Let R be a k-algebra for k a perfect field of characteristic p, or, more generally, a $\mathbb{Z}_{(p)}$-algebra. The de Rham-Witt complex over R is an inverse limit of differential graded algebras. In degree zero, it is the Witt vectors with coefficients in R. The first complex in the inverse limit is the de Rham complex over R. The de Rham-Witt complex provides a complex which is explicit and computable whose hypercohomology agrees with crystalline cohomology.

In the talk, we will define the Witt vectors over R and explain its relation to the special case of Witt vectors over a perfect field. We will do a few simple computations with Witt vectors to get a feel for them, including finding an ideal with divided powers.

Next we will give a careful definition of the de Rham-Witt complex and describe some of its properties. Then we can describe this complex in a few special cases. This is easy for $R = \mathbb{F}_q$, but already for $R = \mathbb{F}_p[x_1, \ldots, x_n]$ or $R = \mathbb{Z}_{(p)}$ the description is not so simple. We'll give as many examples as we have time for.