

On the Impact of Cooperation on Local Delay and Energy Efficiency in Poisson Networks

Libin Liu, Yi Zhong, Wenyi Zhang, and Martin Haenggi

Abstract—Local delay characterizes the number of time slots required for a node to successfully transmit a packet to its target receiver. In this letter, we explore the impact of coordinated transmission on the mean local delay and the energy efficiency, in a Poisson network model. Leveraging tools from point process theory, we derive analytical expressions of the mean local delay and the long-term energy efficiency, without and with coordinated transmission. We find that coordinated transmission greatly decreases the mean local delay, and also improves the long-term energy efficiency under certain network configurations.

Index Terms—Coordinated transmission, energy efficiency, local delay, Poisson point process, stochastic geometry

I. INTRODUCTION

Delay is a crucial performance metric that affects user experience. Recently, tools from point process theory have been applied to study access delay performance in wireless networks. In this letter, we explore potential benefits of coordinated transmission for the local delay of Poisson networks. Meanwhile, as coordinated transmission incurs additional cost in terms of energy consumption, we also conduct a comparative study between the energy efficiencies without and with coordinated transmission.

Related works on local delay are as follows. The general framework for local delay analysis has been established in [1][2][3], wherein a notable observation is that for static networks, local delay may have diverging mean and variance, in the absence of cooperation. The work in [4] evaluated the local delay for the case of finite mobility. In [5], power control policies were proposed to minimize the local delay. In [6], local delay was studied for clustered networks.

II. SYSTEM MODEL

We model the spatial distribution of the transmitters as a Poisson point process (PPP) $\Phi = \{x_i\} \subset \mathbb{R}^2$ of intensity λ . Throughout this letter, the analysis is conditioned on $x_0 \in \Phi$, which is the considered transmitter. We assume that its corresponding receiver is located at the origin o and thus $R = |x_0|$ is the distance between the considered transmitter and receiver.

For simplicity, we assume that the transmit power is P for all transmitters, and that the thermal noise is negligible,

i.e., an interference-limited scenario.¹ Time is divided into discrete time slots with equal duration Δt . Let α be the path loss exponent, and consider spatially and temporally independent Rayleigh fading, i.e., $h_x \sim \mathcal{CN}(0, 1)$ denoting the fading coefficient between transmitter x and the considered receiver. We adopt an SIR-thresholding reception model with a threshold of θ ; that is, as long as the SIR exceeds θ , transmission is successful with spectral efficiency $\log_2(1 + \theta)$ bits/s/Hz. For normalization purposes, we assume that a packet needs exactly one successful transmission, and the amount of information per packet is $W \Delta t \log_2(1 + \theta)$ bits, where W denotes the bandwidth. For coordinated transmission, we adopt a model proposed in [7], in which the signal-to-interference ratio (SIR) of the considered receiver is

$$\text{SIR} = \frac{|\sum_{x \in A} P^{1/2} |x|^{-\alpha/2} h_x|^2}{\sum_{x \in A^c} P |x|^{-\alpha} |h_x|^2}, \quad (1)$$

wherein all the transmitters in the cooperating set A jointly serve the considered receiver by sending the same signal so that their signals are superimposed, and A^c is the set that cause interferences, which are mutually independent and thus are combined as power superposition. We let A consist of $\{x_0\}$ and n transmitters in Φ that are nearest to the considered receiver at the origin. Note that when $A = \{x_0\}$, there is no cooperation and the model degenerates into non-coordinated transmission. We also consider a silencing scheme, in which all transmitters except $\{x_0\}$ in A are silent, and thus in the SIR expression (1) the nominator is simply $P |x_0|^{-\alpha} |h_{x_0}|^2$.

Beyond the SIR model (1) we can further incorporate random access into the transmission model. For tractability we focus on a slotted ALOHA protocol in this letter. Each transmission occurs with probability p , and a retransmission will be attempted, with probability p as well, if the transmission fails. The local delay is defined as the number of time slots for a packet to be successfully received, and in this letter we focus on the mean local delay [1][2][3]. When considering coordinated transmission, the random access protocol is modified so that in a time slot, all transmitters in A simultaneously transmit to the receiver with probability p or keep silent with probability $1 - p$. We call this a *coordinated ALOHA*. The SIR in time slot k in which the transmitters in A are scheduled to transmit thus is

$$\text{SIR}_k = \frac{|\sum_{x \in A} P^{1/2} |x|^{-\alpha/2} h_x|^2}{\sum_{x \in A^c} P |x|^{-\alpha} |h_x|^2 \mathbf{1}(x \in \Phi_k)} \quad (2)$$

L. Liu, Y. Zhong and W. Zhang are with the Department of Electronic Engineering and Information Science, University of Science and Technology of China, Hefei 230027, China (email: {liulibin, geners}@mail.ustc.edu.cn, wenyizha@ustc.edu.cn). M. Haenggi is with the Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN 46556, USA (email: mhaenggi@nd.edu).

¹The effect of noise can be readily addressed; see, e.g., [3]. The resulting expressions will contain an additional scaling factor which depends on the noise power only, thus having relatively mild impact on the main findings of this letter.

where $\Phi_k \in \Phi$ is the transmitting set in time slot k and $\mathbf{1}(\cdot)$ is the indicator function. Note that when $p = 1$, (2) degenerates into (1).

The preceding models may be used to describe ad hoc networks, wireless LAN, or randomly deployed small cells, among other possibilities.

For transmitting a packet, the failed time slots can be classified into two types: those that have not been scheduled by ALOHA and hence in sleep mode,² and those that have been scheduled but failed due to $\text{SIR} < \theta$. Suppose that the mean local delay is $D(p)$. On average, $p(D(p) - 1)$ time slots have been scheduled for transmission but failed, and $(1 - p)(D(p) - 1)$ time slots have been in sleep mode. Eventually, the transmit attempt in the $D(p)$ -th time slot succeeds. The expectation of the total energy consumption for a packet hence is

$$E = [D(p) - 1] [p \cdot P_1 + (1 - p) \cdot P_2] \Delta t + P_1 \Delta t, \quad (3)$$

where P_1 and P_2 are the consumed powers when a transmitter is in transmit and sleep modes, respectively.

The energy efficiency η is defined as the ratio between the amount of information in a packet and the expectation of the total energy consumption for successfully transmitting the packet. For non-coordinated transmission and silencing, we have

$$\eta = \frac{W \log_2(1 + \theta)}{[D(p) - 1] \cdot [p \cdot P_1 + (1 - p) \cdot P_2] + P_1}. \quad (4)$$

Throughout this letter, we will use subscripts n, s, and c to denote the performance metrics for non-coordinated transmission, silencing, and coordinated transmission, respectively. For coordinated transmission, there is an additional factor of $1/(n+1)$, because the energy consumption is multiplied when $(n+1)$ transmitters simultaneously serve one receiver, i.e.,³

$$\eta_c = \frac{1}{n+1} \frac{W \log_2(1 + \theta)}{[D_c(p) - 1] \cdot [p \cdot P_1 + (1 - p) \cdot P_2] + P_1}. \quad (5)$$

In this letter, the numerical study is based on the following parameters: $P_1 = 8.8\text{W}$, $P_2 = 5\text{W}$, $\lambda = 0.0001\text{m}^{-2}$, $n = 1$, $\theta = 0\text{dB}$, and $|x_0| = R$ as a tunable parameter.

III. THE CASE WITHOUT RANDOM ACCESS

The performance for the SIR model (1) without random access is given by the following proposition.

Proposition 1. *Without random access (i.e., $p = 1$), $D_n = D_s = \infty$ while $D_c < \infty$, and correspondingly $\eta_n = \eta_s = 0$ while $\eta_c > 0$.*

The proof of Proposition 1 as well as that of all the propositions in this letter can be found in Appendix.

Proposition 1 reveals that coordinated transmission achieves finite mean local delay. This is a surprising result, in view of the fact that D_n diverges for non-coordinated transmission,

²Here we assume that when a transmitter is not scheduled to transmit it then enters a sleep mode, which is a favorable condition for energy efficiency.

³This scaling also exists when the cooperating transmitters serve $n+1$ receivers through round-robin time division, because then the mean local delay for each receiver is increased by a factor of $n+1$.

which is partly the reason why random access (i.e., $p < 1$) was necessitated in [3]. Furthermore, silencing nearest interferers is ineffective in reducing the mean local delay.

A consequence of divergent D_n and D_s is that the corresponding energy efficiency vanishes since the average energy consumption for successively transmitting a packet also diverges with the mean local delay.

IV. THE CASE WITH RANDOM ACCESS

A. Mean Local Delay

For non-coordinated transmission, the mean local delay has been obtained in [3, Eq (14)], as

$$D_n(p) = \frac{1}{p} \exp\left(\frac{pB}{(1-p)^{1-\delta}}\right) \quad (6)$$

where $B = \lambda \pi R^2 \theta^\delta C(\delta)$, $\delta = 2/\alpha$, and $C(\delta) = 1/\text{sinc}(\delta)$.

For coordinated transmission and silencing, the mean local delay is given by the following proposition.

Proposition 2. *With random access ALOHA, the mean local delay for coordinated transmission is*

$$D_c(p) = \frac{1}{p} \int_{0 < d_1 < \dots < d_n < \infty} \exp\left\{2\pi\lambda \int_{d_n}^{\infty} \frac{p\theta K_c r^{-\alpha}}{1 + (1-p)\theta K_c r^{-\alpha}} r dr\right\} e^{-\lambda\pi d_n^2} (2\pi\lambda)^n \prod_{i=1}^n (d_i \cdot dd_i), \quad (7)$$

where $K_c = (R^{-\alpha} + \sum_{i=1}^n d_i^{-\alpha})^{-1}$; the mean local delay for silencing, $D_s(p)$, is also given by (7), except for replacing K_c with $K_s = R^\alpha$.

B. Analysis of Results

A property of non-coordinated transmission and silencing is given by the following proposition.

Proposition 3. *There exists a unique $p \in (0, 1)$ at which $D_n(p)$ is minimized and thus η_n is maximized. The same statement also holds true for $D_s(p)$ and η_s .*

On the other hand, several properties of coordinated transmission are given by the following proposition.

Proposition 4. *For coordinated transmission, $D_c(p)$ has the following properties:*

- (a) $D_c(p) < D_s(p) < D_n(p)$ for every $p \in (0, 1]$;
- (b) $D_c(p)$ is finite for every $p \in (0, 1]$;
- (c) there exists a critical distance R_0 such that: in the regime of $R \leq R_0$, $D_c(p)$ decreases monotonically with $p \in (0, 1]$, and in the regime of $R > R_0$, $D_c(p)$ is minimized by some $p \in (0, 1)$.

Fig. 1(a) displays the mean local delays in (6) and Proposition 2. We can observe that $D_n(1)$ and $D_s(1)$ go to infinity while $D_c(1)$ remains bounded. Interestingly, the gap between $D_c(p)$ and $D_s(p)$ is largely negligible for the regime of $R \leq R_0$, except for p near one. The gap becomes substantial for the regime of $R > R_0$, implying that then coordinated transmission is more robust against interference correlation than silencing.

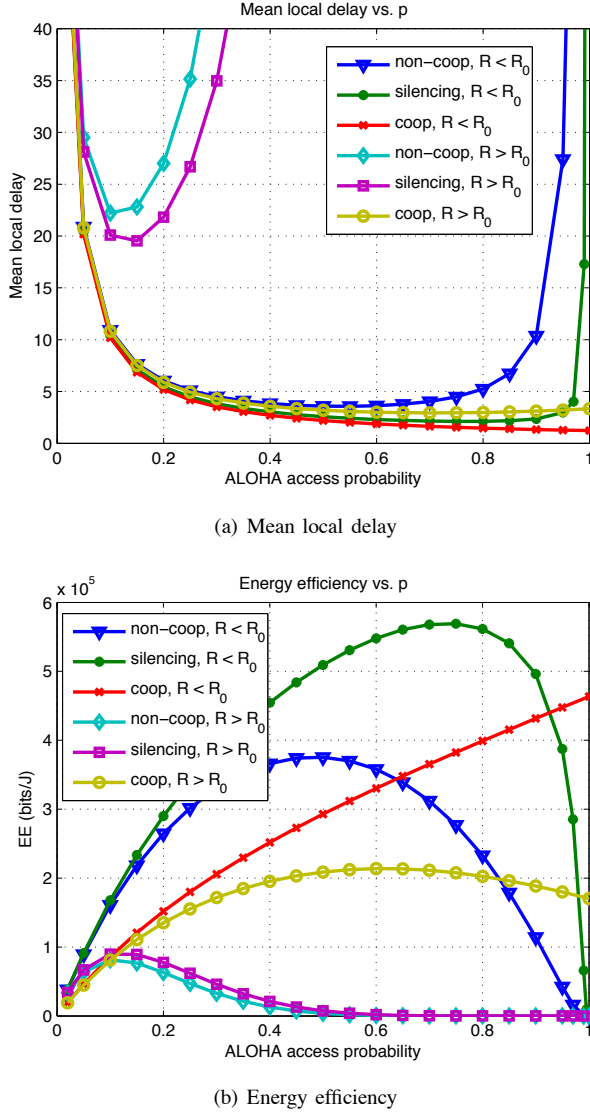


Fig. 1. Comparison of mean local delays and energy efficiencies: non-coordinated transmission vs. coordinated transmission vs. silencing. We set $R = 40\text{m}$ or 120m .

Inspecting Proposition 4, Property (a) coincides with our intuition that coordinated transmission decreases the local delay compared with non-coordinated transmission and silencing.

Property (b) indicates that coordinated transmission always achieves finite values of $D_c(p)$, even for $p = 1$. So coordinated transmission does greatly help improve the local delay performance, in a fundamental way. We also remark that, nevertheless, if the SIR threshold θ is chosen inappropriately, $D_c(1)$ can be finite yet very large. For example, taking θ as 5dB, 10dB, 15dB and 20dB, the corresponding mean local delays are $1.47, 3.52, 88.51$ and 9.18×10^6 time slots, respectively.

As for Property (c), the value of the critical distance R_0 can be calculated numerically. For example, with the parameters listed in the end of Section II, we have $R_0 = 82.2\text{m}$. As illustrated in Fig. 1(a), in the regime of $R \leq R_0$, $D_c(p)$ decreases monotonically, leading to the optimal $p = 1$, i.e., without random access. This regime may be called the “access-

limited” regime, in that the optimal performance demands a full access to the channel. In the regime of $R > R_0$, however, the interference correlation due to non-coordinating transmitters is still substantial as p gets large, so the optimal choice is to tune p appropriately below one, using random access. Therefore, this regime may be called the “interference-limited” regime. Alternatively, when fixing R , there exists a critical intensity λ_0 which divides the network intensity into a “sparse” regime ($\lambda \leq \lambda_0$) and a “dense” regime ($\lambda > \lambda_0$).

Given the mean local delay, the energy efficiency can be evaluated as displayed in Fig. 1(b). We observe that η_n and η_s initially increase but then decrease towards zero with p , while η_c either monotonically increases (in the regime of $R \leq R_0$) or initially increases and then decreases towards a positive value with p (in the regime of $R > R_0$). The curves of η_n (or η_s) and η_c intersect at one point denoted by p_{0n} (or p_{0s}). When $p > p_{0n}$ (or p_{0s}), we have $\eta_c > \eta_n$ (or η_s). But when $p \leq p_{0n}$ (or p_{0s}), the mean local delay does not benefit much from coordinated transmission, while the energy consumption is multiplied due to coordination. Hence the impact of coordinated transmission on energy efficiency becomes indeed negative, and it is preferable to adopt silencing. When $p > p_{0n}$ (or p_{0s}), coordinated transmission becomes necessary to withstand the excess interference correlation, and even though it incurs multiplied energy consumption, such cost is outweighed by the advantage of decreasing the mean local delay.

V. CONCLUSION

In this letter, we have studied the mean local delay and energy efficiency without and with coordinated transmission, in a static Poisson network. We have reached several useful conclusions, as follows.

(a) For non-coordinated transmission, the mean local delay is unbounded and correspondingly the energy efficiency vanishes, unless a random access protocol like ALOHA is employed to mitigate the interference correlation.

(b) Coordinated transmission substantially decreases the mean local delay and increases the energy efficiency. In particular the mean local delay is bounded even without random access. There exists a critical distance between the considered transmitter and receiver, dividing the network configuration into two regimes with distinct behaviors.

(c) Interference silencing is insufficient to reduce the mean local delay from infinity without random access. But when random access is employed, it may lead to improved energy efficiency performance compared with coordinated transmission.

APPENDIX: PROOFS

1) *Proof of Proposition 1:* For non-coordinated transmission, from (6) it is apparent that $D_n(1)$ diverges as $p \rightarrow 1$.

For silencing and coordinated transmission, this proposition is a corollary of Proposition 2. Letting $p = 1$ in (7), we have

$$D_s(1) = \frac{2(\pi\lambda)^n}{(n-1)!} \int_0^\infty \exp\left\{\frac{2\pi\lambda\theta R^\alpha}{\alpha-2} r^{2-\alpha}\right\} e^{-\pi\lambda r^2} r^{2n-1} dr$$

which is divergent because it has a discontinuity point of the second kind at $r = 0$.

Regarding $D_c(1)$, since $K_c = (R^{-\alpha} + \sum_{i=1}^n d_i^{-\alpha})^{-1} \leq (R^{-\alpha} + d_n^{-\alpha})^{-1}$, we have

$$D_c(1) \leq M \int_0^\infty \exp \left\{ \pi \lambda \left[\frac{2\theta}{(\alpha-2)(1+R^{-\alpha}r^\alpha)} - 1 \right] r^2 \right\} r dr$$

where $M = 2(\pi\lambda)^n / (n-1)!$. Letting $r_c = \left(\frac{2\theta}{\alpha-2} - 1 \right)^{1/\alpha} R$, the integral upper bounding $D_c(1)$ above can be divided into

$$D_c(1) < M \int_0^{r_c} (\cdot) dr + M \int_{r_c}^\infty (\cdot) dr. \quad (8)$$

According to the integral mean value theorem, the first integral in (8) is bounded; since the exponent is negative when $r > r_c$ and $\int_0^\infty x^s e^{-tx} dx$ is convergent for all $s, t > 0$, the second integral in (8) is also bounded. This proves that $D_c(1)$ is bounded.

2) *Proof of Proposition 2:* For coordinated transmission, the success probability conditioned on Φ is

$$\begin{aligned} \mathbb{P}(C_\Phi) &= p\mathbb{P}(\text{SIR}_k > \theta|\Phi) \\ &= p\mathbb{P} \left\{ \left| \sum_{x \in A} P^{1/2} |x|^{-\alpha/2} h_x \right|^2 > \right. \\ &\quad \left. \theta \sum_{x \in A^c} P |x|^{-\alpha} |h_x|^2 \mathbf{1}(x \in \Phi_k) \right\} \Big| \Phi \end{aligned}$$

Since $|\sum_{i=1}^n P^{1/2} |x_i|^{-\alpha/2} h_x|^2 \sim \exp((\sum_{i=1}^n P |x_i|^{-\alpha})^{-1})$, we have

$$\begin{aligned} \mathbb{P}(C_\Phi) &= p\mathbb{E} \left\{ \exp \left[-\frac{\theta \sum_{x \in A^c} P |x|^{-\alpha} |h_x|^2 \mathbf{1}(x \in \Phi_k)}{\sum_{d \in A} P |d|^{-\alpha}} \right] \Big| \Phi \right\} \\ &= p \prod_{x \in A^c} \left\{ p\mathbb{E}_h \left[\exp \left(-\frac{\theta |x|^{-\alpha} |h_x|^2}{\sum_{x \in A} |d|^{-\alpha}} \right) \Big| \Phi \right] + 1 - p \right\} \\ &\stackrel{(a)}{=} p \prod_{x \in A^c} \left\{ \frac{p}{1 + \theta K_c |x|^{-\alpha}} + 1 - p \right\} \end{aligned}$$

where $K_c = (R^{-\alpha} + \sum_{i=1}^n d_i^{-\alpha})^{-1}$, and (a) follows from that $|h_x|^2$ are i.i.d $\text{Exp}(1)$. Now let us evaluate the mean local delay $D_c(p) = \mathbb{E}_\Phi(1/\mathbb{P}(C_\Phi))$. For this, let us first condition upon the n nearest transmitters in A , and evaluate the conditional expectation of $1/\mathbb{P}(C_\Phi)$, to obtain

$$\begin{aligned} &\mathbb{E}_\Phi \left(\frac{1}{\mathbb{P}(C_\Phi)} \Big| A \right) \\ &\stackrel{(b)}{=} \frac{1}{p} \exp \left\{ -2\pi\lambda \int_{d_n}^\infty \left[1 - \frac{1}{\frac{p}{1+\theta K_c r^{-\alpha}} + 1 - p} \right] r dr \right\} \end{aligned}$$

where (b) follows from the probability generating functional (PGFL) of the PPP.

Then, let us use the joint distribution of the nearest points in a PPP [9]:

$$\begin{aligned} f(d_1, \dots, d_n) &= e^{-\pi\lambda d_n^2} (2\pi\lambda)^n \prod_{i=1}^n d_i \\ 0 &< d_1 < \dots < d_n < \infty \end{aligned}$$

to obtain

$$D_c(p) = \int_{0 < d_1 < \dots < d_n < \infty} \mathbb{E}_\Phi \left(\frac{1}{\mathbb{P}(C_\Phi)} \Big| A \right) \cdot f(d_1, \dots, d_n) dd_1 \dots dd_n$$

which is exactly (7) after some algebraic manipulations.

For silencing, the evaluation of $D_s(p)$ is analogous, and the only difference is to replace $K_c = (R^{-\alpha} + \sum_{i=1}^n d_i^{-\alpha})^{-1}$ with $K_s = R^\alpha$ due to the lack of coordinated transmitters.

3) *Proof of Proposition 3:* From (6) we observe that $D_n(p)|_{p=0,1} = \infty$, and from (4) we get $\eta_n(p)|_{p=0,1} = 0$. The existence and uniqueness of the optimal $p \in (0, 1)$ is readily verified by taking derivatives. The verification for silencing is similar.

4) *Proof of Proposition 4:* First, for Property (a), in (7) we can see that $D_c(p)$ increases monotonically with K_c . Replacing K_c with K_s and expanding the integration range inside the exponent to $(0, \infty)$, we get exactly $D_n(p)$ [3, Eq. (16)]. Therefore it holds that $D_c(p) < D_n(p)$. Similar bounding techniques can be applied to show $D_s(p) < D_n(p)$ and $D_c(p) < D_s(p)$.

For Property (b), the same technique as that in the proof of the finiteness of $D_c(1)$ in Proposition 1 applies, just by examining an upper bound by ignoring the term $(1-p)\theta K_c r^{-\alpha}$ in the denominator inside the exponent of (7).

For Property (c), as can be verified, $[D_c(p)]'' > 0$, i.e., $[D_c(p)]'$ increases monotonically with p . With $[D_c(p)]'|_{p \rightarrow 0^+} < 0$, the sign of $[D_c(1)]'$ will determine whether $D_c(p)$ decreases monotonically or not. Furthermore, it can be verified that $[D_c(1)]'$ increases monotonically with R . Given that $[D_c(1)]'|_{R=0} < 0$ and $[D_c(1)]'|_{R \rightarrow \infty} > 0$, there exists a unique threshold R_0 which determines the trend of $D_c(p)$ for $p \in (0, 1]$.

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