

Completion of problem in recitation 11 on
Wednesday, Nov 5, 2003

November 6, 2003

In recitation 11, Ljudmila discussed the following problem:

Find the volume inside the sphere $x^2 + y^2 + z^2 = 4$ but outside the double cone $z^2 = x^2 + y^2$.

We didn't complete it, and some of you seemed to want to see how the integration would be carried out, so we will do that here.

This volume is most easily expressed in spherical coordinates. The sphere is of radius 2, so ρ goes from 0 to 2. The double cone has angle $\pi/4$ to the xy -plane, so ϕ goes from $\pi/4$ to $3\pi/4$. Finally, we are going all the way around in the polar direction, so θ goes from 0 to 2π . Thus we have (using $dx dy dz = \rho^2 \sin \phi d\rho d\phi d\theta$):

$$\begin{aligned} \text{Volume} &= \int \int \int_R dx dy dz \\ &= \int_0^{2\pi} \int_{\pi/4}^{3\pi/4} \int_0^2 \rho^2 \sin \phi d\rho d\phi d\theta \\ &= \int_0^{2\pi} \int_{\pi/4}^{3\pi/4} \left. \frac{\rho^3}{3} \right|_{\rho=0}^{\rho=2} \sin \phi d\phi d\theta \\ &= \int_0^{2\pi} \int_{\pi/4}^{3\pi/4} \frac{8}{3} \sin \phi d\phi d\theta \\ &= \int_0^{2\pi} \left. -\frac{8}{3} \cos \phi \right|_{\phi=\pi/4}^{\phi=3\pi/4} d\theta \\ &= \int_0^{2\pi} \frac{8}{3} \sqrt{2} d\theta \\ &= \frac{16}{3} \sqrt{2} \pi \end{aligned}$$