

ME231
Measurements Laboratory
Spring 2003

Basic Signal Analysis

Edmundo Corona ©

So, what is a signal? The word has several meanings, but the Webster's definition that applies here is: *a detectable physical quantity or impulse by which messages or information can be transmitted*. In other words, it is the vehicle that carries information from one time and place to another. Following this reasoning, the ink and paper you are looking at now is the signal that contains the information I am trying to put in your brain. Am I succeeding? Many times, information will travel from its place of origin to its destination using different types of signals. Let's look at an example.

Figure 1 shows a simple schematic of radio communication. Let's say that the broadcaster wants to transmit the words "good morning." The origin of the information is the broadcaster's brain ①, where the message is encoded in an electric signal that travels through nerves ② to muscles in his/her throat and mouth. There, the information is transferred from the nervous electrical signal to an acoustic signal ③, which basically consists of pressure waves propagating through the air. The microphone in front of the broadcaster converts the acoustic signal, and the information it contains, into an electrical signal¹. The radio equipment conditions the electric signal to make it ready for transmission from the antenna. The antenna takes the electrical signal and broadcasts the

¹The details of the transmission of electric signals in nerves and copper wire are different but, for simplicity's sake we will just concern ourselves with the fact that they are electrical.

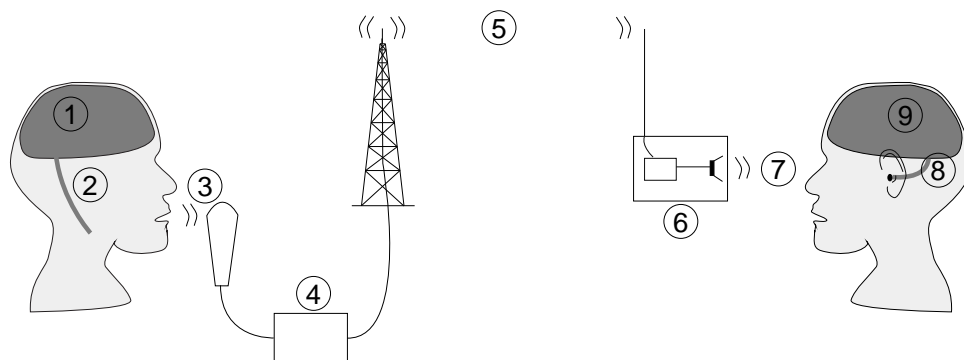


Figure 1: Signals in radio communication.

information using electro-magnetic radiation, in the form of a radio signal ⑤. At the receiving end of the transmission, the antenna of the radio receiver transfers the information from the radio wave to an electrical signal ⑥ and conditions it to drive a speaker, which puts the information into an acoustic signal ⑦. The listener's ear encodes the information contained in the acoustical signal into an electric signal which travels through a nerve ⑧ to his/her brain ⑨, which is the final destination of the information in this example.

From the example above, we can also see that we can perceive some signals with our senses, mainly our eyes (optical signals) and our ears (acoustic signals). Other signals, however, such as radio waves we cannot detect. Of the signals we can detect, some we can interpret rather automatically or with some training, while others we find very difficult to interpret. Many signals encountered in engineering practice belong to the groups of signals we either cannot detect or cannot interpret automatically. Therefore, we must develop tools to find and process the information encoded in many signals of practical interest.

The first such tool is rather simple. It is a plot of the signal as a function of time. One example is shown in Fig. 2. The value of the signal is $y(t)$, along the vertical axis, while time, t , measured in seconds, runs on the horizontal axis.

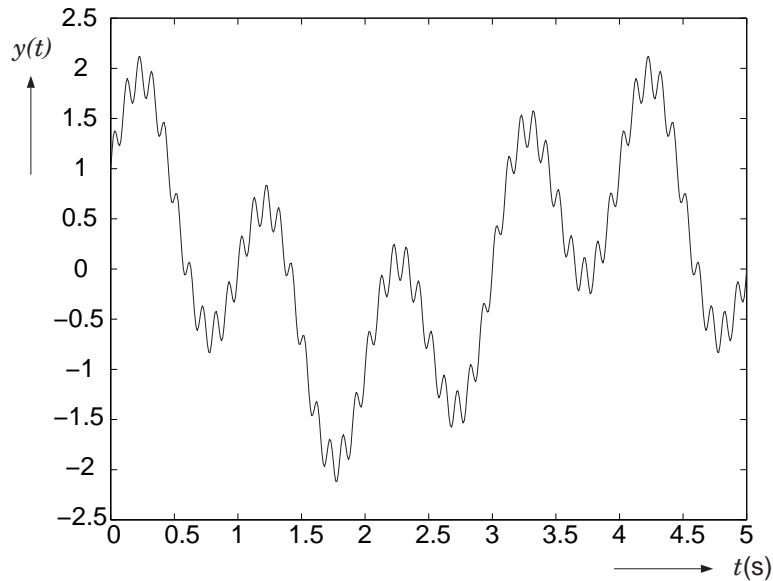


Figure 2: Graph of the signal $y(t)$ vs. time.

Visualizing a signal with a plot is always a good idea. Many times that is all we need to decode the information we are looking for. For example, a doctor looking at the plot of an EKG can find information regarding how well a patients' heart is working. In the example shown in Fig. 2, we can see that the value of the signal in the period of time shown rises to a bit above two and falls to a bit below -2. This is an indication of the *amplitude* of the signal, in other words how large the signal is. Looking at the amplitude can yield important information. For example, looking at the amplitude of the record of a seismographer can give some indication of the severity of an earthquake.

Another quantity we can sometimes assess from graphs of signals that are fairly regular is the frequency at which the signal is oscillating. In the example, we can see that we have a very low frequency oscillation that takes about 4 seconds to complete, in other words it has a *period* of 4 seconds. The *frequency* is the reciprocal of the period and is 0.25 Hertz. In addition we see the signal contains two faster oscillations, one at one Hz and the fastest one at 10 Hz.

In other applications, looking at a graph does not yield the information we are looking for. In those cases, more sophisticated analysis tools are required. We will look at some of these tools later. For now, we will consider two simple quantities which yield some basic information about signals: the mean and the RMS² values.

The mean of a signal over a period of time from t_1 to t_2 is given by

$$\bar{y} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} y(t) dt \quad (1)$$

For example, the mean of a sine wave $y = A \sin(2\pi t)$ over a time interval of one period³—one second in this case—is zero, but its mean over half a period is equal to A/π . The mean of the same function over a very long period of time would be zero. Why?

In the example above it is clear that the mean may not be a very good measure of the amplitude of some signals, such as a sine wave over a long period of time. The reason is that a sine has both positive and negative values and they tend to cancel each other in the calculation of the mean. One trick we can play is to square of the signal and then calculate the mean, that way all the values to be integrated will be positive. The only problem is that the mean we calculate will be related to the square of the amplitude. This can be easily fixed by taking the square root of the mean of the squares. In other words we define the RMS (Root-Mean-Square) of a signal by

$$y_{\text{RMS}} = \sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} y(t)^2 dt}. \quad (2)$$

For example, the RMS value of a sine wave of amplitude A over one period (or over a long period of time) is given by $y_{\text{RMS}} = A/\sqrt{2}$.

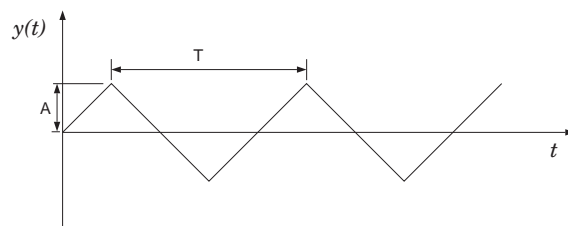
²RMS stands for Root Mean Square.

³Here A is the amplitude of the signal. Note that the argument of the sine will be a multiple of 2π every time t is a multiple of one. This is how we can tell that the period is one second.

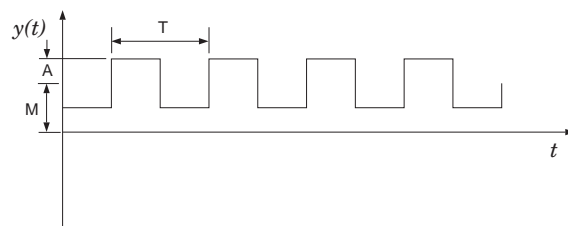
Calculating the mean and the RMS values of a signal are the simplest mathematical operations we can perform in an effort to extract information from a signal. In many cases, they are sufficient for our purposes. Since they do not provide frequency information, however, they may not be sufficient to extract all the information we seek in a signal. We will look at ways of analyzing a signal to extract frequency information in the near future.

Exercises

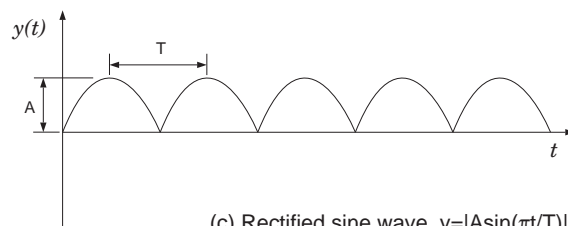
1. Find the mean and RMS values of the periodic signals below over a very long period of time.



(a) Sawtooth wave

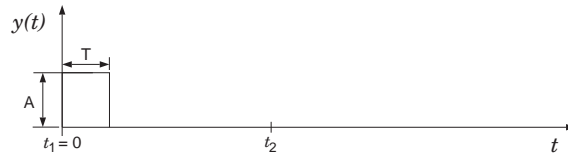


(b) Square wave



(c) Rectified sine wave, $y=|A\sin(\pi t/T)|$

2. Find the mean and RMS values of the signal below as functions of the time period considered. Let t_2 range from zero to infinity.



3. Some instruments calculate the RMS value of the quantity $y - \bar{y}$. This quantity is called the AC value. Calculate the AC value of the signals in problem 1.