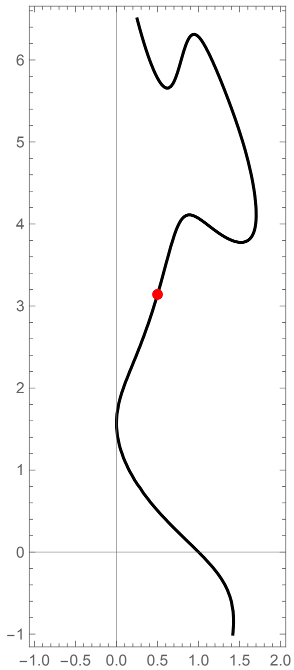
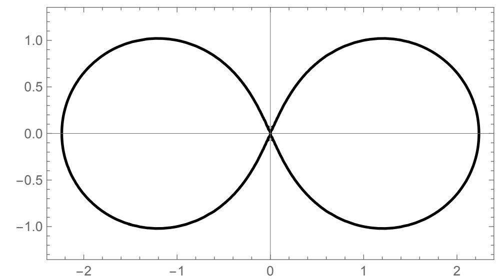
Math 10350 – Example Set 07A
3.8 Implicit Differentiation
3.10 Related Rates

1. Find the equation of the tangent line to the curve $\cos(xy) + 1 = 2x + 2\sin(y)$ at the point $\left(\frac{1}{2}, \pi\right)$.

(Ans: $y = (2 + \pi)x - 1 - \frac{\pi}{2}$)



2. A particle is moving along the curve $(x^2 + y^2)^2 = 5x^2 - y^2$. If the $x-$coordinate of the particle is increasing at the rate of 1 unit per second at $(1, -1)$, how fast is the $y-$coordinate of the particle changing at $(1, -1)$? Is the particle traveling upward or downward at $(1, -1)$?



Math 10350 – Example Set 07A
3.10 Related Rates

3. A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is his shadow on the building decreasing when he is 4 m from the building. (Answer: 0.6 m/s)

Math 10350 – Example Set 07B
3.10 Related Rates

1. Water is leaking out of a conical tank with pointed end down at the rate of 9 ft^3/min. The tank is 4 feet high and the radius at the top is 3 feet. At what rate is the water level changing when the water is 2.5 feet deep? At the same moment, how fast is the area of the water surface changing?

Math 10350 – Example Set 07B
Section 11.1 Parametric Equations (Application of Chain Rule)

The **parametric equations** of a curve is given by the pair of equations $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$

The independent variable t is called the ____________________.

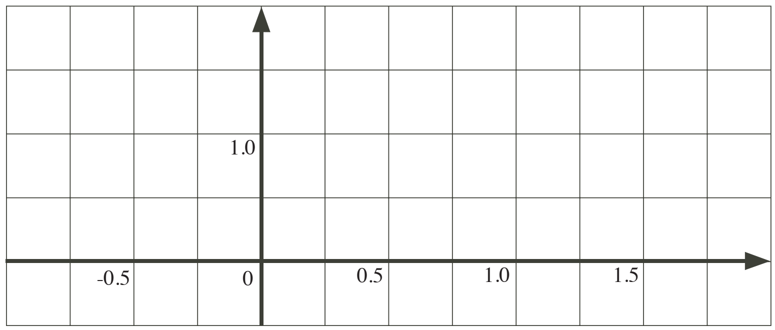
The parametric equations describe the ____________________ traced by a particle on the $xy-$plane as the parameter t ____________________.

This differs from the Cartesian equation (For examples $y = F(x)$ or $x^2 + y^2 = 1$) where the curve is a ____________________ planar curve and does not describe the ____________________ of a particle on the coordinate plane.

2a. (Section 11.1) Find the coordinates at the time $t = 0, 0.5, 1, 1.5, 2, 2.5, 3$ of a particle following the path given by the **parametric equations:** $\quad x = t - 1; \quad y = (t-2)^{2/3}$

t	0.0	0.5	1.0	1.5	2.0	2.5	3.0
x							
y							

2b. Plot the points in (a) and draw the curve given by the parametric equations. Indicate the direction of your path.



2c. By eliminating the parameter, find the cartesian equation for the path you drew.

Remark: Using Chain Rule, obtain the slope formula $\frac{dy}{dx}$ for the parametric equations $x = f(t)$ and $y = g(t)$.

$$\boxed{\frac{dy}{dx} = \underline{\hspace{3cm}} = \underline{\hspace{3cm}}}$$

3. Find the equation of the tangent line at $t = 10$ to the path given by the parametric equations:

$$\begin{cases} x = t - 1 \\ y = (t-2)^{2/3} \end{cases}$$

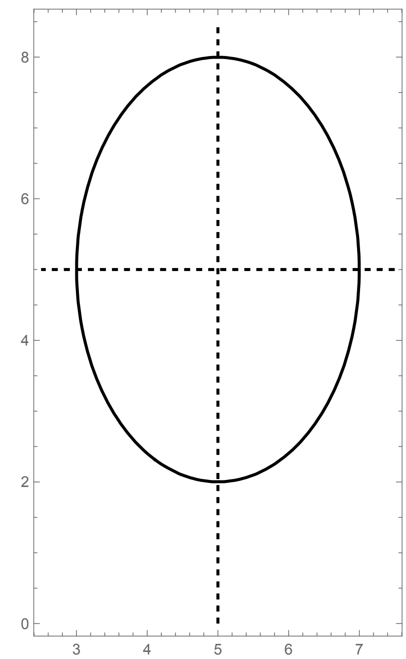
Math 10350 – Example Set 07C
Section 11.1 Parametric Equations (Application of Chain Rule)

1. The population of wolves $w(t)$ and wild boars $p(t)$ in the thousands are given by the equations:

$$p(t) = 2\cos t + 5; \qquad w(t) = 3\sin t + 5.$$

(a) What is the rate of change of w with respect to p at $t = \dfrac{\pi}{4}$ and $t = \dfrac{\pi}{6}$? (b) Find a relation between w and p by eliminating t. (c) The graph of the p and w relationship in a w vs p coordinate plane is below. Describe what is happening between the two populations as time t progresses. Hint: Input different values of t and trace the curve.

(a) $-\frac{3}{2}$, $-\frac{3}{2\sqrt{3}}$; (b) $\frac{(p-5)^2}{4} + \frac{(w-5)^2}{9} = 1$



2. Consider the curve given by the parametric equations: $x = t^2 + 3; \qquad y = \ln(t+2)$

2a. Find the equation of the tangent line at $t = 1$. $\left(y = \frac{1}{6}x + \ln(3) - \frac{2}{3}\right)$

2b. By eliminating the parameter, find the cartesian equation for the curve given by parametric equations. (Would it be easier to solve for t in terms of x or y?)

3. A 5 meter long ladder leaning against a vertical wall such that one end is on the wall and the other end is on the ground about 0.5 meter away. Suppose the top end of the ladder is slipping down the wall at a constant rate of 1/4 meter/min. (a) How fast is the lower end of the ladder on the ground moving away from the wall when the lower end is 3 meters from the bottom of the wall? (b) At what rate the angle of inclination of the ladder changing at the same moment?

(a) $\frac{1}{3}$ m/min; (b) $-\frac{1}{12}$ rad/min