

Math 10360 Review for Exam 3

1. Let $f(x, y) = x^3y + \ln(y - x)$. Find the following limits:

$$(a) \lim_{h \rightarrow 0} \frac{f(1+h, 2) - f(1, 2)}{h}$$

$$(b) \lim_{h \rightarrow 0} \frac{f(1, 2+h) - f(1, 2)}{h}$$

Ans. (a) 5; (b) 2.

2. Let $f(x, y) = \frac{x}{1+y} + e^{xy}$. Find all first and second partial derivatives of f .

$$\frac{\partial f}{\partial x} = \frac{1}{y+1} + ye^{xy}; \quad \frac{\partial f}{\partial y} = -x(1+y)^{-2} + xe^{xy}; \quad \frac{\partial^2 f}{\partial x^2} = y^2e^{xy}; \quad \frac{\partial^2 f}{\partial y^2} = 2x(1+y)^{-3} + x^2e^{xy}; \quad \frac{\partial^2 f}{\partial x \partial y} = -(1+y)^{-2} + e^{xy} + xye^{xy} = \frac{\partial^2 f}{\partial y \partial x}$$

3. Let $z = f(x, y)$ be a function, with $f(1, 2) = 5$, $\frac{\partial f}{\partial x}(1, 2) = 2$, $\frac{\partial f}{\partial y}(1, 2) = -3$. (a) Estimate the value of $f(0.8, 2.1)$, (b) Find the elasticity coefficient of z relative to x at $(1, 2)$ and the elasticity coefficient of z relative to y at $(1, 2)$.

4. Let $f(x, y) = ye^x + 2x - 2y + 5$. Find $f(0, 0)$ and use linear approximation to estimate $f(-0.05, 0.01)$.

Ans. $f(-0.05, 0.01) \approx 4.89$.

5. The population $p(t)$ (in thousands) of cheetah is modeled by the differential equation $\frac{dp}{dt} = p\left(1 - \frac{p}{4}\right)$. If the initial population is 3 thousand, find $p(t)$. You are required to solve the differential equation.

$$(\text{Ans: } p(t) = \frac{12e^t}{1+3e^t} = \frac{12}{e^{-t}+3})$$

6. A cup of warm coffee at 80°C is cooling off outdoors according to the equation

$$\frac{dy}{dt} = k(y - 20)$$

where $y(t)$ is the temperature (in $^\circ\text{C}$) of the coffee at time t minutes. If the temperature of the coffee is 60°C after 10 minutes, what is (i) the value of k , and (ii) the temperature when $t = 15$ minutes?

$$(\text{Ans: (i) } k = \frac{1}{10} \ln\left(\frac{2}{3}\right), \text{ (ii) } y(15) = 60\left(\frac{2}{3}\right)^{3/2} + 20)$$

7. Solve the following initial value problem: $x^2y' = 2xy - 3$; $y(1) = -1$. (Ans: $y(x) = x^{-1} - 2x^2$)

8. The temperature at a point (x, y) on a table is $T(x, y)$, measured in degree Celsius. A bug crawls so that its position on the table after t seconds is given by $x = \sqrt{1+t}$, $y = 2 + (t/3)$, where x and y are measured in centimeters. The temperature function satisfies $T_x(2, 3) = 4$ and $T_y(2, 3) = 3$. How fast is the temperature rising on the bug's path after 3 seconds? (Ans: 2°C/s)

9. The temperature at a point (x, y) on a flat metal plate is given by

$$T(x, y) = \frac{60}{1+x^2+y^2}$$

where T is measured in $^\circ\text{C}$ and x, y in meters. Determine which is bigger: the sensitivity of the temperature T in the x -direction or in the y -direction at the point $(2, 1)$. Discuss the elasticity of T at $(2, 1)$.

10. Find the rate of change of z with respect to y if $10 \cos(x + y + z) = xz^2 + x^2y$

11. $\frac{\partial u}{\partial t}$ and $\frac{\partial u}{\partial s}$ where $u = xyz^2$; $x = \frac{t}{s}$, $y = t^2 + 2s$ and $z = e^{st}$. Find u_t when $t = 1$ and $s = -1$.

12. A tank contains 1000 L of brine with 15 kg of dissolved salt. Brine that contains 0.03 kg of salt per liter of water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank (a) after t minutes and (b) after 20 minutes? (c) What is the limiting concentration? How would your answers change if the brine is drain away at a rate of (i) 5 L/min, and (ii) 20 L/min?

13. The body weight of an animal which initially weighs 10kg is modeled by:

$$\frac{dw}{dt} = -\ln\left(\frac{w}{5}\right); \quad w(0) = 10$$

Use Euler's method with $\Delta t = 0.1$ to estimate $w(0.2)$

14. Let $P(t)$ be the immigrant population (in thousands) of a small country, where t is time in months since the beginning of this year. A demographic study has shown that $P(t)$ is a solution to the following initial value problem:

$$\frac{dP}{dt} = 0.4t \cdot P, \quad P(0) = 2$$

Use Euler's Method with 3 steps to estimate $P(1.5)$.

15. (a) Find **all** possible estimates for the value of $\frac{\partial f}{\partial x}(2, 1)$ and $\frac{\partial f}{\partial y}(2, 1)$ if $f(x, y)$ is given by the table below. (b) How would you estimate the $f(2.1, 1.2)$ using the central difference estimate the partial derivatives at $(2, 1)$?

		x			
		**	1.5	2.0	2.5
y	0.0	36	35	34	
	1.0	38	37	35	
	2.0	44	42	38	

16. The population density of a certain kind of antelope leaving inside the circle $x^2 + y^2 = 100$ is given by the function $\rho(x, y) = x^2 + 3y^2$ antelope per sq. km Find the total population of the antelope.

$$\text{Ans: 16. } \iint_R \rho(x, y) dA = \int_0^{10} \int_0^{2\pi} (r^2 + 2r^2 \sin^2 \theta) r d\theta dr = 10000\pi$$

Math 10360 Review for Exam 3 Answers

12a. $\frac{dy}{dt} = 0.3 - \frac{y}{100}; y(0) = 15$ so $y(t) = 30 - 15e^{-t/100}$

12b. $y(20) = 30 - 15e^{-0.2}$

12c. Concentration at time t , $c(t) = \frac{y(t)}{1000} = 0.03 - 0.015e^{-t/100}$. Therefore $\lim_{t \rightarrow \infty} c(t) = 0.03$

12 (i) a. $\frac{dy}{dt} = 0.3 - \frac{5y}{1000 + 5t} = 0.3 - \frac{y}{200 + t}; y(0) = 15$ so $y(t) = 0.15(200 + t) - \frac{3000}{(200 + t)}$

12 (i) b. $y(20) = 0.15(220) - \frac{3000}{220} = 33 - \frac{150}{11} = \frac{213}{11}$

12 (i) c. Concentration at time t , $c(t) = \frac{y(t)}{1000 + 5t} = 0.03 - \frac{600}{(200 + t)^2}$. Therefore $\lim_{t \rightarrow \infty} c(t) = 0.03$

12 (ii) a. $\frac{dy}{dt} = 0.3 - \frac{20y}{1000 - 10t} = 0.3 - \frac{2y}{100 - t}; y(0) = 15$ so $y(t) = 0.3(100 - t) - \frac{3(100 - t)^2}{2000}$

12 (ii) b. $y(20) = 0.3(80) - \frac{3(80)^2}{2000} = 24 - \frac{3(6400)}{2000} = 24 - \frac{48}{5} = 14\frac{2}{5}$

12 (ii) c. Concentration at time t , $c(t) = \frac{y(t)}{1000 - 10t} = 0.03 - \frac{3(100 - t)}{20000}$. The tank dries out at $t = 100$ minutes. Just before it dries out $c = 0.03$ so saltier than at the start. In fact, $c'(t) > 0$ implies the concentration is increasing and reaches 0.03 just before it dries out.

13. $w(0) = 10; w(0.1) \approx 9.930685282; w(0.2) \approx 9.862066124$.

14. $P(0) = 2; P(0.5) = 2; P(1) = 2.2; P(1.5) = 2.64$.

Math 10360: Calculus B
Exam III
April 21, 2050

Name: _____

Class Time: _____

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- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

Honor pledge. “As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.”:

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	☐ a	☐ b	☐ c	☐ d	☐ e
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Multiple Choice _____

13. _____

14. _____

15. _____

16. _____

Total _____

Name: _____

Class Time: _____

Multiple Choice

1.(5 pts.) Let $P(K, L)$ be the production function for a manufacturing company where K is the amount of capital and L is the amount of labor. Suppose

$$P(20, 30) = 10; \quad \frac{\partial P}{\partial K}(20, 30) = 2; \quad \frac{\partial P}{\partial L}(20, 30) = -3$$

Using the linear approximation of $P(K, L)$ at $(20, 30)$, estimate the amount of production when $K = 20.5$ and $L = 29.5$.

- (a) 7.5
- (b) 12.5
- (c) 11.5
- (d) 8.5
- (e) 10.5

2.(5 pts.) Find the second partial derivative $\frac{\partial^2 f}{\partial y \partial x}$ if $f(x, y) = \ln(3x + 2y)$.

- (a) $\frac{6}{(3x + 2y)^2}$
- (b) $\frac{-4}{(3x + 2y)^2}$
- (c) $\frac{4}{(3x + 2y)^2}$
- (d) $\frac{-9}{(3x + 2y)^2}$
- (e) $\frac{-6}{(3x + 2y)^2}$

Name: _____

Class Time: _____

3.(5 pts.) A 500-gallon tank contains 200 gallons of brine with concentration $1/4$ pounds of salt per gallon. Brine containing $2/5$ pounds of salt per gallon is pumped into the tank at a rate of 5 gallons per minute. The mixture is pumped out of the tank at a rate of 4 gallons per minute. If $y(t)$ is the amount of salt in the tank at time t , find the differential equation modeling the amount of salt in the tank at time t .

(a) $\frac{dy}{dt} = 2 - \frac{y}{50}$

(b) $\frac{dy}{dt} = 2 - \frac{4y}{200 - 4t}$

(c) $\frac{dy}{dt} = \frac{8}{5} - \frac{5y}{200 + t}$

(d) $\frac{dy}{dt} = 2 - \frac{4y}{200 - t}$

(e) $\frac{dy}{dt} = 2 - \frac{4y}{200 + t}$

4.(5 pts.) For the same scenario above, how long it take before the brine overflow?

(a) 100 minutes

(b) 75 minutes

(c) 300 minutes

(d) 500 minutes

(e) 60 minutes

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Class Time: _____

5.(5 pts.) Consider the solution curve of the differential equation below that passing through the point $(-1, 1)$.

$$y' = x^2 + y^2$$

Find the equation of the tangent line to the solution curve at $(-1, 1)$.

- (a) $y = 2x - 3$
- (b) $y = -2x - 1$
- (c) $y = 2x + 3$
- (d) $y = 2x$
- (e) $y = 1$

6.(5 pts.) Let $f(x, y) = e^{x^2y}$. Find the value of the limit

$$\lim_{h \rightarrow 0} \frac{f(2, -1 + h) - f(2, -1)}{h} .$$

- (a) $2e^{-2}$
- (b) $-2e^{-2}$
- (c) Does not exist.
- (d) e^{-4}
- (e) $4e^{-4}$

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7.(5 pts.) Consider the initial value problem:

$$y' = 3x + y, \quad y(0) = 1.$$

Using Euler's method with **TWO** steps of equal size estimate $y(0.2)$

- (a) 1.24
- (b) 4.3
- (c) 1.1
- (d) 1.424
- (e) 2

8.(5 pts.) Find y in terms of t if

$$\ln(1 - y) - \ln(2 - y) = t.$$

- (a) $y = \frac{1 + 2e^t}{1 + e^t}.$
- (b) $y = \frac{1 + e^t}{1 + 2e^t}.$
- (c) $y = \frac{2 - e^t}{1 - e^t}.$
- (d) $y = \frac{1 - 2e^t}{1 - e^t}.$
- (e) $y = \frac{1 - e^t}{2 - e^t}.$

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Class Time: _____

9.(5 pts.) Let $u = 2x^3 - 3y^2$. If $x = \ln(t^2)$ and $y = t^2 - e^2$ find the rate of change of u with respect to t when $t = e$.

- (a) $\frac{24}{e} - 2e$
- (b) $\frac{48}{e}$
- (c) $\frac{48}{e} - 12e^2$
- (d) $\frac{48}{e} - 2e$
- (e) $\frac{24}{e}$

10.(5 pts.) Solve the initial value problem:

$$\frac{dy}{dx} = (2y - 1)x; \quad y(0) = 0$$

- (a) $y = \frac{1}{2}(1 - e^{x^2})$
- (b) $y = \frac{1}{2}(e^{x^2/2} - 1)$
- (c) $y = \frac{1}{2}(e^{x^2} - 1)$
- (d) $y = e^{x^2} - 1$
- (e) $y = \frac{1}{2}(1 - e^{x^2/2})$

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Class Time: _____

11.(5 pts.) Let (x, y) denote the usual rectangular coordinates. Write the following function $f(x, y)$ in polar coordinates (r, θ) .

$$f(x, y) = 2x^2 + 2y^2 + \frac{x}{y}.$$

- (a) $f(r, \theta) = 2r + \tan \theta.$
- (b) $f(r, \theta) = 2r^2 + \tan \theta.$
- (c) $f(r, \theta) = 2\sqrt{r} + \cot \theta.$
- (d) $f(r, \theta) = 2r^2 + \cot \theta.$
- (e) $f(r, \theta) = r^2 \cos(2\theta) + \cot \theta.$

12.(5 pts.) The population density of a sector of a city area is given by

$$\rho(r, \theta) = \frac{\sin \theta}{\sqrt{4 - r^2}} \text{ thousand per sq. km}$$

where $0 \leq r \leq 1$ and $\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$. Find the total population in thousands.

- (a) $\sqrt{2} \cdot \arctan(1/2)$
- (b) $\sqrt{2}(2 - \sqrt{3})$
- (c) $\frac{\pi\sqrt{2}}{12}$
- (d) $\frac{\pi\sqrt{2}}{6}$
- (e) $\frac{\pi\sqrt{2}}{4}$

Name: _____

Class Time: _____

Partial Credit

You must show your work on the partial credit problems to receive credit!

13.(12 pts.) Find the partial derivative of z with respect to y if

$$\frac{x}{(2y+z)^2} = e^{y^2}z - 3z^4$$

Name: _____

Class Time: _____

14.(12 pts.) Consider the initial value problem:

$$y' = (y - t)^2; \quad y(-1) = 1$$

Use Euler's method with two equal steps to estimate $y(0)$.

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Class Time: _____

15.(12 pts.) Solve the following initial value problem:

$$(x^2 + 1) \left(\frac{dy}{dx} - 3 \right) = 2xy; \quad y(1) = 0$$

Name: _____

Class Time: _____

16.(12 pts.) **[Part A.]** Let $u = \sin(x^2 + y^2)$. Find $\frac{\partial u}{\partial s}$ if $x = st$ and $y = \frac{1}{s + 2t}$. Give your answer in terms of s and t .

[Part B. (Unrelated to Part A)] Discuss the elasticity of $z = e^{x^2 + xy^2}$ relative to x and y at $(x, y) = (1, 1)$

Math 10360: Calculus B
Exam III
April 21, 2050

Name: _____
Class Time: ANSWERS

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Multiple Choice _____

13. _____

14. _____

15. _____

16. _____

Total _____

Name: _____

Class Time: _____

Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (12 pts.) Find the partial derivative of z with respect to y if

$$\frac{x}{(2y+z)^2} = e^{y^2} z - 3z^4$$

$$\frac{\partial}{\partial y} \left[x (2y+z(x,y))^{-2} \right] = \frac{\partial}{\partial y} \left[e^{y^2} z(x,y) - 3(z(x,y))^4 \right]$$

$$-2x \cdot (2y+z(x,y))^{-3} \left(2 + \frac{\partial z}{\partial y} \right) = e^{y^2} \frac{\partial z}{\partial y} + 2y e^{y^2} z(x,y)$$

$$- 3 \cdot 4 (z(x,y))^3 \cdot \frac{\partial z}{\partial y}$$

$$-4x(2y+z)^{-3} - 2x(2y+z)^{-3} \frac{\partial z}{\partial y} = e^{y^2} \frac{\partial z}{\partial y} + 2y e^{y^2} z - 12z^3 \frac{\partial z}{\partial y}$$

$$-4x(2y+z)^{-3} - 2y e^{y^2} z = 2x(2y+z)^{-3} \frac{\partial z}{\partial y} + e^{y^2} \frac{\partial z}{\partial y} - 12z^3 \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial y} = \frac{-4x(2y+z)^{-3} - 2y e^{y^2} z}{2x(2y+z)^{-3} + e^{y^2} - 12z^3}$$

$$\underline{\underline{\text{OR}}} \quad \frac{4x(2y+z)^{-3} + 2y e^{y^2} z}{12z^3 - 2x(2y+z)^{-3} - e^{y^2}}$$

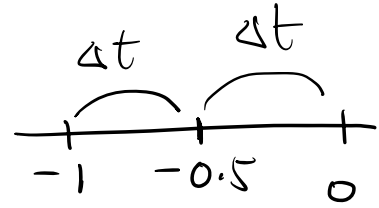
$$\underline{\underline{\text{OR}}} \quad \frac{4x + 2y e^{y^2} z (2y+z)^3}{(12z^3 - e^{y^2})(2y+z)^3 - 2x}$$

Name: _____

Class Time: _____

12.(12 pts.) Consider the initial value problem:

$$y' = (y - t)^2; \quad y(-1) = 1$$

Use Euler's method with two equal steps to estimate $y(0)$.

$$\Delta t = \frac{0 - (-1)}{2} = 0.5$$

$$y(-1) = 1$$

$$\begin{aligned} y(-0.5) &\approx y(-1) + y'(-1)\Delta t \\ &= 1 + 4(0.5) = 3 \end{aligned}$$

$$\begin{aligned} y(0) &\approx y(-0.5) + y'(-0.5)\Delta t \\ &= 3 + \frac{49}{4}(0.5) \\ &= 3 + \frac{49}{8} = \frac{24 + 49}{8} \end{aligned}$$

$$\approx \boxed{\frac{73}{8}}$$

$$y'(t) = (y(t) - t)^2$$

$$\begin{aligned} y'(-1) &= (y(-1) - (-1))^2 \\ &= (1 + 1)^2 = 4 \end{aligned}$$

$$\begin{aligned} y'(-0.5) &= (y(-0.5) - (-0.5))^2 \\ &= (3 + 0.5)^2 = \left(\frac{7}{2}\right)^2 \\ &= \frac{49}{4} \end{aligned}$$

Name: _____

Class Time: _____

13.(12 pts.) Solve the following initial value problem:

$$(x^2 + 1) \left(\frac{dy}{dx} - 3 \right) = 2xy; \quad y(1) = 0$$

$$(x^2 + 1)y' - 3(x^2 + 1) = 2xy$$

$$(x^2 + 1)y' - 2xy = 3(x^2 + 1)$$

$$y' - \frac{2x}{x^2 + 1}y = 3$$

$$\text{Integrating factor} = e^{\int \frac{-2x}{x^2 + 1} dx} \quad \begin{array}{l} u = x^2 + 1 \\ du = 2x dx \end{array}$$

$$= e^{\int -\frac{1}{u} du} = e^{-\ln u} = e^{\ln u^{-1}} = u^{-1} = (x^2 + 1)^{-1}$$

$$(x^2 + 1)^{-1} y' - 2x(x^2 + 1)^{-2} y = 3(x^2 + 1)^{-1}$$

$$\frac{d}{dx} \left[(x^2 + 1)^{-1} y \right] = \frac{3}{x^2 + 1}$$

$$\Rightarrow (x^2 + 1)^{-1} y = \int \frac{3}{x^2 + 1} dx = 3 \arctan(x) + C$$

$$y(1) = 0 \Rightarrow 2^{-1}(0) = 3 \arctan(1) + C \Rightarrow C = -\frac{3\pi}{4}$$

$$(x^2 + 1)^{-1} y = 3 \arctan(x) - \frac{3\pi}{4}$$

$$y = 3(x^2 + 1) \arctan(x) - \frac{3\pi}{4} (x^2 + 1)$$

Name: _____

Class Time: _____

14. (12 pts.) [Part A.] Let $u = \sin(x^2 + y^2)$. Find $\frac{\partial u}{\partial s}$ if $x = st$ and $y = \frac{1}{s+2t}$. Give your answer in terms of s and t .

$$\begin{aligned}
 \frac{\partial u}{\partial s} &= \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial s} \\
 &= 2x \cos(x^2 + y^2) \cdot t + 2y \cos(x^2 + y^2) \cdot \left[-(s+2t)^{-2} \cdot 1 \right] \\
 &= 2st^2 \cos\left[s^2t^2 + \frac{1}{(s+2t)^2}\right] + 2 \cdot \frac{1}{s+2t} \cos\left(s^2t^2 + \frac{1}{(s+2t)^2}\right) \cdot \frac{-s}{(s+2t)^2} \\
 &= \left(2st^2 - \frac{2s}{(s+2t)^3}\right) \cos\left(s^2t^2 + \frac{1}{(s+2t)^2}\right)
 \end{aligned}$$

$x^2 + y^2 = s^2t^2 + \frac{1}{(s+2t)^2}$

[Part B. (Unrelated to Part A)] Discuss the elasticity of $z = e^{x^2+xy^2}$ relative to x and y at $(x, y) = (1, 1)$

- Find the percentage change in z when x increases by 1% and y does not change.

$$\Delta x = \frac{1}{100} \times 1 = \frac{1}{100} \quad ; \quad \Delta y = 0$$

$$\frac{\Delta z}{z(1,1)} \times 100\% \leftarrow ? \quad ; \quad z(1,1) = e^2$$

$$\Delta z \approx \frac{\partial z}{\partial x}(1,1) \cdot \Delta x + \frac{\partial z}{\partial y}(1,1) \cdot \Delta y$$

$$\frac{\partial z}{\partial x}(1,1) = (2x + y^2)e^{x^2+xy^2} \Big|_{\substack{x=1 \\ y=1}} = 3e^2$$

$$\Delta z \approx 3e^2 \cdot \frac{1}{100} = \frac{3e^2}{100}$$

$$\frac{\Delta z}{z(1,1)} \times 100\% = \frac{\frac{3e^2}{100}}{e^2} \times 100\% = \boxed{3\%}$$

A similar computation can be done for elasticity relative to y

$$\begin{aligned}
 Z_y(x,y) &= 2xye^{x^2+xy^2} \\
 Z_y(1,1) &= 2e^2
 \end{aligned}$$

Here $\Delta x = 0$ and $\Delta y = 0.01$

So Δz is approx $2e^2/100$

So elasticity relative to y is 2%

Math 10360: Calculus B
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November 17, 2050

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| 3. | ☐ a | ☐ b | ☐ c | ☐ d | ☐ e |
| 4. | ☐ a | ☐ b | ☐ c | ☐ d | ☐ e |

Please do NOT write in this box.

Multiple Choice _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

Total _____

Name: _____

Class Time: _____

Multiple Choice

1.(5 pts.) The mass density of a 5 cm circular disk is given by

$$\rho(r, \theta) = \theta e^{r^2} \text{ g/cm}^2$$

where $0 \leq r \leq 5$ and $0 \leq \theta \leq 2\pi$. Find the total mass of the disk in grams.

- (a) $\frac{\pi^2 e^{25}}{25}$ grams.
- (b) $\pi^2 (e^{25} - 1)$ grams.
- (c) $\pi^2 e^{25}$ grams.
- (d) $\pi^2 (2e^{25} - 2)$ grams.
- (e) $2\pi^2 e^{25}$ grams.

2.(5 pts.) Using Polar Coordinates, evaluate the following integral:

$$\int_0^2 \int_{-\sqrt{4-y^2}}^0 y \, dx \, dy.$$

- (a) 2
- (b) $\frac{8}{3}$
- (c) $\frac{1}{2}$
- (d) 1
- (e) $\frac{4}{3}$

Name: _____

Class Time: _____

3.(5 pts.) A population of a specie of river dolphins is given, in the thousands, by the function

$$p(t) = \frac{2e^t + 7}{2e^t + 2}$$

where t is time in years. Find the time t for which the population of dolphins is 2 thousand.

- (a) $\ln(2)$ years.
- (b) $\ln(3)$ years.
- (c) $\frac{\ln 3}{\ln 2}$ years.
- (d) $\frac{\ln 3}{2}$ years.
- (e) $\ln\left(\frac{3}{2}\right)$ years.

4.(5 pts.) Find the polar coordinates of the point P whose cartesian coordinates is $(-1, \sqrt{3})$. You may assume that $r \geq 0$ and $0 \leq \theta < 2\pi$.

- (a) $(2, 2\pi/3)$
- (b) $(4, 5\pi/3)$
- (c) $(2, 5\pi/6)$
- (d) $(4, 4\pi/3)$
- (e) $(2, 11\pi/6)$

Name: _____

Class Time: _____

Partial Credit

You must show your work on the partial credit problems to receive credit!

5.(12 pts.) Find $\frac{\partial z}{\partial y}$ if $z^2 \sin(xy) + xy^2 = 2z$.

Name: _____

Class Time: _____

6.(12 pts.) Consider the initial value problem:

$$y' = (y + t)^2; \quad y(0) = 1$$

Use Euler's method with two equal steps to estimate $y(1)$.

Name: _____

Class Time: _____

7.(12 pts.) [A]. Solve the following initial value problem: $y' = y + 2e^{3x}; \quad y(0) = 3$

[B-1 (No relation with the above)]. A tank contains 50 liters of syrup with sugar concentration 0.1 lb/liter. Syrup with sugar concentration 0.2 lb/liter is **pumped in** at a rate of 3 liters/min and syrup is **pumped out** of the tank at rate 5 liters/min. Set up but do not solve the **initial value problem** that gives the amount $y(t)$ in pounds of sugar in the tank at time t . Your answer should have a differential equation and initial condition.

[B-2]. For the mixing problem in Part B-1 above, when will the tank dry out?

Name: _____

Class Time: _____

8.(12 pts.) Consider the differential equation and given conditions:

$$\frac{dy}{dx} = k(y^2 + 1); \quad y(0) = 1; \quad y(2) = \sqrt{3}$$

where k is a constant. Find the constant k and the solution $y(x)$.

Name: _____

Class Time: _____

9.(12 pts.) **a.** Estimates the sensitivity of f relative to x at $(1.5, 2)$ and the sensitivity of f relative to y at $(1.5, 2)$.

		x			
		**	1.5	2.0	2.5
y	0.0	36	35	34	
	1.0	40	47	35	
	2.0	38	42	38	

b. Estimates the elasticity of f relative to x at $(1.5, 2)$ and the elasticity of f relative to y at $(1.5, 2)$. Is f more elastic in x or y at $(1.5, 2)$?

Name: _____

Class Time: _____

10.(12 pts.) **(a)** $\frac{\partial u}{\partial s}$ where $u = e^{xyz}$; $x = s + 2t$, $y = 2s + t^2$ and $z = s^3 + t$.

(b) Find u_t when $t = -1$ and $s = 1$.

Name: _____

Class Time: _____

11.(12 pts.) Let $f(x, y) = e^{2x^2 - y^2 - 1}$

(a) Find the linearization of f at $(1, -1)$.

(b) Using linear approximation of f at $(1, -1)$, estimate the change in f as (x, y) changes from $(1, -1)$ to $(1.2, -1.2)$.

(c) Find the elasticity of f relative to x at $(1, -1)$.

Math 10360: Calculus B
Exam III
November 17, 2050

Name: _____
Class Time: ANSWERS

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour and 15 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

Honor pledge. “As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.”:

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!					
1.	☐ a	☐ ●	☐ c	☐ d	☐ e
2.	☐ a	☐ ●	☐ c	☐ d	☐ e
3.	☐ a	☐ b	☐ c	☐ d	☐ ●
4.	☐ ●	☐ b	☐ c	☐ d	☐ e

Please do NOT write in this box.	
Multiple Choice	_____
5.	_____
6.	_____
7.	_____
8.	_____
9.	_____
10.	_____
11.	_____
Total	_____

Name: _____

Class Time: _____

12.(12 pts.) Consider the initial value problem:

$$y' = (y + t)^2; \quad y(0) = 1$$

Use Euler's method with two equal steps to estimate $y(1)$.

$$\Delta t = \frac{1-0}{2} = 0.5$$



$$y(0) = 1$$

$$y(0.5) \simeq y(0) + y'(0) \cdot \Delta t$$

$$= 1 + 1(0.5) = 1.5$$

$$y(1) \simeq y(0.5) + y'(0.5) \Delta t$$

$$= 1.5 + (4)(0.5)$$

$$= 3.5$$

$$y'(t) = (y(t) + t)^2$$

$$y'(0) = (y(0) + 0)^2$$

$$= 1$$

$$y'(0.5) = (y(0.5) + 0.5)^2$$

$$= (1.5 + 0.5)^2 = 4$$