

## 18.100A: EXAM 2

You may use your book, but nothing else. Cite theorems you use either by their name (e.g. “Cauchy’s criterion for convergence”) or by their number in the book (e.g. “Theorem 8.4B”).

1. Suppose that  $f(x)$  is continuous at  $x_0$ . Show that if  $f(x_0)$  is contained in the interval  $(a, b)$ , then  $f(x)$  is contained in the interval  $(a, b)$  for  $x \approx x_0$ .
2. Show that there are infinitely many  $x$  so that  $\sin x = 1/3$ .
3. Suppose that  $f'(x) = m$  for all  $x \in \mathbb{R}$ . Show that there is a  $b$  such that  $f(x) = mx + b$ .
4. Find the Taylor series for  $f(x) = \frac{1}{(1+x)^2}$ . Show that the Taylor series converges to  $f(x)$  for  $x \in [0, 1)$ .
5. Let  $S$  be a subset of  $\mathbb{R}$ . Show that if  $S$  is sequentially compact, then it contains all of its cluster points.