Multiple Choice

1.(6 pts)Use cylindrical coordinates to evaluate $\iiint_E (x^2+y^2)\,dV$, where

$$E = \{(x,y,z) \mid \sqrt{x^2+y^2} \le z \le 2\}.$$

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$ (c) $\frac{4\pi}{9}$ (d) $\frac{16\pi}{5}$ (e) $\frac{4\pi}{3}$

2.(6 pts) Evaluate $\int_C xy\,ds$, where C is given by $\vec{r}(t) = \langle 4\cos t, 4\sin t, 3t\rangle$ for $0 \le t \le \frac{\pi}{2}$.

(a) 10 (b) 40 (c) 5 (d) 0 (e) -40

3.(6 pts) Find the total mass of the laminated (i.e., thin) region D having density $\rho(x,y) = \sqrt{x^2+y^2}$, where

$$D = \{(x,y) \mid x^2+y^2 \le 4, y \ge 0\}.$$

(a) $\frac{4\pi}{3}$ (b) $\frac{8\pi}{3}$ (c) $\frac{3\pi}{2}$ (d) $\frac{4}{3}$ (e) $\frac{2\pi}{3}$

4.(6 pts) Use spherical coordinates to evaluate $\iiint_E e^{(x^2+y^2+z^2)^{3/2}} dV$, where

$$E = \{(x,y,z) \mid y \ge 0, z \ge 0, x^2+y^2+z^2 \le 1\}.$$

(a) $4\pi e$ (b) 0 (c) $\frac{\pi}{3}e$ (d) $\frac{\pi}{3}(e-1)$ (e) $\frac{4\pi}{3}(e-1)$

5.(6 pts) Let $\vec{F} = \langle xz, xyz, -y^2\rangle$. Compute curl $\vec{F}$.

(a) $\langle -y(2+x), x, yz\rangle$ (b) 0 (c) $z+xy$

(d) $\langle x, -y(2+x), yz\rangle$ (e) $\langle -y(2+x), -x, yz\rangle$

6.(6 pts) Use a double integral to find the area enclosed by one loop of the four-leaved rose $r = \sin(2\theta)$. The region inside the loop is described in polar coordinates by $0 \leq \theta \leq \frac{\pi}{2}$ and $0 \leq r \leq \sin(2\theta)$.

(a) $\frac{\pi}{2}$ (b) 0 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{8}$ (e) $\frac{\pi}{4}$

7.(6 pts) Find the work $\int_C \vec{F} \cdot d\vec{r}$ done by the force field $\vec{F} = \langle xy, yz, zx\rangle$ in moving a particle along the curve C given by $\vec{r}(t) = \langle t, t^2, t^3\rangle$ for $-1 \leq t \leq 1$.

(a) $\frac{1}{4}$ (b) $\frac{5}{7}$ (c) $\frac{1}{2}$ (d) $\frac{10}{7}$ (e) $\frac{27}{28}$

8.(6 pts)Use Green's theorem to evaluate $\int_C \left((3y - e^{x^2})dx + (7x + \sqrt{y^{99} + y + 100})dy\right)$ where C is the circle $x^2 + y^2 = 9$ with the counter-clockwise orientation.

(a) 3π (b) 36π (c) 0 (d) -36π (e) 7π

9.(6 pts) Let $x = 2u$ and $y = -3v$. Then $\int_{-3}^{3}\int_{-2}^{2} f(x,y)dxdy$ can be written as:

(a) $\frac{1}{6}\int_{-1}^{1}\int_{-1}^{1} f(2u,-3v)dudv$ (b) $6\int_{-3}^{3}\int_{-2}^{2} f(2u,-3v)dudv$

(c) $6\int_{-1}^{1}\int_{-1}^{1} f(2u,-3v)dudv$ (d) $-4\int_{-1}^{1}\int_{-1}^{1} f(2u,-3v)dudv$

(e) $-6\int_{-1}^{1}\int_{-1}^{1} f(2u,-3v)dudv$

10.(6 pts)Which of the following vector fields cannot be written as curl $\vec{F}$?

(a) $\langle -x-y+1, xy-1, -xz+y+z\rangle$

(b) $\langle -y,-z,-x\rangle$

(c) $\langle -y\cos(z), -z\cos(x), -x\cos(y)\rangle$

(d) $\langle 2yz, xyz, 3xy\rangle$

(e) $\langle 1-2z, 1-2x, 1-2y\rangle$

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(12 pts.)Let $\vec{F} = \langle y^2+1, 2xy+2y+e^{3z}, 3ye^{3z}+3z^2\rangle$.

(a) Find curl $\vec{F}$.

(b) Find f such that $\nabla f = \vec{F}$.

(c) Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where C is any smooth curve beginning at $(1,0,0)$ and ending at $(0,1,0)$.

12.(12 pts.)Let E be the tetrahedron enclosed by the coordinate planes $x=0$, $y=0$, $z=0$ and the plane $2x+y+z=2$. Assume the density function is $\rho(x,y,z)=1$. Write an iterated integral (with limits) for the moment of the solid E about the yz-plane. (You do NOT need to compute this iterated integral.)

13.(12 pts.)Use the transformation $x=\sqrt{3}u-v$, $y=\sqrt{3}u+v$ to evaluate the integral $\iint_R (x^2-xy+y^2)\,dA$, where R is the region bounded by the ellipse $x^2-xy+y^2=3$.

Name: ______________________

Instructor: ANSWERS

Math 20550, Last Year Exam 3
April 25, 2017

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 minutes
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 4 pages of the test.
- Each multiple choice question is 6 points. Each partial credit problem is 12 points. You will receive 4 extra points.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(•)	(e)
2.	(a)	(•)	(c)	(d)	(e)
3.	(a)	(•)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(•)	(e)
5.	(•)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(•)	(e)
7.	(a)	(b)	(c)	(•)	(e)
8.	(a)	(•)	(c)	(d)	(e)
9.	(a)	(b)	(•)	(d)	(e)
10.	(a)	(b)	(c)	(•)	(e)

Please do NOT write in this box.

Multiple Choice	______
11.	______
12.	______
13.	______
Extra Points.	4
Total:	______

11.
(a)

$$\begin{aligned}
\operatorname{curl} \vec{F} &= \langle \partial x, \partial y, \partial z\rangle \times \langle y^2+1, 2xy+2y+e^{3z}, 3ye^{3z}+3z^2\rangle \\
&= \det \begin{pmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ \partial x & \partial y & \partial z \\ y^2+1 & 2xy+2y+e^{3z} & 3ye^{3z}+3z^2 \end{pmatrix} \\
&= (3e^{3z}-3e^{3z})\vec{\mathbf{i}} - (0-0)\vec{\mathbf{j}} + (2y-2y)\vec{\mathbf{k}} \\
&= \langle 0,0,0\rangle.
\end{aligned}$$

(b) We seek a FUNCTION f such that $\nabla f = \vec{F}$, i.e., such that

$$\begin{aligned}
&(a) \quad f_x = y^2+1 \\
&(b) \quad f_y = 2xy+2y+e^{3z} \\
&(c) \quad f_z = 3ye^{3z}+3z^2.
\end{aligned}$$

Integrating (a) with respect to x we get

(1) $$f(x,y,z) = xy^2 + x + h(y,z)$$

where the functions h may depend on y and z only and does not depend on x.

We differentiate the function f from (1) with respect to y and using (b) set equal to $2xy+2y+e^{3z}$:

$$2xy + h_y(y,z) = 2xy+2y+e^{3z},$$

hence

$$h_y(y,z) = 2y + e^{3z}.$$

Integrating the above expression with respect to y we get

$$h(y,z) = y^2 + ye^{3z} + g(z),$$

where the functions g may depend on z only and does not depend on x, y. From (1) we obtain

(2) $$f(x,y,z) = xy^2 + x + y^2 + ye^{3z} + g(z).$$

We differentiate the function f from (2) with respect to z and using (c) set equal to $3ye^{3z}+3z^2$:

$$3ye^{3z} + g_z(z) = 3ye^{3z} + 3z^2,$$

hence

$$g_z(z) = 3z^2,$$

and

$$g(z) = z^3 + C.$$

We choose $C = 0$ and use (2) to obtain

$$f(x, y, z) = xy^2 + x + y^2 + ye^{3z} + z^3$$

(c) Finally, the fundamental theorem of line integrals tells us that

$$\int_C \vec{F} \cdot d\vec{r} = \int_C \nabla f \cdot d\vec{r} = f(0, 1, 0) - f(1, 0, 0) = 2 - 1 = 1$$

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12. In the xy-plane the condition $2x + y + z = 2$ becomes $2x + y = 2$ so the x-intercept is $x = 1$.

Thus the base of E in the xy-plane is the region D bounded by the lines $x = 0$, $y = 0$, $y = 2 - 2x$, that we can write as $D = \{(x, y)|0 \leq 1, 0 \leq y \leq 2 - 2x\}$.

The region E can be describes as the region that lies above D and below the plane $z = 2 - 2x - y$.

The moment is therefore

$$\iiint_E x\, dV = \int_0^1 \int_0^{2-2x} \int_0^{2-2x-y} x\, dz\, dy\, dx$$

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13. We denote by $f(x, y)$ the function $f(x, y) = x^2 - xy + y^2$.

Thus we need to evaluate

$$\iint_R f(x, y)\, dA.$$

The ellipse $x^2 - xy + y^2 = 3$, which bounds the region R, when written in terms of u and v becomes

$$x^2 - xy + y^2 = (\sqrt{3}u - v)^2 - (\sqrt{3}u - v)(\sqrt{3}u + v) + (\sqrt{3}u + v)^2 = 3(u^2 + v^2) = 3$$

which is the circle S of radius 1. Hence under the change of variables the region R in the (x, y)-plane is transformed to the unit disk D in the (u, v)-plane.

The function $f(x, y)$ in terms of u and v is

$$f = x^2 - xy + y^2 = (\sqrt{3}u - v)^2 - (\sqrt{3}u - v)(\sqrt{3}u + v) + (\sqrt{3}u + v)^2 = 3(u^2 + v^2).$$

The Jacobian is

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \sqrt{3} & -1 \\ \sqrt{3} & 1 \end{vmatrix} = 2\sqrt{3}$$

So

$$\iint_R (x^2 - xy + y^2) dxdy = \iint_D 3(u^2 + v^2)|2\sqrt{3}| dudv$$

$$= 6\sqrt{3} \int_0^{2\pi} \int_0^1 r^2\, r dr d\theta = 6\sqrt{3} \int_0^{2\pi} \frac{1}{4} d\theta = 3\sqrt{3}\pi$$