11. Integrate the acceleration vector:

$$\vec{\mathbf{v}}(t) = \int \vec{\mathbf{a}}(t)\,dt = \langle e^t, 0, -\cos(t)\rangle + \vec{\mathbf{c}},$$

where $\vec{\mathbf{c}}$ is an arbitrary constant vector. Since $\vec{\mathbf{v}}(0) = \langle 1, 0, 1\rangle$, we have

$$\langle 1, 0, -1\rangle + \vec{\mathbf{c}} = \langle 1, 0, 1\rangle.$$

Thus, $\vec{\mathbf{c}} = \langle 0, 0, 2\rangle$, and $\vec{\mathbf{v}}(t) = \langle e^t, 0, -\cos(t) + 2\rangle$.

Integrate the velocity vector:

$$\vec{\mathbf{r}}(t) = \int \vec{\mathbf{v}}(t)\,dt = \langle e^t, 0, -\sin(t) + 2t\rangle + \vec{\mathbf{d}},$$

where $\vec{\mathbf{d}}$ is an arbitrary constant vector. Since $\vec{\mathbf{r}}(0) = \langle 0, 0, 0\rangle$, we have

$$\langle 1, 0, 0\rangle + \vec{\mathbf{d}} = \langle 0, 0, 0\rangle.$$

Thus, $\vec{\mathbf{d}} = \langle -1, 0, 0\rangle$. Putting everything together, we have

$$\vec{\mathbf{r}}(t) = \langle e^t - 1, 0, 2t - \sin(t)\rangle.$$

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12. The first derivatives of $\vec{\mathbf{r}}$ are

$$\vec{\mathbf{r}}'(t) = \langle -\sin(t), 1, \cos(t)\rangle \qquad \vec{\mathbf{r}}''(t) = \langle -\cos(t), 0, -\sin(t)\rangle,$$

and at $t = \frac{\pi}{2}$ they become $\vec{\mathbf{r}}'(\frac{\pi}{2}) = \langle -1, 1, 0\rangle$ and $\vec{\mathbf{r}}''(\frac{\pi}{2}) = \langle 0, 0, -1\rangle$. The binormal vector $\vec{\mathbf{B}}(\frac{\pi}{2})$ is computed first by calculating

$$\vec{\mathbf{b}}(\frac{\pi}{2}) = \vec{\mathbf{r}}'(\frac{\pi}{2}) \times \vec{\mathbf{r}}''(\frac{\pi}{2}) = \langle -1, 1, 0\rangle \times \langle 0, 0, -1\rangle = \langle -1, -1, 0\rangle.$$

and letting $\vec{\mathbf{B}}(\frac{\pi}{2}) = \frac{1}{|\vec{\mathbf{b}}(\frac{\pi}{2})|}\vec{\mathbf{b}}(\frac{\pi}{2}) = \frac{1}{\sqrt{2}}\langle -1, -1, 0\rangle$.

Finally, the osculating plane is the plane normal to $\vec{\mathbf{b}}(\frac{\pi}{2}) = \langle -1, -1, 0\rangle$ through $\vec{\mathbf{r}}(\frac{\pi}{2}) = (0, \frac{\pi}{2}, 1)$ is $-x - (y - \pi/2) = 0$, or equivalently $x + y = \pi/2$.

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13. The direction vectors for the lines L_1, L_2 are respectfully $\vec{v}_1 = \langle 1, 2, -3\rangle$, $\vec{v}_2 = \langle 1, 3, 2\rangle$. The lines are not parallel since $\vec{v}_1 \neq c\vec{v}_2$ for any constant c.

Checking to see if there is any point of intersection, we write the lines in parametric equations. (Note that we need different parameters for the lines.)

$$\begin{array}{cc} L_1: & L_2: \\ x = 2 + t & x = 1 + s \\ y = 3 + 2t & y = -2 + 3s \\ z = 1 - 3t & z = 4 + 2s \end{array}$$

A point of intersection (x_0, y_0, z_0) would satisfy both equations. We have

$$2 + t = x_0 = 1 + s$$

for some values s and t. So $s = 1 + t$. We have also that

$$3 + 2t = y_0 = -2 + 3s,$$

and so $3 + 2t = -2 + 3(1 + t)$. Hence $t = 2$, which also gives us $s = 3$. These values for s and t give us $x_0 = 4$ and $y_0 = 7$ that satisfy the first couple lines of the system of equations. We check what z-value they each give:

$$L_1 : z = 1 - 3t = 1 - 6 = -5$$
$$L_2 : z = 4 + 2s = 4 + 6 = 10$$

The points $(4, 7, -5)$ and $(4, 7, 10)$ are different. So the lines do not intersect and are not parallel. They are skew.