Math 10350 – Example Set 03A
Sections 2.2, 2.3 & 2.4

Discuss the different ways a variable x can approach a fixed value c.

Limit of a function. What happens to $f(x)$ as x gets as close to a fixed value c as we want? This question is answered with the concept of the limit of a function.

Explain what each of the following limits means.

$L = \lim_{x \to c^-} f(x)$ ____

We call this the ____________________ limit of $f(x)$ as x ____________________ c.

$L = \lim_{x \to c^+} f(x)$ ____

We call this the ____________________ limit of $f(x)$ as x ____________________ c.

$L = \lim_{x \to c} f(x)$ ____

We call this the (____________) limit of $f(x)$ as x ____________________ c.

1. The graph of a function f is shown in Figure 1. By inspecting the graph, find each of the following values and limits if it exists. If the limit does not exist, explain why.

$\lim_{x\to -1^-} f(x) \stackrel{?}{=}$

$\lim_{x\to -1^+} f(x) \stackrel{?}{=}$

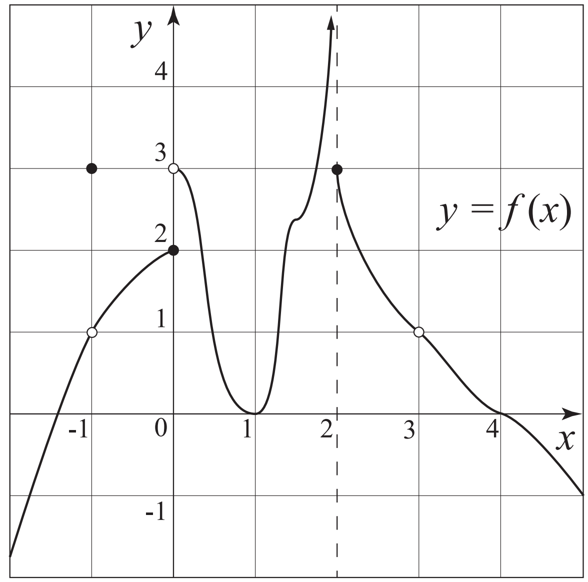


Figure 1

$\lim_{x\to -1} f(x) \stackrel{?}{=}$

$f(-1) \stackrel{?}{=}$

$\lim_{x\to 0^-} f(x) \stackrel{?}{=}$

$\lim_{x\to 0^+} f(x) \stackrel{?}{=}$

$\lim_{x\to 0} f(x) \stackrel{?}{=}$

$f(0) \stackrel{?}{=}$

$\lim_{x\to 2} f(x) \stackrel{?}{=}$

$f(2) \stackrel{?}{=}$

$\lim_{x\to 3} f(x) \stackrel{?}{=}$

$f(3) \stackrel{?}{=}$

$\lim_{x\to 2^-} f(x) \stackrel{?}{=}$

$\lim_{x\to 2^+} f(x) \stackrel{?}{=}$

$\lim_{x\to 3^-} f(x) \stackrel{?}{=}$

$\lim_{x\to 3^+} f(x) \stackrel{?}{=}$

Theorem 1 $\lim_{x\to c} f(x)$ exists $\iff$ ______________ and ______________ both exist as numbers and are equal. Specifically, $\lim_{x\to c} f(x) = L \iff$ ______________ $= L =$ ______________.

Remark If any of the following are true:

$$\lim_{x\to c^-} f(x) = \infty; \qquad \lim_{x\to c^-} f(x) = -\infty; \qquad \lim_{x\to c^+} f(x) = \infty; \qquad \lim_{x\to c^+} f(x) = -\infty$$

then the graph of $f(x)$ has a ____________________________ at $x = c$.

Properties of Limits. Suppose $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$ exist. Then we have the following statements:

(1) $\lim_{x\to c} k \cdot f(x) =$

(2) $\lim_{x\to c}[f(x) + g(x)] =$

(3) $\lim_{x\to c}[f(x) - g(x)] =$

(4) $\lim_{x\to c} f(x) \cdot g(x) =$

(5) $\lim_{x\to c} \dfrac{f(x)}{g(x)} =$ provided $\lim_{x\to c} g(x) \neq 0$.

(6) $\lim_{x\to c}[f(x)]^n =$

(7) $\lim_{x\to c} \sqrt[n]{f(x)} =$ assume $\lim_{x\to c} f(x) \geq 0$ if n is even.

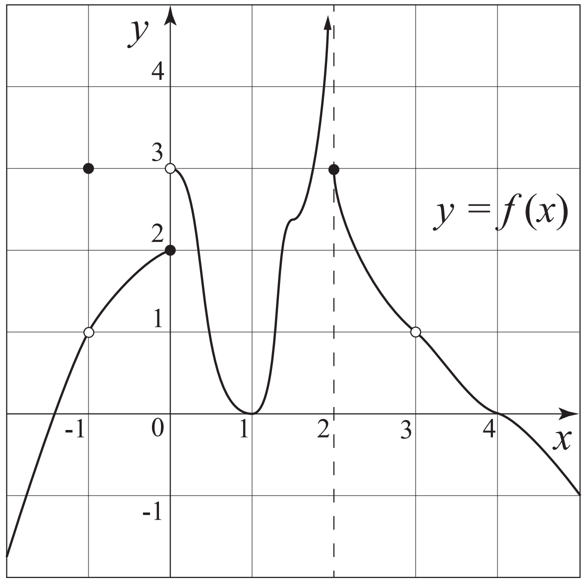


Figure 1

2. The graph of $f(x)$ is given in Figure 1 and $g(x) = 3x + 2$. By thinking about the values each function $f(x)$ and $g(x)$ approaches in the expressions below deduce the value of each limits:

$(a)\ \lim\limits_{x\to 3} [2f(x) + 3g(x)] \stackrel{?}{=}$

$(d)\ \lim\limits_{x\to 0} [f(x) - g(x)]^4 \stackrel{?}{=}$

$(b)\ \lim\limits_{x\to 2^+} [f(x) \cdot g(x)] \stackrel{?}{=}$

$(e)\ \lim\limits_{x\to 0} \sqrt{f(x)} \stackrel{?}{=}$

$(c)\ \lim\limits_{x\to 0^-} \dfrac{g(x)}{f(x) + 4} \stackrel{?}{=}$

3. **(Limit Concept)** Consider the **piecewise defined** function $f(x) = \begin{cases} 2-x & -\infty < x < 0 \\ x^2 & 0 \leq x < 1 \\ 1 & 1 \leq x < +\infty \end{cases}$

(a) Find the following values and limits if they exist without sketching the graph.

i. $f(-1)$

ii. $f(3)$

iii. $\lim\limits_{x \to 1^-} [2f(x) - 5]$

iv. $\lim\limits_{x \to 0} (f(x) - 1)^2$

v. $\lim\limits_{x \to -5} (f(x))^2$

Math 10350 – Example Set 03B
Sections 2.2, 2.3 & 2.4
Computing Limits and Continuity

• A function $f(x)$ has a **jump** discontinuous at $x = c \iff$ ________________ $\neq$ ________________.

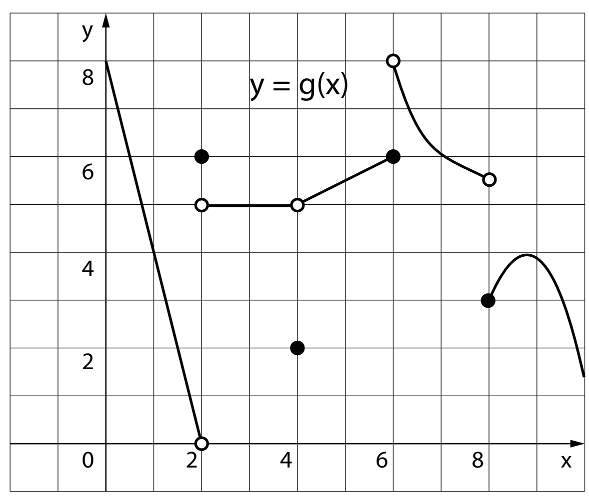
Graphically, $f(x)$ having a **jump** discontinuity at $x = c$ means there is a ________________ in the graph of $f(x)$ at $x = c$

• A function $f(x)$ has a **removable** discontinuous at $x = c \iff \lim_{x \to c} f(x)$ exist but ________________ $\neq f(c)$.

Graphically, $f(x)$ having a **removable** discontinuity at $x = c$ means there is a ________________ in the graph of $f(x)$ at $x = c$

1. Consider the piecewise defined function $g(x)$ given by the graph above for $0 \leq x \leq 10$.

(a) State the values of x where $g(x)$ has a jump discontinuity for $0 < x < 10$. Explain your answers using limits.



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(b) State the values of x where $g(x)$ has a removable discontinuity for $0 < x < 10$. Explain your answers using limits.

- A function $f(x)$ is continuous at $x = c \iff f(c)$ is defined and ________________ $= f(c)$.

Graphically, $f(x)$ being continuous at $x = c$ means ____

- A function $f(x)$ said to be continuous for $a < x < b \iff f(x)$ is ________________ for ________________

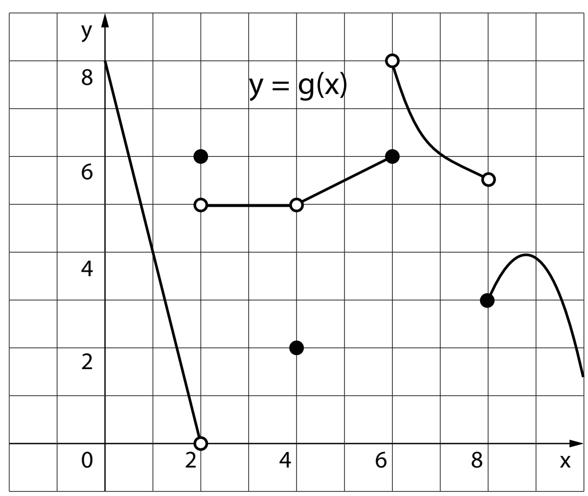
__________________________ .

Give some common examples of continuous functions:

- A function $f(x)$ is **left** continuous at $x = c \iff f(c)$ is defined and ________________ $= f(c)$.

- A function $f(x)$ is **right** continuous at $x = c \iff f(c)$ is defined and ________________ $= f(c)$.

2. Comment on the continuity of $g(x)$ for $0 < x < 10$ given by the graph below. State also the values of x where $g(x)$ is not continuous but is left or right continuous or neither.

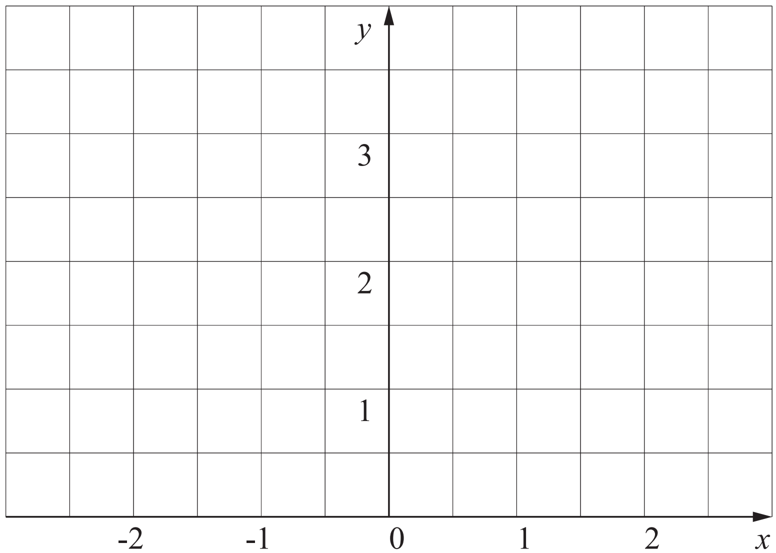


1. (Limit Concept) Consider the **piecewise defined** function

$$f(x) = \begin{cases} 2 - x & -\infty < x < 0 \\ x^2 & 0 \leq x < 1 \\ 1 & 1 \leq x < +\infty \end{cases}$$

(a) Without sketching the graph of $f(x)$, determine the continuity of $f(x)$ at (i) $x = 0$ and (ii) $x = 1$.

(b) Sketch the graph of $f(x) = \begin{cases} 2 - x & -\infty < x < 0 \\ x^2 & 0 \le x < 1 \\ 1 & 1 \le x < +\infty \end{cases}$



Math 10350 – Example Set 03C
Sections 2.2, 2.3 & 2.4
Computing Limits and Continuity

1. Using limits find k such that the function $f(x)$ (a) is continuous at $x = 2$ and (b) is discontinuous at $x = 2$. Sketch a graph clearly depicting the nature of $f(x)$ at $x = 2$ in (a) and (b).

$$f(x) = \begin{cases} \dfrac{x^2 + x - 6}{x - 2} & x \neq 2 \\ k & x = 2 \end{cases}$$

2. Determine the constants a and b such that the following function is continuous on the entire real line

$$f(x) = \begin{cases} 2 & -\infty < x \leq -1 \\ ax^2 + b & -1 < x < 3 \\ -2 & 3 \leq x < +\infty \end{cases}$$

3. Consider the piecewise continuous function

$$g(x) = \begin{cases} \dfrac{2 - x - x^2}{x+2} & -\infty < x < -2 \text{ or } -2 < x < 0 \\ -1 & x = -2 \\ x^2 - 4 & 0 \le x < +\infty \end{cases}$$

Discuss the continuity of $g(x)$ at $x = -2$ and at $x = 0$.