University of Notre Dame
Department of Electrical Engineering
EE 30344 – Signals and Systems I
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Remarks on Problem 5.50 (b)

From the given input-output pair, we find

$$H(e^{j\omega}) = \frac{(1-\frac{1}{2}e^{-j\omega})^2}{2(1-\frac{1}{4}e^{-j\omega})^2}\,.$$

The FT of the new output is

$$Y(e^{j\omega}) = \frac{\frac{1}{2}e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}}\,.$$

From $X(e^{j\omega}) = Y(e^{j\omega})/H(e^{j\omega})$ we have

$$X(e^{j\omega}) = \frac{Y(e^{j\omega})}{H(e^{j\omega})} = e^{-j\omega}\frac{(1-\frac{1}{4}e^{-j\omega})^2}{(1+\frac{1}{2}e^{-j\omega})(1-\frac{1}{2}e^{-j\omega})^2}\,.$$

"Forgetting" the pre-factor $e^{-j\omega}$ for a moment, let $v := e^{-j\omega}$ and concentrate on

$$G(v) = \frac{(1-\frac{1}{4}v)^2}{(1+\frac{1}{2}v)(1-\frac{1}{2}v)^2}\,.$$

Now, you can either one of the following methods for the partial fraction expansion (PFE):

Method A: Write

$$G(v) = \frac{A}{1+\frac{1}{2}v} + \frac{B}{1-\frac{1}{2}v} + \frac{C}{(1-\frac{1}{2}v)^2} \tag{1}$$

and determine A, B, C from the identity

$$A(1-\frac{1}{2}v)^2 + B(1+\frac{1}{2}v)(1-\frac{1}{2}v) + C(1+\frac{1}{2}v) \equiv (1-\frac{1}{4}v)^2$$

which gives you three equations (one for v^2, one for v, and one for the constant term), from which you can determine the three unknowns A, B, C. The equation for the constant term is, for example, $A + B + C = 1$. You find $A = 9/16$, $B = 5/16$, and $C = 1/8$.

Method B: Calculate the PFE using the so-called residue method. Since $G(v)$ is given as in equation (A.57) on page 916 in the textbook, we apply the formula (A.59) to obtain the coefficients A, B, and C.

$$A = \Big[(1+\frac{1}{2}v)G(v)\Big]_{v=-2} = \frac{(1-\frac{1}{4}(-2))^2}{(1-\frac{1}{2}(-2))^2} = \frac{(3/2)^2}{2^2} = \frac{9}{16}$$

$$B = (-2)\Big[\frac{d}{dv}\big((1-\frac{1}{2}v)^2G(v)\big)\Big]_{v=2} = (-2)\Big[\frac{(v-4)(v+8)}{8(v+2)^2}\Big]_{v=2} = \frac{5}{16}$$

$$C = \Big[(1-\frac{1}{2}v)^2G(v)\Big]_{v=2} = \frac{(\frac{1}{2})^2}{2} = \frac{1}{8}\,.$$

Method B is convenient for A and C, a bit less for B. The fastest way to get the PFE would probably be to determine only A and C with method B, and then use $A + B + C = 1$ from method A.

continued on back

With the coefficients A, B, C, we have (including the "forgotten" factor $e^{-j\omega}$):

$$X(e^{j\omega}) = e^{-j\omega}\left(\frac{\frac{9}{16}}{1+\frac{1}{2}e^{-j\omega}} + \frac{\frac{5}{16}}{1-\frac{1}{2}e^{-j\omega}} + \frac{\frac{1}{8}}{(1-\frac{1}{2}e^{-j\omega})^2}\right),$$

which, with the unit delay factor, transforms to

$$x[n] = \frac{9}{16}\left(-\frac{1}{2}\right)^{n-1} u[n-1] + \frac{5}{16}\left(\frac{1}{2}\right)^{n-1} u[n-1] + \frac{1}{8}n\left(\frac{1}{2}\right)^{n-1} u[n-1].$$

For the third term, we have use the relationship found in Example 5.13 for the case $\alpha = \beta$, see equation (5.58) on page 386 and the "differentiation in frequency property" in Table 5.1 on page 391.