absolute.

$$T_L = 60 + 273.15 = 333.15 \text{ K}.$$

$$T_R = 100 + 273.15 = 373.15 \text{ K}.$$

So

$$T_F = \frac{(1 \text{ kg})\,(333.15 \text{ K}) + (2 \text{ kg})\, 373.15 \text{ K}}{(1 \text{ kg}) + (2 \text{ kg})} = \boxed{359.817 \text{ K.}}$$

Note, if we had not converted to absolute temperature, we would have found $T_F = 86.6667°\text{C} = 359.15$ K. Now the ideal gas law gives

$$v_L = \frac{RT_L}{P_L}, \quad v_R = \frac{RT_R}{P_R}.$$

We check the tables and find $R = 0.4882$ kJ/kg/K for ammonia. So

$$v_L = \frac{\left(0.4882 \ \frac{\text{kJ}}{\text{kg K}}\right)(333.15 \text{ K})}{100 \text{ kPa}} = 1.62644 \ \frac{\text{m}^3}{\text{kg}}, \quad v_R = \frac{\left(0.4882 \ \frac{\text{kJ}}{\text{kg K}}\right)(373.15 \text{ K})}{200 \text{ kPa}} = 0.910859 \ \frac{\text{m}^3}{\text{kg}}.$$

Thus, with $V = mv$, we get

$$V_L = \frac{m_L R T_L}{P_L}, \quad V_R = \frac{m_R R T_R}{P_R}.$$

So

$$V_L = \frac{(1 \text{ kg})\left(0.4882 \ \frac{\text{kJ}}{\text{kg K}}\right)(333.15 \text{ K})}{100 \text{ kPa}} = 1.62644 \text{ m}^3,$$

$$V_R = \frac{(2 \text{ kg})\left(0.4882 \ \frac{\text{kJ}}{\text{kg K}}\right)(373.15 \text{ K})}{200 \text{ kPa}} = 1.82172 \text{ m}^3.$$

Now

$$V_F = V_L + V_R = 1.62644 \text{ m}^3 + 1.82172 \text{ m}^3 = 3.44816 \text{ m}^3.$$

Thus

$$v_F = \frac{V_F}{m_F} = \frac{3.44816 \text{ m}^3}{3 \text{ kg}} = 1.14939 \ \frac{\text{m}^3}{\text{kg}}.$$

Thus

$$P_F = \frac{R T_F}{v_F} = \frac{\left(0.4882 \ \frac{\text{kJ}}{\text{kg K}}\right)(359.817 \text{ K})}{1.14939 \ \frac{\text{m}^3}{\text{kg}}} = \boxed{152.832 \text{ kPa.}}$$

(b) *non-ideal gas:* We check the tables and find

$$u_L = 1441.5 \ \frac{\text{kJ}}{\text{kg}}, \quad v_L = 1.6156 \ \frac{\text{m}^3}{\text{kg}}, \quad u_R = 1508.2 \ \frac{\text{kJ}}{\text{kg}}, \quad v_R = 0.9028 \ \frac{\text{m}^3}{\text{kg}}.$$

We solve

$$m_L(u_L - u_F) + m_R(u_R - u_F) = 0,$$

to get

$$u_F = \frac{m_L u_L + m_R u_R}{m_L + m_R}.$$

Substitute numbers to get

$$u_F = \frac{(1 \text{ kg})\left(1441.5 \ \frac{\text{kJ}}{\text{kg}}\right) + (2 \text{ kg})\left(1508.2 \ \frac{\text{kJ}}{\text{kg}}\right)}{(1 \text{ kg}) + (2 \text{ kg})} = 1485.97 \ \frac{\text{kJ}}{\text{kg}}.$$

Now we easily see that

$$V_F = V_L + V_R,$$

$$V_F = m_L v_L + m_R v_R,$$

$$V_F = (1 \text{ kg})\left(1.6156 \ \frac{\text{m}^3}{\text{kg}}\right) + (2 \text{ kg})\left(0.9028 \ \frac{\text{m}^3}{\text{kg}}\right) = 3.4212 \text{ m}^3.$$

Thus

$$v_F = \frac{V_F}{m_F} = \frac{3.4212 \text{ m}^3}{3 \text{ kg}} = 1.1404 \ \frac{\text{m}^3}{\text{kg}}.$$

We now know two properties at the final state. We can double interpolate the tables to find the final temperature and pressure. This reveals that

$$T_F = \boxed{86.35^\circ\text{C},} \qquad P_F = \boxed{129.2 \text{ kPa.}}$$

The temperature is nearly identical to that given by the ideal gas law, but the pressure is lower.