Math 10360 – Example Set 01A
Derivative and Integration Review

Basic Properties of Derivatives:

$[f(x) + g(x)]' \stackrel{?}{=}$

$[f(x) - g(x)]' \stackrel{?}{=}$

$[c \cdot f(x)]' \stackrel{?}{=}$

Product/Quotient/Chain Rule. Let $f(x)$ and $g(x)$ be differentiable functions. Derive formulas for the derivatives of $p(x) = f(x) \cdot g(x)$ and $q(x) = \dfrac{f(x)}{g(x)}$.

Product Rule:

$$\frac{d}{dx}(f(x)g(x)) = (f(x)g(x))' =$$

Chain Rule:

$$\frac{d}{dx}\left(f(g(x))\right) = [f(g(x))]' =$$

Quotient Rule: $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \left(\dfrac{f(x)}{g(x)}\right)' =$

Some Common Derivatives. For any numbers k and n:

$\dfrac{d}{dx}(k) \stackrel{?}{=}$

$\dfrac{d}{dx}(x^n) \stackrel{?}{=}$ (Power Rule)

$\dfrac{d}{dx}(\sin(x)) \stackrel{?}{=}$

$\dfrac{d}{dx}(\cos(x)) \stackrel{?}{=}$

$\dfrac{d}{dx}(\tan(x)) \stackrel{?}{=}$

$\dfrac{d}{dx}(\csc(x)) \stackrel{?}{=}$

$\dfrac{d}{dx}(\sec(x)) \stackrel{?}{=}$

$\dfrac{d}{dx}(\cot(x)) \stackrel{?}{=}$

1. Find the following derivatives

a. $\dfrac{d}{dx}(x^3 \tan(x)) \stackrel{?}{=}$

b. $\dfrac{d}{dx}\left(\sqrt[3]{2x^2 - 5x + 3}\right) \stackrel{?}{=}$

2. Find the equation of the tangent line to the curve $x \cos(1 + 2y) = 2y^2 - 8$ at the point $(0, 2)$.

$\left(\text{Check } \frac{dy}{dx} = \frac{\cos(1+2y)}{4y+2x\sin(1+2y)}\right)$

Basic Integrals. For any numbers k and n:

$\displaystyle\int x^n \, dx \stackrel{?}{=}$ (Power Rule)

$\displaystyle\int \sin(x) \, dx \stackrel{?}{=}$

$\displaystyle\int \cos(x) \, dx \stackrel{?}{=}$

$\displaystyle\int \sec^2(x) \, dx \stackrel{?}{=}$

$\displaystyle\int \csc^2(x) \, dx \stackrel{?}{=}$

$\displaystyle\int \csc(x)\cot(x) \, dx \stackrel{?}{=}$

$\displaystyle\int \sec(x)\tan(x) \, dx \stackrel{?}{=}$

Method of Substitution

3. Find a formula for the function $f(x)$ if its slope is given by the $x\sin(x^2+1)$ and the graph of $f(x)$ passes through the point $(1, 2)$.

4. Evaluate $\displaystyle\int_0^1 \frac{x^2+2}{\sqrt{x^3+6x+5}}\,dx.$

Math 10360 – Example Set 01B
Derivative of Exponential & Logarithmic Functions: Section 3.9

Definition of the Natural Log. Consider the area function $f(x) = \displaystyle\int_1^x \frac{1}{t}\,dt$ for $x > 0$. We call $f(x)$ the logarithm function and denote it by $f(x) = \ln x$. Discuss (a) through (f) below.

a. What can you say about $\ln(1)$?

b. $f'(x) = \dfrac{d}{dx}[\ln x] = \dfrac{d}{dx}\left[\displaystyle\int_1^x \frac{1}{t}\,dt\right] \stackrel{?}{=}$ ____________ $(x > 0)$

c. $\dfrac{d}{dx}[\ln|x|] \stackrel{?}{=}$ ____________ $(x \neq 0)$. This gives $\displaystyle\int \frac{1}{x}\,dx \stackrel{?}{=}$ ____________

e. Give a sketch of the graph of $y = \ln x$. State clearly the domain and range of $\ln x$. What are the values of $\displaystyle\lim_{x\to 0^+} \ln x$ and $\displaystyle\lim_{x\to\infty} \ln x$? Define the value of e using the definition of the natural logarithm.

d. Using the Fundamental Theorem of Calculus, show that $\ln(ax) = \ln(a) + \ln(x)$.

1. Find the area under the graph of $y = \dfrac{-2}{4x-3}$ for $0 \le x \le 1/2$.

Definition of the Natural Exponential Function. Discuss (f) through (i).

f. The inverse $g(x)$ of $f(x) = \ln x$ exists. Why? Sketch the graph of $g(x) = \exp(x)$. In can be shown that $\exp(x) = e^x$ for all real value x.

g. Explain why we may write: (i) $\ln(e^x) = x$ for all x, and $e^{\ln y} = y$ for $y > 0$. Show $\dfrac{d}{dx}(e^x) = e^x$.

h. Using $e^{\ln b} = b$ $(b > 0)$, show that $\dfrac{d}{dx}(b^x) = b^x \ln b$.

i. Using the change of base formula $\log_b x = \dfrac{\ln x}{\ln b}$, show that $\dfrac{d}{dx}(\log_b x) = \dfrac{1}{x \ln b}$.

2. Find the equation of the tangent line to the curve $y = 4 - 2e^x + \ln\left(\dfrac{1-x^2}{1+x^2}\right)$ at $x = 0$.

Review Exercise. Complete the following formulas:

Logarithmic Properties

$$\ln(ab) \stackrel{?}{=}$$

$$\ln(a^n) \stackrel{?}{=}$$

$$\ln\left(\frac{a}{b}\right) \stackrel{?}{=}$$

$$\ln(e) \stackrel{?}{=}$$

$$\ln 1 \stackrel{?}{=}$$

$$\ln(e^x) \stackrel{?}{=}$$

$$e^{\ln x} \stackrel{?}{=}$$

Exponential Rules

$$a^n \cdot a^m \stackrel{?}{=}$$

$$\frac{a^n}{a^m} \stackrel{?}{=}$$

$$a^n \cdot b^n \stackrel{?}{=}$$

$$\frac{a^n}{b^n} \stackrel{?}{=}$$

Derivative and Anti-derivative Rules

$$\frac{d}{dx}(\ln x) \stackrel{?}{=}$$

$$\frac{d}{dx}(e^x) \stackrel{?}{=}$$

$$\frac{d}{dx}(\log_b x) \stackrel{?}{=}$$

$$\frac{d}{dx}(b^x) \stackrel{?}{=}$$

$$\int \frac{1}{x}\, dx \stackrel{?}{=}$$

$$\int e^x\, dx \stackrel{?}{=}$$

$$\int b^x\, dx \stackrel{?}{=}$$

Math 10360 – Example Set 01C
Derivative of Exponential & Logarithmic Functions: Section 3.9
Derivative of Inverse Trig Function: Section 5.8

1. By restricting the domain of $\sin x$, $\cos x$, and $\tan x$ define their inverse functions ($\arcsin x$, $\arccos x$, and $\arctan x$). Sketch the graph of each of the inverse functions stating their range and domain.

2. Using chain rule, obtain the derivative of $\arcsin(x)$, $\arccos(x)$, and $\arctan(x)$.

Key Formulas:

$$\frac{d}{dx}(\arcsin x) \stackrel{?}{=} \qquad \frac{d}{dx}(\arccos x) \stackrel{?}{=} \qquad \frac{d}{dx}(\arctan x) \stackrel{?}{=}$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx \stackrel{?}{=} \qquad \int \frac{1}{1+x^2}\,dx \stackrel{?}{=}$$

3a. Find the derivative of (i) $(1+2x)^e$ and (i) $(1+2e)^{x^2}$

3a. (Logarithmic Differentiation) Using the log function, find the derivative of $y = (1+2x)^{\arctan x}$.

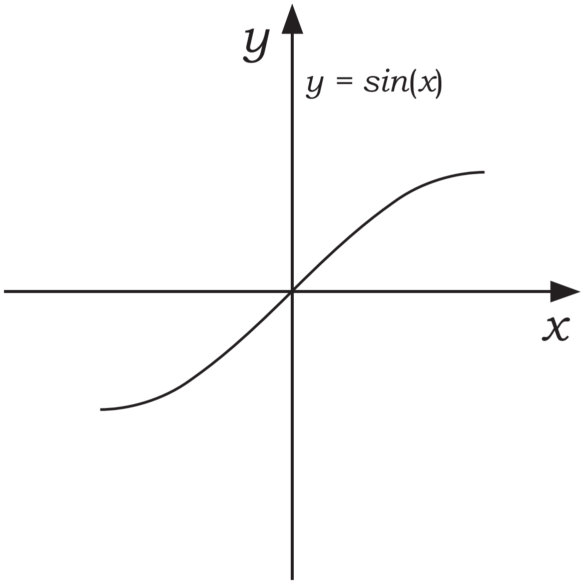
4a. Find the derivative of $\arcsin(2x + y^2)$ with respect to x treating y as a constant.

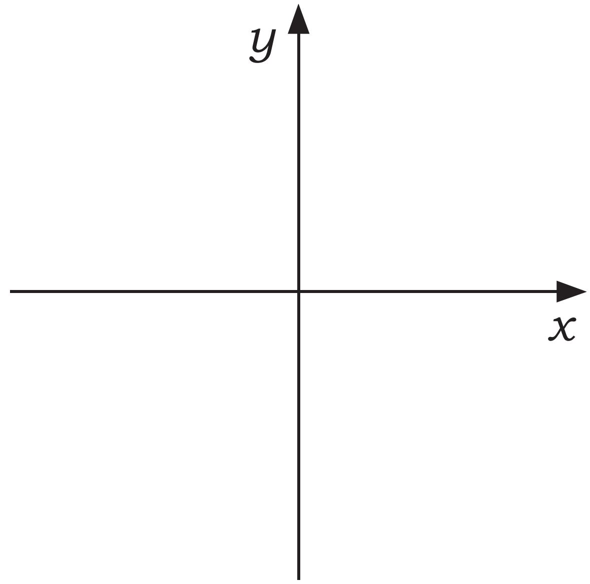
4b. Find the derivative of $\arcsin(2x + y^2)$ with respect to y treating x as a constant.

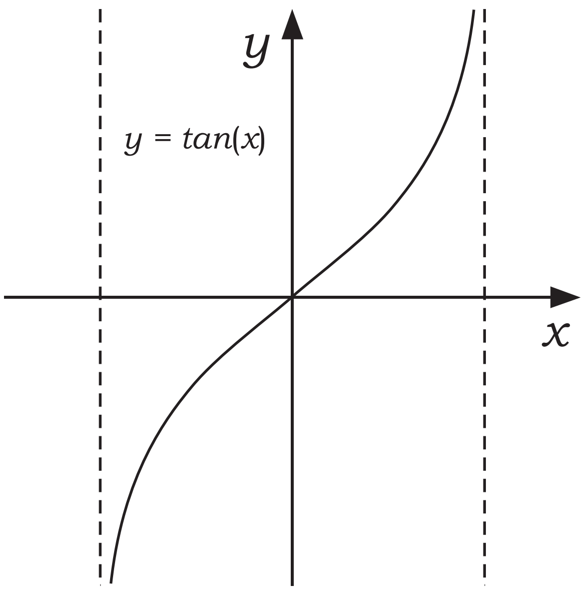
5. A population $y(t)$ (in units of millions) of bacteria grows according to the rate $\dfrac{dy}{dt} = \dfrac{1}{1+4t^2}$. Find the total change in the size of the population over the time duration $0 \leq t \leq 1/2$.

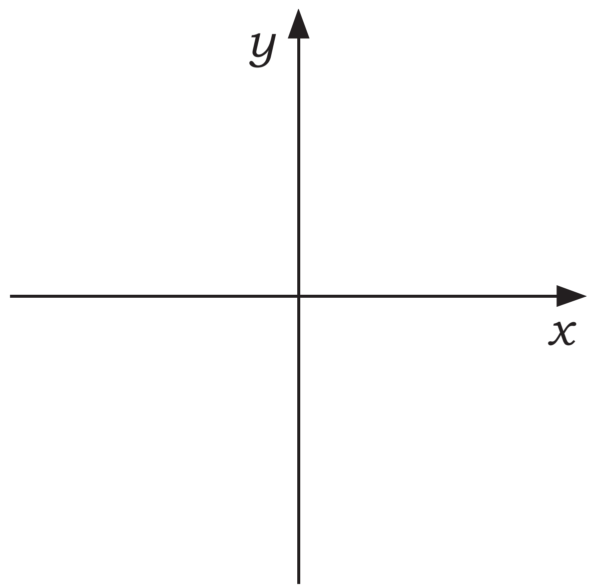
6. (Review) Without using a calculator, find the value of each of the following expressions:

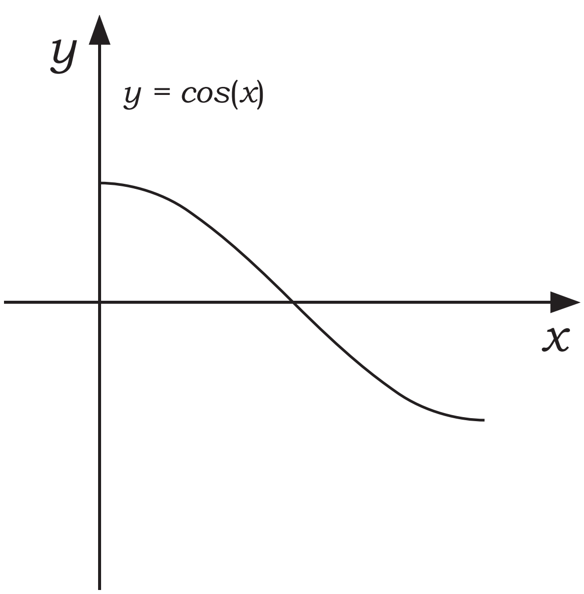
a. $\arcsin(\sqrt{3}/2)$ **b.** $\arcsin(-\sqrt{3}/2)$ **c.** $\arccos(0.5)$ **d.** $\arctan(-1)$

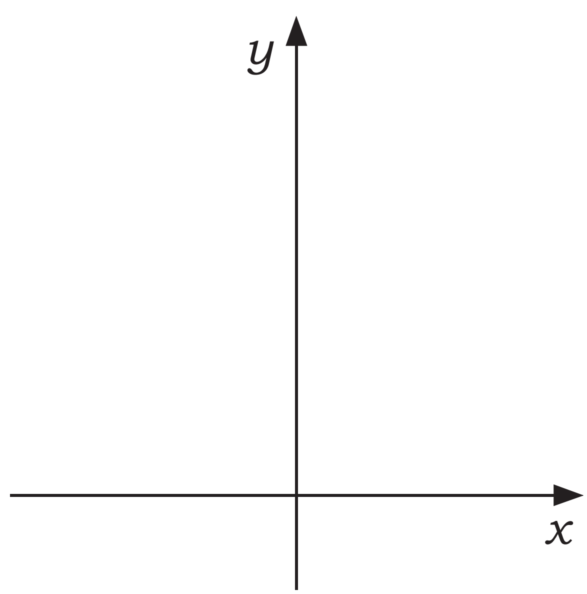

y
y = sin(x)
x


y
x


y
y = tan(x)
x


y
x


y
y = cos(x)
x


y
x