Multiple Choice

1.(6 pts) Find the volume of the solid that lies under $z = x^3 + y^3$ and above the region in the xy-plane bounded by $y = x^2$ and $x = y^2$.

(a) $\dfrac{3}{16}$ (b) $\dfrac{1}{9}$ (c) $\dfrac{1}{16}$ (d) $\dfrac{1}{18}$ (e) $\dfrac{5}{18}$

2.(6 pts) Let E be the part of the ball $x^2 + y^2 + z^2 \leq 9$ that lies in the first octant. Determine which integral computes the mass of E if the density is $\delta(x, y, z) = x^2 + y^2$.

(a) $\int_0^{\pi/2} \int_0^{\pi/2} \int_0^3 \rho^4 \sin^3\phi \, \mathrm{d}\rho \, \mathrm{d}\phi \, \mathrm{d}\theta$

(b) $\int_0^{\pi/2} \int_0^{\pi/4} \int_0^3 \rho^3 \sin^2\phi \cos\phi \, \mathrm{d}\rho \, \mathrm{d}\phi \, \mathrm{d}\theta$

(c) $\int_0^{\pi/2} \int_0^{\pi/2} \int_0^3 \rho^2 \cos\phi \, \mathrm{d}\rho \, \mathrm{d}\phi \, \mathrm{d}\theta$

(d) $\int_0^{\pi/2} \int_0^{\pi/2} \int_0^3 \rho^3 \sin^2\phi \, \mathrm{d}\rho \, \mathrm{d}\phi \, \mathrm{d}\theta$

(e) $\int_0^{\pi/2} \int_0^{\pi/4} \int_0^3 \rho^4 \sin\phi \cos\phi \, \mathrm{d}\rho \, \mathrm{d}\phi \, \mathrm{d}\theta$

3.(6 pts) Which of the following computes $\iiint_E y \, \mathrm{d}V$, where E is the solid that lies between cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ above the xy-plane and below the plane $z = x + 4$?

(a) $\int_0^{2\pi} \int_1^2 \int_0^{r\cos\phi+4} r^2 \sin\phi \, \mathrm{d}z \mathrm{d}r \mathrm{d}\phi$

(b) $\int_0^{2\pi} \int_1^2 \int_0^r r \sin\phi \, \mathrm{d}z \mathrm{d}r \mathrm{d}\phi$

(c) $\int_0^{2\pi} \int_1^2 \int_0^{r\cos\phi+4} r \sin\phi \, \mathrm{d}z \mathrm{d}r \mathrm{d}\phi$

(d) $\int_0^{2\pi} \int_1^2 \int_0^r r^2 \sin\phi \, \mathrm{d}z \mathrm{d}r \mathrm{d}\phi$

(e) $\int_0^{2\pi} \int_1^2 \int_0^{r\sin\phi} (r\cos\phi + 4) \, \mathrm{d}z \mathrm{d}r \mathrm{d}\phi$

4.(6 pts) Evaluate $\int_C 4 \, \mathrm{d}s$, where C is the helix $x = 2\sin t$, $y = 2\cos t$, $z = 3t$, $0 \leq t \leq 2\pi$.

(a) $4\sqrt{13}\pi$ (b) $8\sqrt{13}$ (c) $8\sqrt{13}\pi^2$ (d) $\sqrt{13}$ (e) $8\sqrt{13}\pi$

5.(6 pts)Use Fundamental Theorem of line integrals to evaluate $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$, where

$$\mathbf{F}(x, y) = 2x\mathbf{i} + 2y\mathbf{j}$$

and C is given by $\mathbf{r}(t) = \langle t^2 \cos(\pi t), 2^{t-1}\sqrt{t} \rangle$, $1 \leq t \leq 2$.

(a) 3 (b) 18 (c) 24 (d) 22 (e) 0

6.(6 pts) Find the curl of the vector field

$$\mathbf{F}(x, y, z) = xz^2\mathbf{i} + \cos(yz)\mathbf{j} + (x + yz)\mathbf{k}.$$

(a) $\frac{1}{2}z^2(x^2 + y^2) + xz + \frac{1}{z}\sin(yz)$

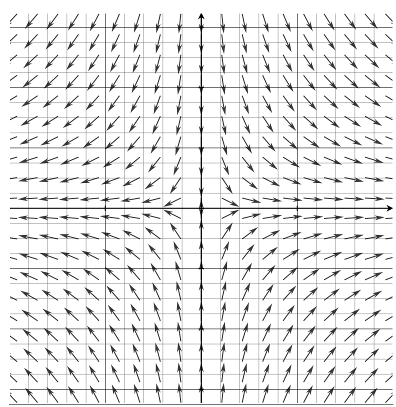
(b) $(z + y\sin(yz))\mathbf{i} + (2xz - 1)\mathbf{j}$

(c) $y + z^2$

(d) $z^2\mathbf{i} + y\mathbf{k}$

(e) $(y + z\sin(yz))\mathbf{i} + (z^2 - y)\mathbf{j} + (-z\sin(yz) - z^2)\mathbf{k}$

7.(6 pts) Which of the following could be the vector field depicted below?



(a) $\mathbf{F} = x\mathbf{i} + \mathbf{j}$ (b) $\mathbf{F} = x\mathbf{i} - y\mathbf{j}$ (c) $\mathbf{F} = x^2\mathbf{i} + y^2\mathbf{j}$

(d) $\mathbf{F} = y\mathbf{i} - x\mathbf{j}$ (e) $\mathbf{F} = -x^2\mathbf{i} - y^2\mathbf{j}$

8.(6 pts) Compute the Jacobian $\dfrac{\partial(x, y)}{\partial(u, v)}$ of the map $x = 5v\sin u$, $y = 4v\cos u$.

(a) $9v$ (b) $-20v\sin u\cos u$ (c) $20v$

(d) $9v^2$ (e) v

9.(6 pts) Evaluate the line integral $\int_C xy\,\mathrm{d}x$, where C is the part of $y = x^2$ form $(0,0)$ to $(2,4)$.

(a) 4 (b) -4 (c) 0 (d) 2 (e) -2

10.(6 pts) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = xy\mathbf{i} + e^{x^3}\mathbf{j}$ and C is the line segment from $(2,0)$ to $(4,0)$.

(a) -2 (b) 2 (c) 4 (d) -4 (e) 0

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(12 pts.) Let $\mathbf{F} = (x + yz)\mathbf{i} + xz\mathbf{j} + (z + xy)\mathbf{k}$ be a vector field.
(a) Find a function $f(x,y,z)$ such that $\mathbf{F} = \nabla f$.
(b) Compute $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{t}$, where C is the curve $\mathbf{r}(t) = \langle t, e^t, te^{t^3} \rangle$, $0 \leq t \leq 1$.

12.(12 pts.) Use the transformation $x = u + v$, $y = v$ to compute $\iint_D 2\mathrm{d}A$ where D is the region bounded by $x^2 - 2xy + 2y^2 = 1$.

13.(12 pts.) Use Green's Theorem to compute $\int_C (e^x - y)\mathrm{d}x + (5x + \cos y)\mathrm{d}y$, where C is the curve $x^2 - 2xy + 2y^2 = 1$ with positive orientation.

Name: ______________________

Instructor: ANSWERS

Math 20550, Practice Exam 3
April 25, 2017

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 4 pages of the test.
- Each multiple choice question is 6 points. Each partial credit problem is 12 points. You will receive 4 extra points.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(•)	(c)	(d)	(e)
2.	(•)	(b)	(c)	(d)	(e)
3.	(•)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(•)
5.	(a)	(b)	(c)	(•)	(e)
6.	(a)	(•)	(c)	(d)	(e)
7.	(a)	(•)	(c)	(d)	(e)
8.	(a)	(b)	(•)	(d)	(e)
9.	(•)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(•)

Please do NOT write in this box.

Multiple Choice	________
11.	________
12.	________
13.	________
Extra Points.	4
Total:	________

11.(a) We need to find a function $f(x, y, z)$ with

$$(1)\ f_x = x + yz; \quad (2)\ f_y = xz; \quad (3)\ f_z = (z + xy).$$

First let's assume $f_x = x + yz$, then

$$(I) \quad f = \int (x + yz)\, dx = \frac{1}{2}x^2 + xyz + h(y, z),$$

where $h(y, z)$ does not contain x.

To satisfy (2) we need to have $f_y = xz$. Differentiating the above function with respect to y we find that $xz + h_y(y, z)$ must be equal to xz, so $xz + h_y(y, z) = xz$ and $h_y(y, z) = 0$. Solving $h_y(y, z) = 0$ we obtain that $h(y, z) = g(z)$, where $g(z)$ does not contain neither x nor y. Replacing $h(y, z)$ by $g(z)$ in (I) we obtain that our candidate for f has form

$$(II) \quad f = \frac{1}{2}x^2 + xyz + g(z).$$

To satisfy (3) the partial derivative of f with respect to z must be equal to $z + xy$. Differentiating our candidate with respect to z we obtain an equation

$$z + xy = xy + g_z(z)$$

or

$$g_z(z) = z$$

Integrating with respect to z we get $g(z) = \frac{1}{2}z^2 + C$, where C is any constant. For simplicity we choose $C = 0$, and from (II) we get an answer.

$$f = \frac{1}{2}x^2 + xyz + \frac{1}{2}z^2.$$

(b) By the Fundamental Theorem of Line Integrals

$$\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{t} = f(\mathbf{r}(1)) - f(\mathbf{r}(0)) = f(1, e, e) - f(0, 1, 0) = \frac{1}{2} + e^2 + \frac{1}{2}e^2 - 0 = \frac{1}{2} + \frac{3}{2}e^2.$$

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12. First we compute the Jacobian

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$$

Then we see that our region D bounded by $x^2 - 2xy + 2y^2 = 1$ corresponds to the region S bounded by $u^2 + v^2 = 1$ under this transformation.

So our integral becomes

$$\iint_D 2dA = \iint_S 2 \cdot 1\, dA = 2\pi$$

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13. Let D be the region bounded by the curve C. By Green's theorem we have

$$\int_C (e^x - y)\mathrm{d}x + (5x + \cos y)\mathrm{d}y = \iint_D (5 - (-1)\, dA = 6 \iint_D \, dA.$$

Completing squares we rewrite C as

$$(x - y)^2 + y^2 = 1.$$

We use substitution $x - y = u$, $y = v$, or $x = u + v$, $v = 1$.

Under this substitution the curve C corresponds to the circle $u^2 + v^2 = 1$, and the region D corresponds to the unit disk S.

The Jacobian is

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1,$$

and our integral becomes

$$6 \iint_S \, dA = 6 \text{ times the area of } S = 6\pi.$$