Multiple Choice

1.(6 pts) Which of the following vectors has the same direction as $\boldsymbol{v} = \langle -1, 2, 2\rangle$ but has length 6?

(a) $\langle -2, 4, 4\rangle$ (b) $\langle 2, 4, 4\rangle$ (c) $\langle 4, 2, 4\rangle$

(d) $\langle -\sqrt{2}, 2\sqrt{2}, 2\sqrt{2}\rangle$ (e) $\langle 0, 6, 0\rangle$

2.(6 pts) Compute the vector projection of the vector $\langle 1, 0, 0\rangle$ on the vector $\langle 2, -1, 1\rangle$.

(a) $\frac{1}{3}\langle 1, 2, 3\rangle$ (b) $\frac{1}{3}\langle 2, -1, 1\rangle$ (c) $\langle 1, 1, -1\rangle$

(d) $\langle 2, -1, 1\rangle$ (e) $\langle 3, 1, 4\rangle$

3.(6 pts) Determine which of the following expressions gives the length of the curve defined by $\boldsymbol{r}(t) = 2t\boldsymbol{i} + \cos(t)\boldsymbol{j} + 2\sin(t)\boldsymbol{k}$ between the points $(0, 1, 0)$ and $(2\pi, -1, 0)$.

(a) $\int_0^{\pi} \sqrt{4t^2 + \cos^2(t) + 4\sin^2(t)}dt$ (b) $\int_0^{2\pi} \sqrt{4 + \sin^2(t) + 4\cos^2(t)}dt$

(c) $\int_0^{2\pi} \sqrt{4t^2 + \cos^2(t) + 4\sin^2(t)}dt$ (d) $\int_0^{\pi} (2t\boldsymbol{i} + \cos(t)\boldsymbol{j} + 2\sin(t)\boldsymbol{k})dt$

(e) $\int_0^{\pi} \sqrt{4 + \sin^2(t) + 4\cos^2(t)}dt$

4.(6 pts) Find the volume of the parallelopiped determined by the vectors $\langle 3, 1, 4\rangle$, $\langle 2, 0, 2\rangle$ and $\langle -3, -1, 0\rangle$.

(a) 8 (b) -4 (c) 20 (d) $\langle 2, -6, -2\rangle$ (e) 4

5.(6 pts) What is the center of the sphere given by the equation

$$x^2 - 2x + y^2 - 4y + z^2 - 6z + 25 = 10$$

(a) $(3, 2, 1)$ (b) $(1, 2, 3)$

(c) This is not the equation of a sphere (d) $(0, 0, 0)$

(e) $(2, 1, 3)$

6.(6 pts) The two curves

$$\boldsymbol{r}(t) = \langle t^2 - 1, \ln(t^2), t^4 - t^3 - t^2 + t \rangle$$
$$\boldsymbol{s}(t) = \left\langle 2\sqrt{t+3} - 4, \cos(\pi t) + 1, \sqrt{t} - 1 \right\rangle$$

intersect at the origin when $t = 1$. What is the cos of the angle of intersection?

(a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\frac{1}{2}$ (e) 1

7.(6 pts) The plane S contains the points $(0, 1, 3)$, $(2, 2, 2)$, and $(3, 2, 1)$. Which of the following is an equation for S?

(a) $2x - 4y + 3z = 5$ (b) $x + y + z = 6$ (c) $x + 3y + z = 6$

(d) $y + z = 4$ (e) $x - y + z = 2$

8.(6 pts) For which of the following function does the contour plot consist of concentric circles?

(a) $f(x, y) = 8 - x - y$ (b) $f(x, y) = e^{-(x^2+y^2)}$ (c) $f(x, y) = e^{4x^2-y}$

(d) $f(x, y) = \sin(x + y)$ (e) $f(x, y) = x^2 - y^2$

9.(6 pts) Find symmetric equations for the tangent line to the helix $\boldsymbol{r}(t) = \langle 2\cos(t), \sin(t), t \rangle$ at the point $(0, 1, \pi/2)$.

(a) $\dfrac{x}{-2} = \dfrac{z - \frac{\pi}{2}}{1}$ and $y = 0$

(b) $\dfrac{x}{-2} = \dfrac{z - \frac{\pi}{2}}{1}$ and $y = 1$

(c) $\dfrac{x}{2} = \dfrac{y-1}{2} = \dfrac{z - \frac{\pi}{2}}{1}$

(d) $\dfrac{x}{-2} = \dfrac{y-1}{1} = \dfrac{z - \frac{\pi}{2}}{1}$

(e) $\dfrac{x}{2} = \dfrac{z - \frac{\pi}{2}}{1}$ and $y = 1$

10.(6 pts) The position vector of a flying cardinal at second t is given by $\boldsymbol{r}(t) = \langle 4t, \cos(3t), \sin(-3t) \rangle$. What is the normal component of its acceleration, a_N, at time t?

(a) $9t$

(b) $\cos(3t)$

(c) 1

(d) 9

(e) -9

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(12 pts.) Find the area of the triangle with vertices $P(1,4,6)$, $Q(-2,5,-1)$ and $R(1,-1,1)$.

12.(12 pts.) Let $z = z(x,y)$ be the function of x, y given implicitly by the equation

$$x^2 + y^3 + z^4 + 2xyz = 1$$

Find $\dfrac{\partial x}{\partial y}$ and $\dfrac{\partial y}{\partial z}$.

13.(12 pts.) The acceleration of a particle at time t is

$$\boldsymbol{a} = \left\langle 36t^2, e^t, \cos(t) \right\rangle$$

Determine its location at time $t = 1$ if it is known that at time $t = 0$ the particle was passing through the origin with velocity $\langle 1, 1, 1 \rangle$.

Name: ______________________________

Instructor: ANSWERS

Math 20550, Old Exam 1
February 21, 2017

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 4 pages of the test.
- Each multiple choice question is 6 points. Each partial credit problem is 12 points. You will receive 4 extra points.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(•)	(b)	(c)	(d)	(e)
2.	(a)	(•)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(•)
4.	(•)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(•)	(d)	(e)
6.	(a)	(b)	(c)	(•)	(e)
7.	(a)	(b)	(c)	(d)	(•)
8.	(a)	(•)	(c)	(d)	(e)
9.	(a)	(•)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(•)	(e)

Please do NOT write in this box.

Multiple Choice	________
11.	________
12.	________
13.	________
Extra Points.	4
Total:	________

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11. The are of the triangle PQR is the half of the are of the parallelogram spanned by vectors $\mathbf{PQ}$ and $\mathbf{PR}$, hence

$$\text{Area}(\Delta PQR) = \frac{1}{2}\|\mathbf{PQ} \times \mathbf{PR}\|.$$

We have $\mathbf{PQ} = \langle -3, 1, -7 \rangle$, $\mathbf{PR} = \langle 0, -5, -5 \rangle$, and $\mathbf{PQ} \times \mathbf{PR} = \langle -40, -15, 15 \rangle$.
Hence

$$\text{Area}(\Delta PQR) = \frac{1}{2}\|\langle -40, -15, 15 \rangle\| = \frac{5}{2}\sqrt{82}.$$

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12. To find $\dfrac{\partial x}{\partial y}$ we view $x = x(y, z)$ as a function of y and z. Since

$$x^2 + y^3 + z^4 + 2xyz = 1$$

we have

$$\frac{\partial}{\partial y}(x^2 + y^3 + z^4 + 2xyz) = \frac{\partial}{\partial y}(1) = 0.$$

On the other hand

$$\frac{\partial}{\partial y}(x^2 + y^3 + z^4 + 2xyz) = 2x\frac{\partial x}{\partial y} + 3y^2 + 2\frac{\partial x}{\partial y}yz + 2xz.$$

Thus we have

$$2x\frac{\partial x}{\partial y} + 3y^2 + 2\frac{\partial x}{\partial y}yz + 2xz = 0.$$

Solving for $\dfrac{\partial x}{\partial y}$ we obtain

$$\frac{\partial x}{\partial y} = -\frac{3y^2 + 2xz}{2x + 2yz}.$$

Similarly for $\dfrac{\partial y}{\partial z}$ we view $y = y(x, z)$ as a function of y and z. We have

$$\frac{\partial}{\partial z}(x^2 + y^3 + z^4 + 2xyz) = \frac{\partial}{\partial z}(1) = 0.$$

Computing we get

$$\frac{\partial}{\partial z}(x^2 + y^3 + z^4 + 2xyz) = 3y^2\frac{\partial y}{\partial z} + 4z^3 + 2x\frac{\partial y}{\partial z}z + 2xy,$$

hence

$$3y^2\frac{\partial y}{\partial z} + 4z^3 + 2x\frac{\partial y}{\partial z}z + 2xy = 0.$$

Solving for $\frac{\partial y}{\partial z}$ we obtain

$$\frac{\partial y}{\partial z} = -\frac{4z^3 + 2xy}{3y^2 + 2xz}.$$

13.

Given $\mathbf{r}(0) = \langle 0, 0, 0, \rangle$, $\mathbf{v}(0) = \langle 1, 1, 1, \rangle$, and $\mathbf{a}(t) = \langle 36t^2, e^t, \cos(t) \rangle$, we need to find $\mathbf{r}(1)$.

Integrating we get

$$\mathbf{v}(t) = \int \langle 36t^2, e^t, \cos(t) \rangle \mathrm{d}t = \langle 12t^3, e^t, \sin(t) \rangle + \mathbf{c},$$

where $\mathbf{c}$ is a constant vector.

We are given $\mathbf{v}(0) = \langle 1, 1, 1 \rangle$, and also from the above formula we have $\mathbf{v}(0) = \langle 0, 1, 0 \rangle + \mathbf{c}$. Hence $\mathbf{c} = \langle 1, 0, 1 \rangle$ and

$$\mathbf{v}(t) = \langle 12t^3 + 1, e^t, \sin(t) + 1 \rangle.$$

Integrating we get

$$\mathbf{r}(t) = \int \langle 12t^3 + 1, e^t, \sin(t) + 1 \rangle \mathrm{d}t = \langle 3t^4 + t, e^t, -\cos(t) + t \rangle + \mathbf{c}.$$

We are given $\mathbf{r}(0) = \langle 0, 0, 0 \rangle$, and from the above formula we also have $\mathbf{r}(0) = \langle 0, 1, -1 \rangle + \mathbf{c}$. Thus $\mathbf{c} = \langle 0, -1, 1 \rangle$, and

$$\mathbf{r}(t) = \langle 3t^4 + t, e^t, -\cos(t) + t \rangle + \langle 0, -1, 1 \rangle.$$

Thus

$$\mathbf{r}(1) = \langle 4, e - 1, 2 - \cos(1) \rangle.$$