Math 10350 – Example Set 01A
Functions Review: Sections 1.1, 1.2, & 1.3

Algebra Review - Arithmetic Rules & Laws of Exponent

Complete the Arithmetic Operations below:

$a(b+c) =$

$\dfrac{a}{b} + \dfrac{c}{d} =$

$a \times b =$

$a(bc) =$

$\dfrac{a+b}{c} =$

$\dfrac{a}{b} \times \dfrac{c}{d} =$

$a \div \dfrac{b}{c} =$

$\dfrac{\frac{a}{b}}{\frac{c}{d}} =$

$(a+b)(c+d) =$

$(a+b)(a-b) =$

$(a+b)^2 =$

$(a-b)^2 =$

Complete the Laws of Exponents below:

$a^m \cdot a^n =$

$(ab)^m =$

$\dfrac{a^m}{a^n} = \qquad ; a \neq 0$

$a^0 = \qquad ; a \neq 0$

$a^{1/m} =$

$\left(\dfrac{a}{b}\right)^m = \qquad ; b \neq 0$

$\dfrac{1}{b^m} =$

$(a^m)^n =$

1. Simplify the following expression giving your answer in the form $(x^2+2)^m(2x+1)^n(Ax^2+Bx+C)$.

$(x^2+2)^3 \cdot 4(2x+1)^3 \cdot 2 + 3(x^2+2)^2 \cdot 2x(2x+1)^4$

Ans: $(x^2+2)^2(2x+1)^3(20x^2+6x+16)$

2. Simplify the following expression giving your answer in the form $\dfrac{(3x+2)^m(Ax^3+Bx^2+Cx+D)}{(2x^3+5)^n}$.

$$\frac{(2x^3+5)^{1/3} \cdot 4(3x+2)^3 \cdot 3 - \frac{1}{3}(2x^3+5)^{-2/3} \cdot 6x(3x+2)^4}{(2x^3+5)^{2/3}}$$

Ans: $\dfrac{(3x+2)^3(24x^3-6x^2-4x+60)}{(2x^3+5)^{4/3}}$

$\boxed{\text{Definition}}$ A **function** is a rule that assigns a (single) value y (in the range) to each value x (in the domain).

(Basic Functions) Give an example for each type of basic functions below and give their general form:

A. Power Function:

An example: ____________________

General form: ____________________

Monomial: ____________________

B. Polynomial Function:

An example: ____________________

General form: ____________________

Special Cases

Linear functions: ____________________

Quadratic functions: ____________________

C. Rational Function:

An example: ____________________

General form: ____________________

D. Exponential Function:

An example: ____________________

General form: ____________________

E. Logarithmic Function:

An example: ____________________

General form: ____________________

F. Trigonometric Function:

Math 10350 – Example Set 01A
Functions Review: Sections 1.1, 1.2, & 1.3

Price, Revenue, Cost & Profit. Write an equation that connects the revenue from the sale of a certain product, the number of the product sold (or demand), and selling price of one unit of the product. How does revenue differ from the profit from the sale of the product?

3. (An application of Functions) A electronic company decides to set the sale price of a sound card at \$60 a piece for a monthly demand of 100 pieces. The sale price drops to \$50 a piece for a monthly demand of 200 pieces.

3a. Assuming that the sale price for one sound card is a linear function of the size of the monthly demand, find a formula for the sale price s dollars per sound card in terms of the size x of the monthly demand. What is the revenue function from the sales of the sound card? $\left(-\frac{x}{10}+70; -\frac{x^2}{10}+70x\right)$

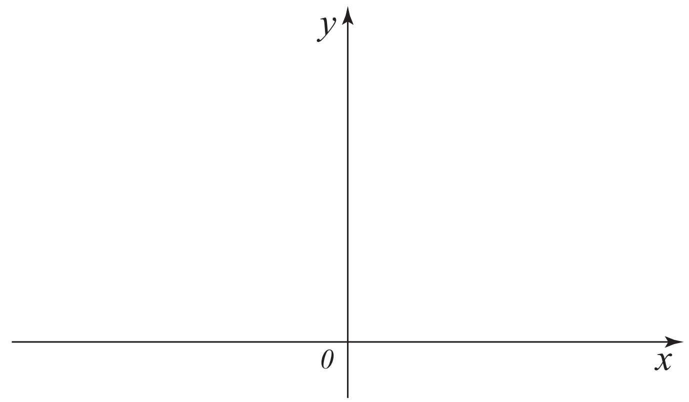
3b. Suppose further that the company has a monthly overhead cost of \$5000 for producing the sound cards and a cost of \$10 for producing each piece of the sound card. What is the monthly profit from the sales of the sound card in terms of month production assuming that all items produced are sold? $\left(-\frac{x^2}{10}+60x-5000\right)$

Math 10350 – Example Set 01B
Exponential & Logarithmic Function: Section 1.6

Graph of $y = a^x$

Case 1: $a > 1$

For example, $y = 2^x$.



(i) Complete the table below:

x	-2	-1	0	1	2
2^x					

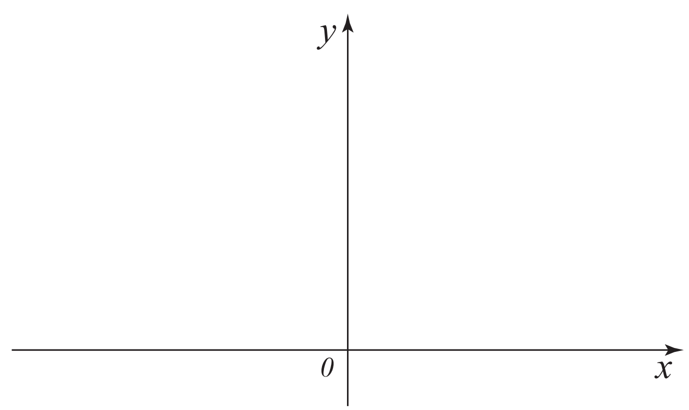
(iii) **Properties of a^x when $a > 1$:**

- $a^0 \stackrel{?}{=}$
- domain $\stackrel{?}{=}$ range $\stackrel{?}{=}$
- As x gets unboundedly large and positive, the value of a^x gets ____________________. $\lim_{x\to\infty} a^x \stackrel{?}{=}$
- As x gets unboundedly large but negative, the value of a^x gets ____________________. $\lim_{x\to-\infty} a^x \stackrel{?}{=}$
- Asymptote:

Graph of $y = a^x$

Case 1: $0 < a < 1$

For example, $y = (1/2)^x$.



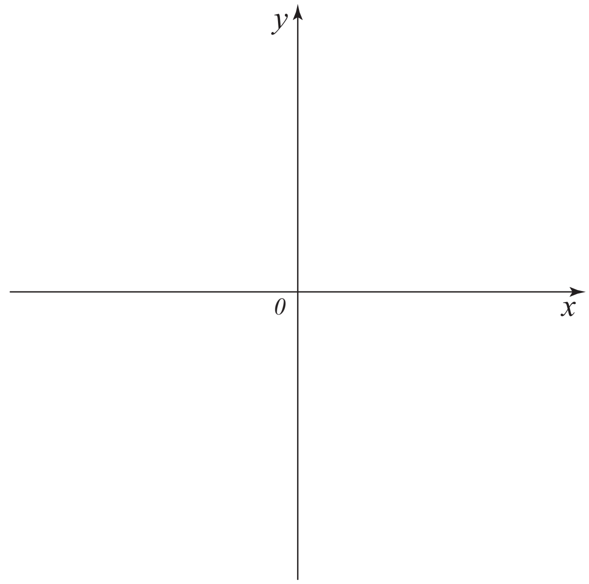
(i) Complete the table below:

x	-2	-1	0	1	2
$(1/2)^x$					

(iii) **Properties of a^x when $0 < a < 1$:**

- $a^0 \stackrel{?}{=}$
- domain $\stackrel{?}{=}$ range $\stackrel{?}{=}$
- As x gets unboundedly large and positive, the value of a^x gets ____________________. $\lim_{x\to\infty} a^x \stackrel{?}{=}$
- As x gets unboundedly large but negative, the value of a^x gets ____________________. $\lim_{x\to-\infty} a^x \stackrel{?}{=}$
- Asymptote:

Sketch the graph of the inverse of a^x for $a > 1$. State its domain and range. We call the inverse of a^x the logarithm function to the base a.

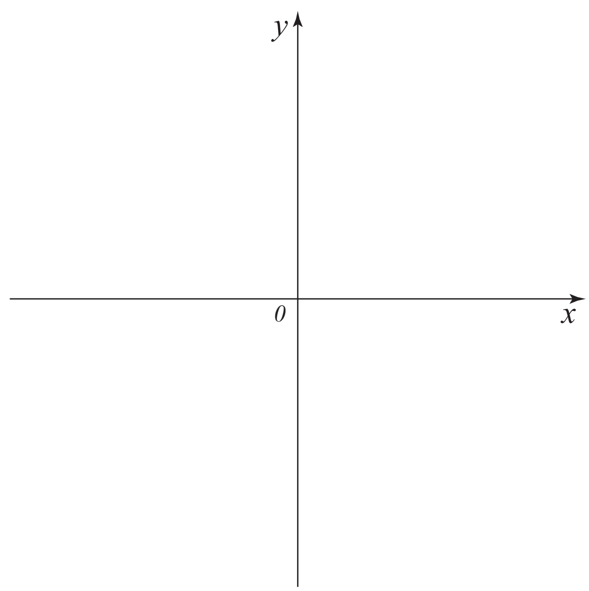


Graph of $y = \log_a x$

Case 1: $a > 1$

(iii) **Properties of** $\log_a x$ **when** $a > 1$**:**

- $\log_a 1 \stackrel{?}{=}$
- domain $\stackrel{?}{=}$ range $\stackrel{?}{=}$
- As x gets unboundedly large and positive, the value of $\log_a x$ gets ____________________ . $\lim\limits_{x \to \infty} \log_a x \stackrel{?}{=}$
- As x gets closer and closer to 0 but positive, the value of $\log_a x$ gets ____________________ . $\lim\limits_{x \to 0^+} \log_a x \stackrel{?}{=}$
- Asymptote:



Graph of $y = \log_a x$

Case 2: $0 < a < 1$

(iii) **Properties of** $\log_a x$ **when** $0 < a < 1$**:**

- $\log_a 1 \stackrel{?}{=}$
- domain $\stackrel{?}{=}$ range $\stackrel{?}{=}$
- As x gets unboundedly large and positive, the value of $\log_a x$ gets ____________________ . $\lim\limits_{x \to \infty} \log_a x \stackrel{?}{=}$
- As x gets closer and closer to 0 but positive, the value of a^x gets ____________________ . $\lim\limits_{x \to 0^+} \log_a x \stackrel{?}{=}$
- Asymptote:

1. Complete the following properties of the logarithm function to the base a below. Could you prove some of them?

i. Change to log base b: $\log_a x =$

ii. $a^{\log_a x} \stackrel{?}{=}$

iii. $\log_a(a^x) \stackrel{?}{=}$

iv. $\log_a(a) \stackrel{?}{=}$

v. $\log_a 1 \stackrel{?}{=}$

vi. $\log_a(xy) \stackrel{?}{=}$

vii. $\log_a(x^n) \stackrel{?}{=}$

viii. $\log_a\left(\frac{x}{y}\right) \stackrel{?}{=}$

Commonly used natural log properties: $\ln(e^x) \stackrel{?}{=}$; $e^{\ln x} \stackrel{?}{=}$

2. (Sect 1.6) A quantity y is said to grow or decay exponentially with time t if $y(t) = k \cdot a^t$. A . It is known that the amount of a medication in a patient reduces from an initial amount of 100 mg to 40 mg after three hours. Assuming that the amount of medication decays exponentially, write a formula for the amount of the medication $y(t)$ as a function of time t in hours. What is the half life of the medication in the body? Draw a graph for $y(t)$.

Math 10350 – Example Set 01C
Basic Exponential Equations: Section 1.6

1. Solve the following equations:

a. $4^x = \dfrac{1}{8}$

b. $3 \cdot 9^{x+1} = 81^x$

c. $3(4^{x-1}) = 5$ (Give your answer in base e)

d. $4e^{x-2} = 3e^{2x}$

2. Given that $\ln(x) = p$ and $\ln(y) = q$, write the following expressions in term of p and q

(i) $\ln(5x^2y^3) \stackrel{?}{=}$

(ii) $\ln\left(\sqrt[5]{\dfrac{e^4x}{y}}\right) \stackrel{?}{=}$

3. Solve for x for each of the following equations.

(a) $2\ln(x-3) = 3$

(b) $\log_2(2x+3) = \log_2(x+1) + 2$.

4a. It is said that one dog year equals seven human years. What kind of function is H in terms of D? Chart a graph of dog years D verses human years H.

4b. A new genomics research that studies aging in mammals (**) says that the equation between dog years and humans years is much more complicated: $H = 16\ln(D) + 31$. (i) What is the human years of a dog when it is one year old? (ii) What is the dog age of a 21 year old human? (**): Quantitative Translation of Dog-to-Human Aging by Conserved Remodeling of the DNA Methylome