NAME: SOLUTION
AME 20231, Thermodynamics
Examination 1
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1. (5) A volume of 10 m^3 contains 1 kmole of N_2 at 300 K. Assuming an ideal gas, find the mass contained in the volume and its pressure.

Solution

The molecular mass of N_2 is 28.013 kg/kmole. As we have 1 kmole, the mass contained in the volume is

$$m = \boxed{28.013 \text{ kg.}}$$

From the ideal gas law, we have

$$P = \frac{n\overline{R}T}{V}.$$

So

$$P = \frac{(1 \text{ kmole})\left(8.314 \frac{\text{kJ}}{\text{kmole K}}\right)(300 \text{ K})}{10 \text{ m}^3} = \boxed{249.42 \text{ kPa.}}$$

2. (15) N_2 has $P = 100$ kPa and $T = 105$ K. Estimate the specific volume via
   (a) the ideal gas law, and
   (b) the superheated N_2 tables.

Solution

For N_2, Table A.5 tells us $R = 0.2968$ kJ/kg/K. Thus for the ideal gas approximation of v, we find

$$v = \frac{RT}{P} = \frac{\left(0.2968 \frac{\text{kJ}}{\text{kg K}}\right)(105 \text{ K})}{100 \text{ kPa}} = \boxed{0.31164 \frac{\text{m}^3}{\text{kg}}.}$$

The superheated N_2 tables may be interpolated to estimate v as

$$v = \left(0.29103 \frac{\text{m}^3}{\text{kg}}\right) + \frac{\left(0.35208 \frac{\text{m}^3}{\text{kg}} - 0.29103 \frac{\text{m}^3}{\text{kg}}\right)}{120 \text{ K} - 100 \text{ K}}(105 \text{ K} - 100 \text{ K}) = \boxed{0.306293 \frac{\text{m}^3}{\text{kg}}.}$$

3. (30) N_2 has initially $T_1 = 70$ K, $x_1 = 0.001$.
   (a) Find P_1 and v_1 at the initial state.
   (b) The N_2 is heated in an isochoric process until a saturated state is reached with no gas present. Find the final P_2, T_2, and v_2.
   (c) Determine if the final phase is a solid, liquid, or gas.
   (d) Determine the work per unit mass done in the process.
   (e) Give an accurate sketch of the process in the $P-v$, $T-v$, and $P-T$ planes, including the vapor dome and critical point.

Solution

We find the initial P from Table B.6.1. It is

$$P_1 = \boxed{38.6 \text{ kPa.}}$$

We find the initial v from Table B.6.1 via

$$v_1 = v_f + x_1 v_{fg} = 0.001191 \ \frac{\text{m}^3}{\text{kg}} + 0.001 \left(.52513 \ \frac{\text{m}^3}{\text{kg}} \right) = \boxed{0.00171613 \ \frac{\text{m}^3}{\text{kg}}.}$$

For the final state for the isochoric process:

$$v_2 = v_1 = \boxed{0.00171613 \ \frac{\text{m}^3}{\text{kg}}.}$$

Table B.6.1 tells us that $v_c = 0.003194 \text{ m}^3/\text{kg}$. So $v_2 < v_c$. We deduce then that the final state must be a

$$\boxed{\text{saturated liquid.}}$$

The final temperature must be interpolated between 110 K and 115 K. It is

$$T_2 = 110 \text{ K} + \frac{115 \text{ K} - 110 \text{ K}}{0.001729 \ \frac{\text{m}^3}{\text{kg}} - 0.001610 \ \frac{\text{m}^3}{\text{kg}}} \left(0.00171613 \ \frac{\text{m}^3}{\text{kg}} - 0.001610 \ \frac{\text{m}^3}{\text{kg}} \right) = \boxed{114.459 \text{ K.}}$$

The final pressure must be interpolated between 1467.6 kPa and 1939.3 kPa. It is

$$P_2 = 1467.6 \text{ K} + \frac{1939.3 \text{ kPa} - 1476.6 \text{ kPa}}{0.001729 \ \frac{\text{m}^3}{\text{kg}} - 0.001610 \ \frac{\text{m}^3}{\text{kg}}} \left(0.00171613 \ \frac{\text{m}^3}{\text{kg}} - 0.001610 \ \frac{\text{m}^3}{\text{kg}} \right) = \boxed{1889.26 \text{ kPa.}}$$

Because the process is isochoric, there is no work:

$$_1w_2 = \boxed{0 \ \frac{\text{kJ}}{\text{kg}}.}$$

Figure 1 shows process in the various planes. Note that we must have $v_1 = v_2 < v_c$. State 2 must lie on the dome in the $T - v$ and $P - v$ planes, not above or below the dome. In the $P - T$ plane, the path must always lie on the liquid-vapor equilibrium curve.

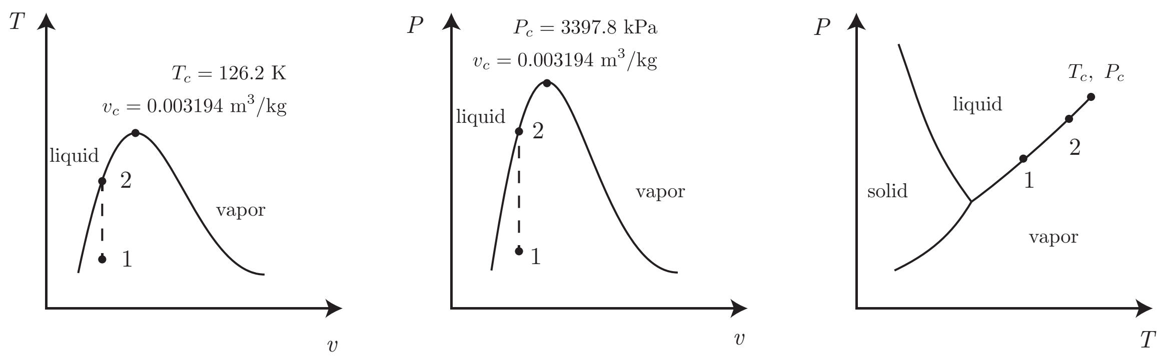

Figure 1: Isochoric compression of N_2 in the $T - v$, $P - v$, and $P - T$ planes (not to scale).

4. (50) N_2 is initially at $T_1 = 100$ K, $P_1 = 100$ kPa. It is isothermally compressed to $P_2 = 600$ kPa. It is then isochorically decompressed to $P_3 = 100$ kPa. It is then isobarically expanded to its initial state of $P_4 = P_1 = 100$ kPa, $T_4 = T_1 = 100$ K.

   (a) Determine T_3 and the net work per unit mass of the cycle under the assumption that the ideal gas equation of state is valid for the entire cycle.

(b) Determine T_3 and the net work per unit mass of the cycle under the assumption that the N_2 tables give the equation of state for the entire cycle.

(c) Give an accurate sketch of the process in the $P-v$, $T-v$, and $P-T$ planes, including the vapor dome and critical point.

Solution

First make the assumption of an ideal gas. For N_2, we have $R = 0.2968$ kJ/kg/K. We have $T_1 = 100$ K, $P_1 = 100$ kPa. So

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(0.2968 \frac{\text{kJ}}{\text{kg K}}\right)(100 \text{ K})}{100 \text{ kPa}} = 0.2968 \frac{\text{m}^3}{\text{kg}}.$$

For the isothermal compression, we have $T_2 = T_1 = 100$ K. And $P_2 = 600$ kPa. So

$$v_2 = \frac{RT_2}{P_2} = \frac{\left(0.2968 \frac{\text{kJ}}{\text{kg K}}\right)(100 \text{ K})}{600 \text{ kPa}} = 0.0494667 \frac{\text{m}^3}{\text{kg}}.$$

Now for the isochoric decompression, we have $P_3 = 100$ kPa, $v_3 = v_2 = 0.0494667$ m^3/kg. So using the ideal gas law, we get

$$T_3 = \frac{P_3 v_3}{R} = \frac{(100 \text{ kPa})\left(0.0494667 \frac{\text{m}^3}{\text{kg}}\right)}{0.2968 \frac{\text{kJ}}{\text{kg K}}} = \boxed{16.6667 \text{ K.}}$$

Indeed this is a very low value and tells us the ideal gas assumption is not a good one, as the nitrogen has begun a phase transition. Let us proceed however under the ideal gas assumption.

We are now ready to calculate the work per unit mass under the ideal gas assumption. It is

$$w_{cycle} = {}_1w_2 + {}_2w_3 + {}_3w_1.$$

$$w_{cycle} = \int_1^2 P\, dv + \int_2^3 P\, dv + \int_3^1 P\, dv.$$

Now for the ideal gas, the isothermal 1 to 2 has $P = RT_1/v$. The isochoric 2 to 3 has zero work, and the isobaric 3 to 1 has $P = P_1$. So we get

$$w_{cycle} = RT_1 \int_1^2 \frac{dv}{v} + P_1 \int_3^1 dv.$$

Integrating gives

$$w_{cycle} = RT_1 \ln \frac{v_2}{v_1} + P_1(v_1 - v_3).$$

Substituting numbers, we get

$$w_{cycle} = \underbrace{\left(0.2968 \frac{\text{kJ}}{\text{kg K}}\right)(100 \text{ K}) \ln \frac{0.0494667 \frac{\text{m}^3}{\text{kg}}}{0.2968 \frac{\text{m}^3}{\text{kg}}}}_{-53.1794 \frac{\text{kJ}}{\text{kg}}} + \underbrace{(100 \text{ kPa})\left(0.2968 \frac{\text{m}^3}{\text{kg}} - 0.0494667 \frac{\text{m}^3}{\text{kg}}\right)}_{24.7333 \frac{\text{kJ}}{\text{kg}}}.$$

$$w_{cycle} = \boxed{-28.4461 \frac{\text{kJ}}{\text{kg}}.}$$

Now make the assumption of a non-ideal gas and use Table B.6 to assist. We find

$$v_1 = 0.29103 \frac{\text{m}^3}{\text{kg}}.$$

We know $T_2 = 100$ K and $P_2 = 600$ kPa. The superheated N_2 table tells us that $v_2 = 0.04299$ m^3/kg. Then for the isochoric decompression, we have $v_3 = v_2 = 0.04299$ m^3/kg. The tables tell us this is a two-phase liquid-vapor mixture. We interpolate to find

$$T_3 = (75 \text{ K}) + \frac{77.3 \text{ K} - 75 \text{ K}}{(101.3 \text{ kPa} - 76.1 \text{ kPa})}(100 \text{ kPa} - 76.1 \text{ kPa}) = \boxed{77.1813 \text{ K.}}$$

We are now ready to calculate the work per unit mass under the non-ideal gas assumption. It is

$$w_{cycle} = {}_1w_2 + {}_2w_3 + {}_3w_1.$$

$$w_{cycle} = \int_1^2 P\, dv + \int_2^3 P\, dv + \int_3^1 P\, dv.$$

Now for the non-ideal gas, the isothermal 1 to 2 has $P(v)$ from the tables. We have values at four pressures, 100 kPa, 200 kPa, 400 kPa, and 600 kPa. The isochoric 2 to 3 has zero work, and the isobaric 3 to 1 has $P = P_1$. So we get

$$w_{cycle} = \int_1^2 P\ dv + P_1 \int_3^1 dv.$$

$$w_{cycle} = \int_1^2 P\ dv + P_1(v_1 - v_3).$$

We approximate the isothermal portion by the trapezoidal rule. We can summarize the calculations in Table 1. We see

Table 1: Tabular calculation of work.

P	v	P_i^{ave}	Δv_i	$P_i^{ave}\Delta v_i$
kPa	m^3/kg	kPa	m^3/kg	kJ/kg
100	0.29103	150	-0.14851	-22.2765
200	0.14252	300	-0.07446	-22.338
400	0.06806	500	-0.02507	-12.535
600	0.04299	-	-	-
				-57.1495

$$w_{cycle} = -57.1495\ \frac{\text{kJ}}{\text{kg}} + (100\ \text{kPa})\left(0.29103\ \frac{\text{m}^3}{\text{kg}} - 0.04299\ \frac{\text{m}^3}{\text{kg}}\right).$$

$$w_{cycle} = -57.1495\ \frac{\text{kJ}}{\text{kg}} + 24.804\ \frac{\text{kJ}}{\text{kg}}.$$

$$w_{cycle} = \boxed{-32.3455\ \frac{\text{kJ}}{\text{kg}}.}$$

Figure 2 shows the cycle in the various planes. It is important to have the cycle correctly positioned in

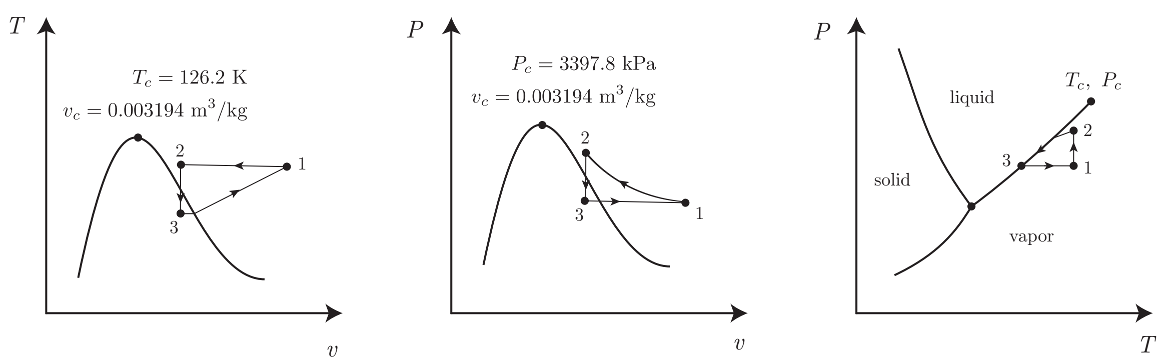

Figure 2: Cycle for N_2 in the $T-v$, $P-v$, and $P-T$ planes (not to scale).

the various planes. Certainly, a portion of the cycle must be in the vapor state, and a portion must be under the dome.