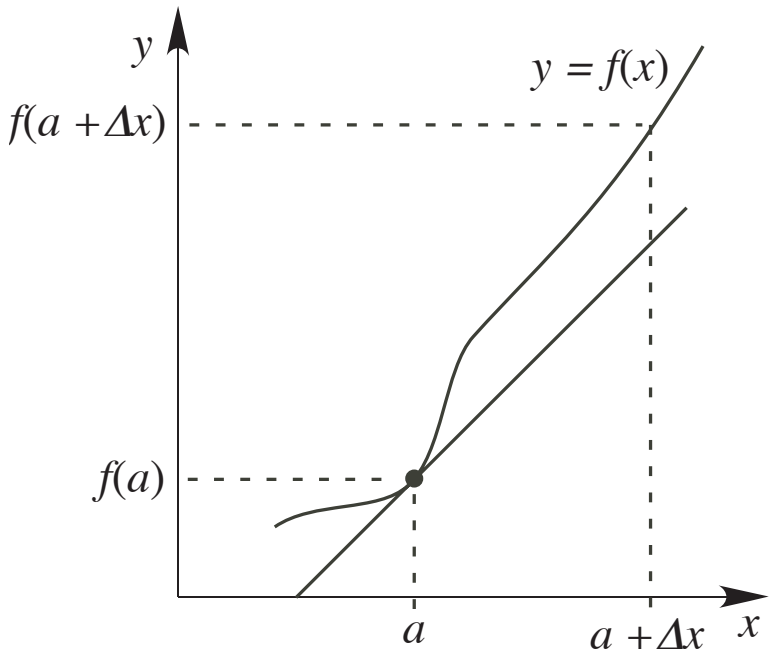
Math 10350 – Example Set 08A
Section 4.1 Linear Approximation and Applications



The linear approximation (or _____ or _____)

of a differentiable function $f(x)$ at $x = a$ is given by the function of the _____ to the graph of $f(x)$ at $x = a$.

$f(x) \approx L(x) =$ _____

(a) The point of tangency $x = a$ is also called the _____ of the linear approximation.

(b) As x changes from $x = a$ (_____) to $x = a + \Delta x$ (_____)

the value of $f(x)$ changes from _____ to _____ .

Exact value of the change in f is $\Delta f =$ _____

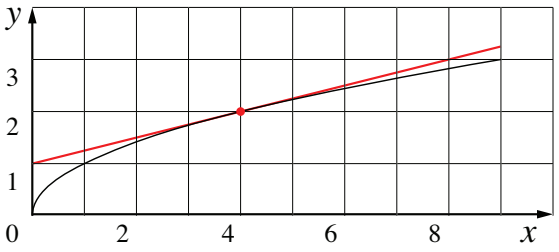
(c) For small Δx, the linear approximation of $f(x)$ at $x = a$ gives

$$\Delta f \approx \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

(d) Such estimates for Δf are also used to approximate percentage change.

1a. Find the tangent line to $f(x) = \sqrt{x}$ at $x = 4$.

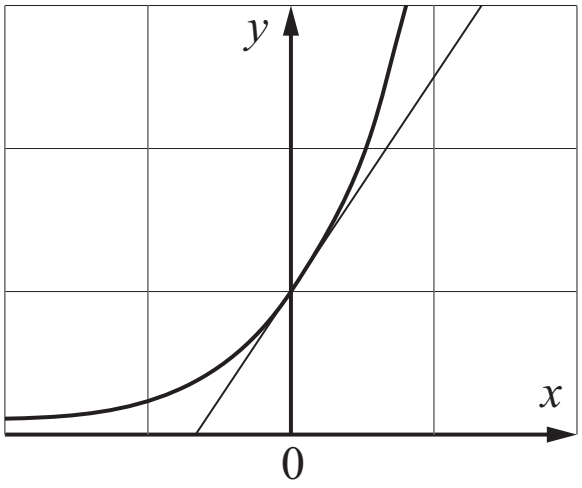
1b. Write down the linearization (linear approximation) of $f(x) = \sqrt{x}$ at $x = 4$.



1c. Using your answer in (b), estimate the following values and comment on their accuracy with a calculator:

(i) $f(4.05) \overset{?}{\approx}$ (ii) $f(3.9) \overset{?}{\approx}$ (iii) $f(5) \overset{?}{\approx}$

2. Find the linearization (tangent line approximation) of e^{2x} at $x = 0$. Estimate $e^{0.03}$. Draw a graph to illustrate your estimation. Is your estimate an overestimate or underestimate?



3.

t sec	0.5	3.5	6.5	9.5
$s(t)$ meters	5.0	6.5	5.8	7.0

The position $s(t)$ of a particle moving on a straight line at selected time t are given in the table above.

a. Use the central difference formula to estimate the velocity of the particle at $t = 3.5$ seconds.

b. Use linear approximation of $s(t)$ at $t = 3.5$ seconds to estimate the position of the particle at $t = 3.2$ second.

Math 10350 – Example Set 08B

Let $f(x)$ be differentiable at $x = a$. Then the linear approximation for the change Δf in $f(x)$ when x changes from a (anchor point) to $a + \Delta x$:

$$\Delta f = f(a + \Delta x) - f(a) \approx f'(a)\Delta x$$

1. The diameter of a circular disk is given as 10 cm with a maximum error in measurement of 0.2 cm. Use linear approximation to estimate the maximum error (ΔA) and percentage error in the calculated area of the disk. If the disk is made with an expensive titanium sheet that costs \$50 per cm^2, estimate an upper limit for your budget in making a disk of 10 cm diameter (Upper limit for budget is $\$(1250\pi + 50\pi) = \$1300\pi$).

2. A vessel is in the shape of an inverted cone. The radius of the top is 5 cm and the height is 8 cm. Water is poured in to a height of x cm. Find an expression for the volume V of the water in the vessel in terms of x. Use linear approximate to estimate the increased in V when x increases from 4 cm to 4.08 cm. Give units for your answer. (Answer: $\pi/2$ cm^3)