HOMEWORK 8

DUE: TUESDAY, APRIL 6

1. Let X be a CW complex, with skeletal filtration X^k. The skeletal filtration induces a filtration on the singular chain complex of X:

$$F_s C_*^{sing}(X) = C_*^{sing}(X^s).$$

Let $\{E^r_{s,t}, d_r\}$ be the spectral sequence associated to this filtered complex. Identify the E^1 term as a relative homology group. Argue that (E^1, d_1) *is* the cellular chain complex of X, and conclude that the spectral sequence collapses to give $E^2 = E^\infty$. Conclude that this recovers the isomorphism

$$H_*^{CW}(X) \cong H_*^{sing}(X).$$

2. Suppose that

$$E^r_{s,t} \Rightarrow A_{s+t}$$

is a first quadrant spectral sequence of homological type. Suppose that $E^2_{s,*} = 0$ except for $s \in \{s_1, s_2\}$ for fixed s_1, s_2 satisfying $s_2 - s_1 > 1$ (such a spectral sequence is called a "two line spectral sequence"). Argue that there is a long exact sequence relating $E^2_{s_1,*}$, $E^2_{s_2,*}$, and A_*.

3. Deduce from problem 2, and the existence of the Serre spectral sequence, that if

$$F \to E \to S^n$$

is a fiber sequence with $n \geq 2$, then there is a long exact sequence whose terms are given by $H_*(F)$ and $H_*(E)$. This long exact sequence is called the "Gysin Sequence".