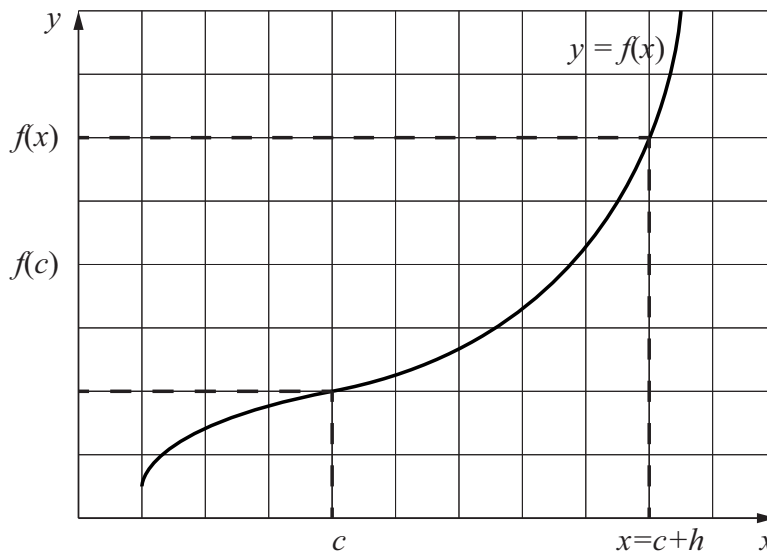


Math 10350 – Example Set 04A
 Sections 3.1& 3.2
 Differentiability & Derivative of a Function

Summary: Average Rate and Instantaneous Rate of Change



- The average rate of change of $f(x)$ over the time interval between x and c is given by:

- The difference quotient of $f(x)$ at $x = c$ is given by:

- The instantaneous rate of change of $f(x)$ at $x = c$ is given by:

$$f'(c) =$$

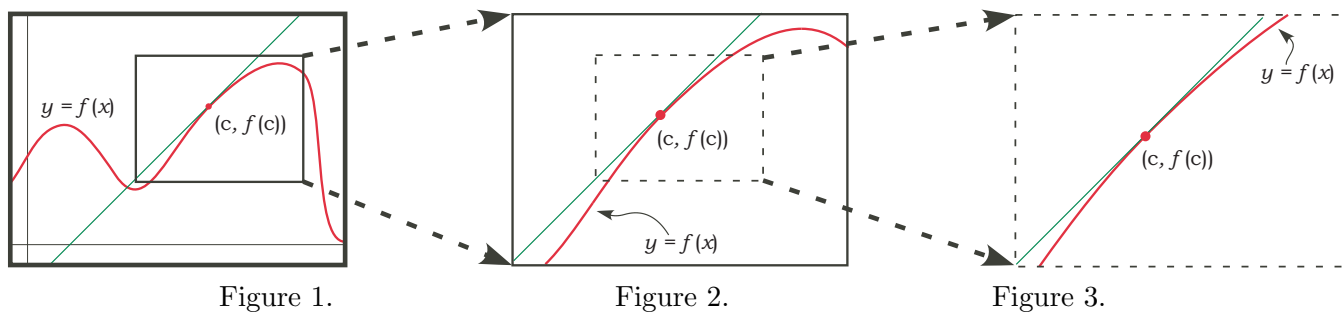
$f'(c)$ is also called the _____

and it gives the _____ to the graph of $f(x)$ at $x = c$.

Definition A function $f(x)$ is said to be differentiable at $x = c$ provided the following limit exist:

This means that the slope at $x = c$ of the graph is a _____ number. We denote this number by $f'(c)$.

Graphically, differentiable means that each small segment of the graph of $f(x)$ is almost identical to a straight line. This is illustrated in Figure 1 through 3 below. As you zoom into the point $(c, f(c))$, the segment of the graph of $f(x)$ near point c becomes more and more like its tangent line at $x = c$.



Theorem 1 If $f(x)$ be differentiable at $x = c$, then $f(x)$ is _____ at $x = c$.

Graphical Interpretation: If a $f(x)$ is differentiable $f(x)$ on $x = c$ than we say that

- (1) the graph of $f(x)$ is locally _____ at $x = c$ ie the graph of $f(x)$ looks line a _____ near $x = c$.
- (2) the graph of $f(x)$ has a unique _____ line at $x = c$.
- (3) the graph of $f(x)$ looks _____ at $x = c$ ie no sharp _____ at $x = c$.

Remark: We say that a function $f(x)$ is differentiable on $a < x < b$ if $f(x)$ is differentiable for all $x = c$ in $a < x < b$.

Derivative of a function. The derivative of the function $f(x)$ is given by the following limit:

$$f'(x) =$$

Setting $\Delta x = h$ and $\Delta y = f(x + h) - f(x)$ gives the notation:

$$f'(x) = \frac{f(x) - f(a)}{x - a} = \frac{f(x) - f(b)}{x - b}$$

Notation: If $y = f(x)$ is a differentiable function. Write down all standard notations of the derivative of $y = f(x)$.

1. Let $f(x) = \frac{1}{x^2}$. Compute the derivative or the slope function of $f(x)$ using limits by following steps below.

- 1a.** Find the average rate of change of f over the interval between x and $x + h$ assuming that $h \neq 0$.

This is also called the

1b. Using (a), find the derivative of $f(x)$ (with respect to x) using the limit definition.

1c. What is the instantaneous rate of change of $f(x)$ with respect to x ? _____

1d. Infer from the above derivative formula for x^{-2} , a derivative formula for a general power function x^n

$\frac{d}{dx}(x^n) = (x^n)' =$ _____ . (We call this the _____ Rule)

1e. Find the equation of the tangent line to the graph of $f(x) = \frac{1}{x^2}$ at $x = 2$. Draw a graph to illustrate the curve and the tangent line.

Some Common Derivatives. For any numbers k and n :

$$\frac{d}{dx}(k) \stackrel{?}{=} \qquad \frac{d}{dx}(x^n) \stackrel{?}{=} \qquad \text{(Power Rule)}$$

Basic Properties of Derivatives:

$$[f(x) + g(x)]' \stackrel{?}{=} \qquad [f(x) - g(x)]' \stackrel{?}{=} \qquad [c \cdot f(x)]' \stackrel{?}{=}$$

2. Find the derivative of each of the following functions with respect to the :

a. $f(x) = \sqrt{x} + \frac{\pi}{\sqrt{x}}$

b. $y = \frac{x^3 + 5x + 6}{x}$

c. $h(t) = (2 + \sqrt{t}) t^2$

Math 10350 – Example Set 04B

1. Find the equation(s) of the tangent line(s) to the graph of $y = x^3 + 2$ is parallel to the line $24x - 2y = 3$.

2. Use the fact $\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = \underline{\hspace{1cm}}$ to obtain formulas for $\frac{d}{dx}(e^x)$ and $\frac{d}{dx}(a^x)$.

3. The position (in feet) of a particle moving on a straight line is given by the function

$$s(t) = \frac{5}{t} + t^e + 2e^t + 3^t.$$

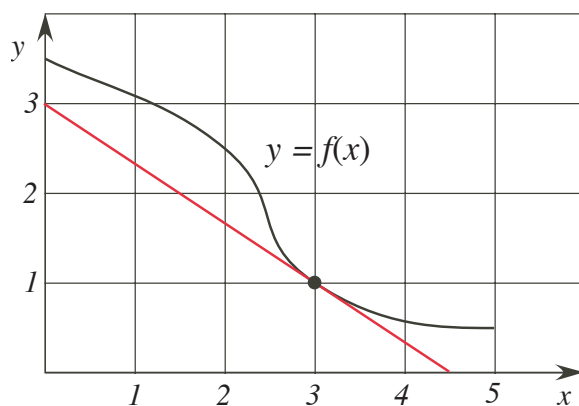
Find an expression for the (instantaneous) velocity $v(t)$. What is the velocity of the particle when $t = \ln 2$ seconds?

Math 10350 – Example Set 04C

1. If $f'(a) = \lim_{h \rightarrow 0} \frac{(3+h)^{10} - 3^{10}}{h}$, what is a possible $f(x)$ and the value of a ?

$f(x) \stackrel{?}{=} \underline{\hspace{2cm}}$ and $a \stackrel{?}{=} \underline{\hspace{2cm}}$

2.



The figure above describes the graph of $y = f(x)$ and its tangent line at $x = 3$. Answer the problems below:

- a. Estimate the average rate of change of $f(x)$ over the interval $[0, 5]$.

b. $f(3) \stackrel{?}{=} \underline{\hspace{2cm}}$ and $f'(3) \stackrel{?}{=} \underline{\hspace{2cm}}$

- c. Find the equation of the tangent line at $x = 3$. Give your answer in slope-intercept form.

- 3.** The slope of the curve $y = ax^2 + bx$ at the point $(2, 4)$ is -8 . Calculate the values of a and b .

4. Find the values of x for which both the graphs of the functions $f(x) = x^3 - 3x^2 + 7x + 8$ and $g(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 5x - 3$ have parallel tangent lines at x . Pick one such location on the graph of $f(x)$ and find the equation of the tangent line there.

5. A military craft made with a new technology that could change its velocity on demand in a moment was test driven on a long straight road. The graph of its position $s(t)$ for eight seconds of travel is given below. Sketch in the given axes below the velocity function $v(t)$ indicating clearly places where velocity is undefined.

