ESTIMATING NONLINEAR CHANGE MODELS IN HETEROGENEOUS POPULATIONS WHEN CLASS MEMBERSHIP IS UNKNOWN: DEFINING AND DEVELOPING THE LATENT CLASSIFICATION DIFFERENTIAL CHANGE MODEL

A Dissertation

Submitted to the Graduate School

of the University of Notre Dame

in Partial Fulfillment of the Requirements

for the Degree of

Doctor of Philosophy

by

Kenneth Kelley III, B.A., M.A.

Scott E. Maxwell, Director

Ke-Hai Yuan, Director

Graduate Program in Psychology

Notre Dame, Indiana

April 2005

ESTIMATING NONLINEAR CHANGE MODELS IN HETEROGENEOUS POPULATIONS WHEN CLASS MEMBERSHIP IS UNKNOWN: DEFINING AND DEVELOPING THE LATENT CLASSIFICATION DIFFERENTIAL CHANGE MODEL

Abstract

by

Kenneth Kelley III

Standard methods for analyzing change generally assume that the population of interest is homogeneous or that heterogeneity is known. When a population consists of unknown subpopulations, the parameters within each of the latent classes may be unique to that particular class. In such a situation the results of standard techniques for analyzing change are misleading, because such methods ignore unobserved heterogeneity and treat the population as if it were homogeneous. The *growth mixture model* (GMM; Muthén, 2001a; Muthén, 2001b; Muthén, 2002) partly addresses the problem of unknown heterogeneity because the parameters of the GMM are conditional on latent class membership. However, the GMM is necessarily restricted to models of change linear in their parameters (such as polynomial change models). The *latent classification differential change* (LCDC) model allows change to follow a nonlinear functional form and parameter estimates to be conditional on latent class membership. Due to the integration of nonlinear multilevel models and finite mixture models, neither or which have closed form solutions, analytic solutions do not generally exist for the LCDC model. Five methods of parameter estimation are developed and evaluated with a Monte Carlo simulation study. The simulation showed that the parameters of the LCDC model can be accurately estimated with each of the proposed methods, and that the method of choice depends on the particular question of interest. In situations where a different functional form of change or a different error structure is desired across the latent classes, Method 3.1 is recommended. In situations where the functional form of change and the error structure are constant across class, and an interest exists in cross-class hypothesis testing and/or cross-class constraints, Method 3.2 is recommended. In situations where the functional form of change and fixed effect model specification is constant, with or without holding the error structure constant across class, Method 4 is recommended. Method 4 was shown to be the most accurate method in recovering the population fixed effect values in straightforward applications of the LCDC model. The LCDC model provides a novel method of modeling and understanding change when change is governed by a nonlinear functional form in populations where latent classes exist.

DEDICATION

For her love and support throughout this long and arduous process, I dedicate this document to Heather, a good person, wonderful mother, and terrific wife.

CONTENTS

FIGURES

TABLES

ACKNOWLEDGMENTS

I would like to thank my advisor and dissertation codirector, Scott E. Maxwell, for his mentorship over the last five years. He has helped teach me how to think like a methodologist and how important methodological issues are to scientific research. I would also like to thank Steven M. Boker and my dissertation codirector, Ke-Hai Yuan, for the many times they assisted me in this and other endeavors over the past five years. Thanks also to Cindy S. Bergeman and David A. Smith for their help in making this dissertation a more general and stronger work by providing unique perspectives on the topics addressed in this dissertation. Finally, I would like to thank Joseph R. Rausch for his thoughts on this dissertation, the numerous discussions that we have that we have routinely had over the last four years, and for his always poignant questions that ultimately lead to a better understanding of the topic at hand.

INTRODUCTION

An overarching theme permeating the whole of the scientific enterprise is the study of change. Change can be defined as the transition from one state of being to another state of being, with a special case of change being constancy, a condition where no change occurs. Although true for science in general, understanding the process of change in the behavioral sciences is especially difficult. The difficulties that arise when attempting to understand the process of change motivates abundant methodological work in the behavioral sciences. Even though so much time and effort has been dedicated to gaining a better understanding of the process of change, state-of-the-art methods presently used to analyze and understand change may not be adequate when unknown heterogeneity exists in a population where change is governed by some nonlinear functional form. That is to say, when the transition from one state to another state is complicated and thus cannot be adequately represented by a simple model, modern methods of analyzing change do not address the fact that different groups of individuals might be governed by a different set of parameter values when the grouping structure of those individuals is unknown.[1]

More than 100 years ago Davies acknowledged that understanding "change is by no means a simple affair" (1900, p. 506). One of the earliest solutions to the difficulties

[1]The term "growth" is oftentimes used synonymously with "change" in the behavioral sciences. For example, even if a trend declines over time, its model of change is often called a "growth curve" and can be defined by some mathematical "functional form of growth." Although the general term "change" generally seems more appropriate than growth in such situations, growth is the term most commonly used. A decaying trend can be conceptualized as negative growth and a trend exhibiting constancy (i.e., no change) can be thought of as no growth. It is therefore the case that decay and constancy are special cases of growth. Because change seems more appropriate than growth when discussing a transition, change will be the term generally employed throughout the work. The exception will be when a model of change has traditionally been referred to with growth in its name. Examples are the latent growth curve, the growth mixture model, the Gompertz growth curve, et cetera.

in analyzing change in the behavioral sciences was published in 1924 when Thomson gave a correction for the correlation between change and the initial value. Criticisms of the study of change in the behaviorial sciences picked up markedly in the 1950s. Lord, for example, warned researchers interested in the study of change that "the fact that test scores are not perfectly reliable often makes this 'obvious' procedure [of using the difference score to measure change] produce absurd results" (1956, p. 421). Because no competent researcher desires to produce "absurd results," Lord's work was a major impetus behind 30 years of subsequent methodological work warning of the supposed dangers and misconceptions when measuring change (see also a follow-up to his 1956 paper, Lord, 1958, as well as the motivating works criticizing the analysis of change contained in the volume edited by Harris, 1963). One of the most well-known criticisms of the measurement of change, particularly the use of difference scores, was leveled by Cronbach and Furby (1970). Cronbach and Furby questioned whether change should be measured at all, going so far as to suggest that researchers interested in the measurement of change "frame their questions in other ways" (1970, p. 80). However, framing research questions in "other ways" is not always possible when what is of interest is the process of change, as doing so may fundamentally alter the research question of interest. Mandating that alterations be made in the study of change would many times lead to important topics being overlooked.

Although a generation's worth of literature developed through the 1950s, 1960s, and 1970s questioning the validity of the study and analysis of change, the 1980s began a revolution of sorts in which the importance of the study of change was again acknowledged and new methods based on solid statistical and conceptual frameworks began to develop. Methodologists began to realize that fundamental problems existed in many of the arguments set forth by those who critiqued the study and analysis of change. Rather than focusing on interindividual differences in change at various measurement occasions, the new approach emphasized intraindividual differences in change over time (e.g., Bryk & Raudenbush, 1987; Collins, 1996; Mehta & West, 2000; Raudenbush, 2001a; Rogosa, Brandt, & Zimowski, 1982; Rogosa & Willett, 1985; Willett, 1988). Intraindividual change is now widely considered to be the appropriate starting point for the study and analysis of change. Along these same lines, a logical prerequisite before measuring or attempting to understand interindividual differences in change or group differences is the precise and valid measurement of intraindividual change (Collins, 1996).

Focusing on the individual is central to modern conceptualizations of change. However, examining only an individual without regard to the population to which the individual belongs would lead to a science of specifics rather than one of generality. Yet, overgeneralization of the effect the population has on an individual can also lead to inappropriate conclusions, as the population may contain distinct classes of individuals, where each class might somehow be different from the other classes. The population may not represent a single class of homogeneous individuals; heterogeneity may exist in the form of a finite number of known or unknown groups. Such a discussion is closely related to the longstanding debate in the behavioral sciences between idiographic and nomothetic conceptualization of behavior.

As has been historically understood in the behavioral sciences since Allport (1937; but see Lamiell, 1998, for theoretical clarification of Windelband's, 1921, originating work on the distinction between idiographic and nomothetic conceptualizations), an idiographic approach to understanding behavior involves focusing on the individual and acknowledges that individuals may well be qualitatively or quantitatively different from one another, whereas the nomothetic approach to understanding behavior attempts to establish universal laws that generalize over individuals (see also Dunn, 1994). Idiographic and nomothetic conceptualizations of behavior are traditionally considered antithetical (Dunn, 1994, p. 377). Because idiographic and nomothetic conceptualizations are often thought of as being mutually exclusive, potential benefits of combining the approaches in an integrated fashion often go unrecognized. What has not often been realized in practice, however, is that combining the idiographic and nomothetic conceptualizations of behavior into a single model of change may lead to a more realistic and powerful model of behavior than either approach alone.

In order to help integrate idiographic and nomothetic approaches to understanding behavior, it is helpful to distinguish between variable centered and pattern centered statistical methods. Muthén and Muthén (2000) discuss that, at least in general, statistical methods designed to help researchers answer questions about behavior are either variable centered or pattern-centered.[2] Statistical methods that are variable-centered focus on relationships among variables, generally in an effort to predict the dependent variable(s) from the independent variable(s). Examples of variable-centered statistical methods are regression analysis, standard applications of exploratory and confirmatory factor analysis, and structural equation modeling. Statistical methods that are pattern-centered focus on the relationships among individuals, such that individuals can potentially be classified into various classes, where within a class individuals are relatively similar and between classes individuals are relatively dissimilar. Examples of pattern-centered statistical methods are cluster analysis, finite mixture modeling, latent class analysis, and latent transition analysis. Pattern-centered approaches are more consistent with the "weak concept of development" (Burchinal & Appelbaum, 1991), which allows for interindividual differences in intraindividual change (see also Baltes & Nesselroade, 1979).

[2]Like many developmental psychologists, Muthén and Muthén (2000) use the term person-centered rather than the term pattern-centered. In an effort to be more general, like Schulenberg, O'Malley, Bachman, Wadsworth, and Johnston (1996a), the present work will use the term pattern-centered. When the "pattern" applies to a "person," the pattern-centered approach is equivalent to the person-centered approach. The pattern, however, may refer to an animal, a group, or some other entity of interest. Depending on the particular circumstances, the term "person-centered" may not always be appropriate.

When the nomothetic conceptualization of behavior holds, all individuals are regarded as being “of the same kind,” and thus what is of interest is the relationship among variables. The nomothetic conceptualization is consistent with the “strong concept of growth,” because this view states that “a single developmental function can adequately describe the change of all individuals from some population” (Burchinal & Appelbaum, 1991, p. 25). Because all individuals are supposed to follow the same trend, perhaps with variation around the “prototype” function (Wohlwill, 1973, p. 25), the relationship among variables is then appropriately examined using a variable-centered statistical technique. However, when an idiographic conceptualization of behavior holds, there is no “general law” that governs everyone, and thus what is of interest is centered on the individuals themselves. It may be the case that what is of interest is the classification of individuals who exhibit similar patterns of change. Such a conceptualization is explored by pattern-centered techniques crossed with variable-centered techniques. Muthén and Muthén (2000) argue that some research questions demand an integration of variable-centered and pattern-centered statistical techniques (see also Burchinal & Appelbaum, 1991; Magnusson & Bergmen, 1988; Muthén et al., 2002; Schulenberg et al., 1996a). Such a combination of statistical techniques is appropriate when behavior is thought to be a combination of idiographic and nomothetic conceptualizations.

When studying change, it may be reasonable to hypothesize that not all individuals change in the same fashion. In fact, it may be the case that there are underlying classes of individuals, where the different classes each have a unique set of parameters governing the process of change or even follow different functional forms of change altogether. If such classes exist, standard methods of longitudinal data analysis may not yield meaningful results. The reflexive application of nomothetic techniques to mixed nomothetic-idiographic situations may not accurately reflect any of the component distributions that are mixed together, which potentially results in misleading conclusions.

Presumably the questions of interest regarding change would best be applied within each of the classes (i.e., conditional on class membership) and not across all of the classes simultaneously. Rather than only the parameters governing the process of change differing for each of the classes, it could be the case that the functional form of change is fundamentally different for some or all of the classes. In such a scenario, fitting a single model of change to all of the individuals would yield parameter estimates from a misspecified model, as the correct model would allow for multiple functional forms of change.

In a series of works Muthén has proposed a general latent variable technique that encompasses the growth mixture model (GMM; Muthén, 2001a, 2001b, 2002; Muthén & Shedden, 1999). Within the GMM context is that different individuals are nested within unknown and thus latent classes, and within these latent classes there is variability around the class specific population coefficients of change. Note that the latent class growth model, a discussion of which begins on page 64, is similar to the GMM, with the major difference being that no variability is assumed around each of the class specific fixed effects (Land & Nagin, 1996; Nagin, 1999). The model is analogous to a multiple group analysis, but group membership is itself unknown and must be estimated. Although Muthén's GMM is capable of combining variable-centered and pattern-centered approaches to studying change (Muthén & Muthén, 2000), the GMM is limited in several respects. The limitations of the GMM are delineated in a later section beginning on page 61.

In addition to beginning to integrate nomothetic and idiographic approaches to understanding behavior, recent work reflecting increased appreciation of models of change nonlinear in their parameters is beginning to gain ground in the behavioral sciences. Methodologists have been giving models nonlinear in the parameters increased attention because of the benefits they provide. Nonlinear models of change are models where parameters enter the mathematical function defining the trajectories in a nonlinear fashion.[3] As has been delineated elsewhere (e.g. Browne & du Toit, 1991; Cudeck, 1996; Davidian & Giltinan, 1995; Pinheiro & Bates, 2000), the benefits of nonlinear models in scientific research can be tremendous. Nonlinear models have many advantages over their linear counterparts. For example, nonlinear models need not exhibit unlimited growth or decay, oftentimes nonlinear models of change require fewer parameters than do linear models for change that is asymptotic and/or sigmoidal ("S-shaped"), and the parameters of nonlinear models oftentimes have "real-world" interpretations. Detailed benefits of selected nonlinear models will be given in a future section beginning on page 34.

The present work attempts to combine the standard nomothetic approach to the analysis of change with the idiographic approach, specifically in the context of nonlinear models of change. In doing so, variable-centered and pattern-centered techniques can be combined more effectively into a unified treatment, where the focus is not on either a completely homogeneous or a completely heterogeneous population, but rather on some combination of the two.[4] If one were to accept the premise that some behavior or pattern of behavior is best explained by relying on both idiographic and nomothetic conceptualizations, the method of data analysis should be flexible enough to allow such a rich theory to be incorporated into the statistical model. Specifically, the goal of the

[3]The difference between linear and nonlinear models of change is how the parameters enter the model. If the parameters enter linearly (i.e., they are only raised to the zeroth or first power), the model is linear. If the parameters enter nonlinearly (i.e., enter some way besides being raised to the zeroth or first power), the model is nonlinear. A linear model of change may exhibit something other than a straight-line relationship over time. This occurs when time itself (or some function of time) is manipulated (e.g., squaring, cubing, taking the logarithm of, etc.) and is included as a predictor in the model with some nonzero regression coefficient. Thus, a linear model need not illustrate a straight-line relationship between time and the dependent variable. The technical definition of a linear model is one where none of the partial derivatives of the function with respect to each of the parameters depends on any of the other parameters (Bates & Watts, 1988, chapter 2).

[4]It can be argued that standard models of longitudinal data analysis, specifically the multilevel model (discussion of which begins on page 9) and the latent growth curve (discussion of which begins on page 16), represent the interaction of nomothetic and idiographic methods. This is true to some extent, because individuals are generally allowed to have unique effects around some set of population parameters. However, forcing all individuals to follow the same functional form with the same set of parameters of change implies that the individuals are "of the same kind" and may be too nomothetic for some situations.

present work is to develop and examine the effectiveness of the latent classification differential change (LCDC) model, which is a general model of heterogeneous change introduced in a future section beginning on page 77.

The LCDC model is conceptually similar to Muthén's GMM as well as Pauler and Laird's fully Bayesian mixture model for hierarchical nonlinear models, however the model flexibility and estimation procedures that define the LCDC model for classification and parameter estimation are fundamentally different from the former models. There is greater flexibility in the functional forms of change that can be modeled with the LCDC model, because unlike Muthén's GMM the LCDC model is not restricted to functional forms linear in their parameters. Unlike the Pauler and Laird model, there are no rigorous prior distributions that need to be specified, the Bayesian perspective need not be accepted, and it is not necessary to use Markov chain Monte Carlo methods for parameter estimation. When a process changes in a nonlinear fashion over time, and unknown heterogeneity exists, the LCDC model provides a conceptually appealing method of parameter estimation that is more flexible than current methods for understanding known heterogeneity in the context of longitudinal data analysis.

Because the analysis of change is such an important component of behavioral science research, it is important for novel methodological contributions to be formally developed in a statistical context, compared and contrasted with existing techniques, and shown to be effective by analytic proofs and/or via Monte Carlo simulation methods. The goal of this work is to formally develop and then evaluate the LCDC model in such a manner. In order to place the LCDC model into the larger analysis of change framework, the present work begins with a discussion of widely used statistical methods for both the analysis of change and mixture modeling before formally introducing the LCDC model and its properties.

STANDARD APPROACHES FOR THE ANALYSIS OF CHANGE

How to study change has been hotly debated in the behavioral sciences. Many of the traditional criticisms behavioral science methodologists cited to avoid studying change were remedied with more modern conceptualizations of change. In particular, behavioral science methodologists and applied researchers argue that multilevel models (MLM) and special applications of structural equation models (SEM) that have been termed latent growth curve (LGC) analysis are effective ways to analyzing change. Modeling change with the MLM or with the LGC provides estimates of fixed effects parameters, as well as information about both interindividual variability and covariation around those fixed effects. Fixed effects are the theoretical means of the individuals' coefficients of change. The variability around each of the fixed effects relates to interindividual differences. It is generally of interest to infer if the population value of fixed effects and covariance parameters differ from zero. The individual variability around the fixed effects addresses the extent to which intraindividual variability is or is not present for the particular parameter. The next two sections outline how the MLM and the LGC address the issue of analyzing change, with a follow-up section detailing the similarities and differences between the two models.

The Multilevel Model Approach to the Analysis of Change

The most general MLMs are those that allow parameters to enter the model in a nonlinear fashion. A special case of a nonlinear model is a linear model. General MLMs allowing nonlinearities have been proposed in several works. Examples of important review works that discuss the topic of nonlinear MLMs in book length treatments are Davidian and Giltinan (1995), Pinheiro and Bates (2000), and Vonesh and Chinchilli (1997). Apparently some of the first developments of nonlinear MLMs came from the pharmacokinetic work of Sheiner and Beal (1980; see Pinheiro & Bates, 2000, p. 310). Like Pinheiro and Bates (2000), the nonlinear MLM used throughout the present work is a generalization of the formulation proposed by Lindstrom and Bates (1990).

The general (univariate) nonlinear MLM as developed by Lindstrom and Bates (1990) is given as follows:

$$y_{ijt} = f(\boldsymbol{\psi}_{ij}, \mathbf{x}_{ij})_t + \varepsilon_{ijt}, \tag{1}$$

where y_{ijt} is the response from the ith individual ($i = 1, \ldots, N$) in the jth group ($j = 1, \ldots, J$), at the tth timepoint ($t = 1, \ldots, T$), $f(\cdot)$ is some functional form of change relating the r parameters in the vector $\boldsymbol{\psi}_{ij}$ and the predictors in the vector $\mathbf{x}_{ij}$, and ε_{ijt} is the error term of the ith individual in the j group at the tth measurement occasion.[5] The parameter vector is potentially specific to the individual and such uniqueness is incorporated into the model by defining $\boldsymbol{\psi}_{ij}$ as

$$\boldsymbol{\psi}_{ij} = \mathbf{A}_{ij}\boldsymbol{\beta} + \mathbf{Z}_{ij}\boldsymbol{\upsilon}_{ij}, \tag{2}$$

where $\boldsymbol{\beta}$ is a vector of length P ($p = 1, \ldots, P$) of population fixed effect parameters, $\boldsymbol{\upsilon}_{ij}$ is a vector of length P with q nonzero elements representing unique effects associated with the ith individual in the jth group, $\mathbf{A}_{ij}$ is an r by P design matrix associated with the fixed effects, and $\mathbf{Z}_{ij}$ is an r by P design matrix with q nonzero columns associated with the unique effects.

[5]The $\mathbf{x}_{ij}$ variables have been defined generically as “predictor” variables. These predictor variables can be thought of as independent variables manipulated by the researcher (e.g., group assignment, amount or duration of treatment, etc.) or concomitant variables thought to be related to the outcome variable (e.g., socioeconomic status, some measure of intelligence, gender, etc.).

Let $\mathbf{y}_{ij} = [y_{ij1}, \ldots, y_{ijT}]'$, $\mathbf{x}_{ij} = [\mathbf{x}_{ij1}, \ldots, \mathbf{x}_{ijN}]'$, and $\boldsymbol{\varepsilon}_{ij} = [\varepsilon_{ij1}, \ldots, \varepsilon_{ijT}]'$. Equation 1 can then be written as

$$\mathbf{y}_{ij} = f(\boldsymbol{\psi}_{ij}, \mathbf{x}_{ij}) + \boldsymbol{\varepsilon}_{ij}. \tag{3}$$

Combining Equations 2 and 3, the full MLM can be written as

$$\mathbf{y}_{ij} = f(\mathbf{A}_{ij}\boldsymbol{\beta} + \mathbf{Z}_{ij}\boldsymbol{\upsilon}_{ij}, \mathbf{x}_{ij}) + \boldsymbol{\varepsilon}_{ij}. \tag{4}$$

Application of the MLM generally places no restriction on $\mathbf{x}_{ij}$, $\mathbf{A}_{ij}$, or $\mathbf{Z}_{ij}$. Thus, unique measurement occasions (i.e., unbalanced data) pose no special problems, nor does data missing at random or completely at random (provided the appropriate assumptions are met; see Little & Rubin, 1987, Schafer, 1997, and Schafer & Graham, 2002, for a discussion of data missing at random versus completely at random, along with Schafer, 2001, and Twisk and de Vente, 2002, for missing data issues in the context of longitudinal data analysis). Although not strictly necessary, applications of the MLM often assume that ε_{ijt} are independent and normally distributed with a mean of zero and common variance σ^2_ε. Thus, a commonly assumed condition is that $\mathrm{Cov}(\varepsilon_{ijt}, \varepsilon'_{ijt})_{\varepsilon_{ijt} \neq \varepsilon'_{ijt}} = 0$ for all values of j and t, where $\mathrm{Cov}(\cdot, \cdot)$ represents the covariance of the quantities in parentheses. Furthermore, it is generally assumed that conditional on the fixed effects and the predictors, the unique effects are multivariate normally distributed with mean zero and finite covariance matrix. The reader is referred to Davidian and Giltinan (1995), Vonesh and Chinchilli (1997), and Pinheiro and Bates (2000) for detailed treatments of the nonlinear MLM, including methods of parameter estimation, assumptions of the model, and inferential techniques applied to the fixed effects and variance components in order to make decisions based on probabilistic evidence.

Although writing the general MLM in the form of Equation 4 is mathematically elegant, much of the intuitive appeal of the MLM is lost because the hierarchical structure of the data is not explicitly represented. Suppose that the function $f(\cdot)$ from Equation 4 is the identity function (which means the equation is left unaltered and thus the arbitrary function $f(\cdot)$ is one linear in its parameters). Equation 4 could then be expressed as

$$\mathbf{y}_{ij} = \mathrm{I}(\mathbf{A}_{ij}\boldsymbol{\beta} + \mathbf{Z}_{ij}\boldsymbol{\upsilon}_{ij}, \mathbf{x}_{ij}) + \boldsymbol{\varepsilon}_{ij}, \tag{5}$$

which is the general linear MLM, where $\mathbf{A}_{ij}$ and $\mathbf{Z}_{ij}$ are the design matrices for the fixed effects (βs) and unique effects (υs), respectively. In fact, Equation 5 is equivalent to the model defined in the classic work of Laird and Ware (1982).[6]

The components of Equation 5 can be further reduced, such that they can be expressed in the classic hierarchical form of the MLM. Such a hierarchy, where occasions of measurement are nested within individuals, who in turn are potentially nested within higher-order organizational structures, can be written explicitly by application of Equation 5:

$$y_{ijt} = \pi_{0ij} + \pi_{1ij}a_{ijt} + \pi_{2ij}a_{ijt}^2 + \ldots + \pi_{kij}a_{ijt}^K + \varepsilon_{it}, \tag{6}$$

where π_{kij} is the kth polynomial coefficient of change ($k = 0, \ldots, K$) and a_{ijt} is some basis of time (e.g., time itself, age, measurement occasion, etc.) for the ith individual in the jth group at the tth measurement occasion. Equation 6 is known as the Level 1 model, and this particular model is linear in its parameters. Notice that the coefficients of change in the Level 1 model of Equation 6 have i and j subscripts. Each of the coefficients of change in Equation 6 are thus specific to the individual and thus allow individual differences to be explicitly taken into consideration in the statistical model.

The hierarchical nature of the MLM is evidenced when the coefficients of change in Equation 6 are themselves modeled at a second level. Each coefficient of change from the

[6] Rather than explicitly including the $\mathbf{x}$ variables into the design matrices as the Laird and Ware (1982) specification, the Lindstrom and Bates (1990) specification uses identifiers that are conditional on the $\mathbf{x}$ variables. Although the two models are parameterized in a slightly different manner, they are in fact equivalent. The Laird and Ware (1982) work was previously called a “classic.” Justification for this is apparent given the number of citations the article has received. According to the Web of Science, as of April 2005, the Laird and Ware (1982) article has been cited 1,867 times.

Level 1 model is a dependent variable in the Level 2 equation. As dependent variables, the π_{kij}s from Equation 6 can be explained by an overall fixed effect, one or more predictors, and potentially a unique effect associated with the individual or organizational structure. Such a conceptualization can be written as

$$\pi_{kij} = \beta_{k0} + \beta_{k1}x_{k1ij} + \beta_{k2}x_{k2ij} + \ldots + \beta_{kMij}x_{kMij} + \upsilon_{kij}, \tag{7}$$

where β_{km} represents the effect of the mth predictor variable ($m = 1, \ldots, M$) x on the kth individual specific change coefficient, x_{km}. The M x variables are predictor variables thought to affect the value of the particular π_{ki} and/or other parameters in the model. Although only a two level model has been illustrated for the general linear MLM, the Level 2 predictors can themselves be the outcome variables at Level 3, where the Level 3 predictors can be the outcome variables at Level 4, et cetera. Provided appropriate data exist, there is theoretically no limit to the number of levels of the MLM.[7]

In the context of longitudinal data analysis, one of the most common models explored in the behavioral sciences is the straight-line change model. In fact, Mehta and West (2000) state that "linear growth is virtually the only [functional] form of growth that is investigated by substantive researchers" in the behavioral sciences (p. 40). A special case of Equations 6 and 7 is the conditional (where the Level 1 parameters are conditional on the Level 2 predictors) straight-line change model, which can be written as

$$y_{it} = \pi_{0i} + \pi_{1i}a_{it} + \varepsilon_{ijt}, \tag{8}$$

where

$$\pi_{0i} = \beta_{00} + \beta_{01}x_i + \upsilon_{0i} \tag{9}$$

[7]Note that when predictor variables are used in the Level 2 (or higher levels) equation, the model is said to be conditional. The model is conditional because the parameters in the Level 1 model are regressed on the particular variable(s) in the Level 2 model. When no x variables are included in the Level 2 (or higher) model, the model is said to be unconditional. An unconditional model would thus consist of fixed effects and unique effects (either of which can be set to zero if desired), neither of which depend on any predictor variables.

and

$$\pi_{1i} = \beta_{10} + \beta_{11}x_i + \upsilon_{1i}. \tag{10}$$

When combining the Level 2 model (Equations 9 and 10) with the Level 1 model (Equation 8), the full model can be written as follows:

$$y_{it} = \beta_0 + \beta_{01}x_i + u_{0i} + a_{it}(\beta_{10} + \beta_{11}x_i + u_{1i}) + \varepsilon_{it}. \tag{11}$$

The statistical model of change governed by Equation 11 is a special case of Equation 5, which itself is a special case of Equation 1. This popular model of change is desirable, presumably in large part because of its parsimony. However, many relationships in the behavioral sciences cannot be represented adequately by such a model. As Cudeck states, “in the study of human behavior . . . many responses are inherently nonlinear and cannot be treated by a linear mixed [i.e., multilevel] model” (1996, p. 372). What is oftentimes more appropriate is a model where the parameters enter the model in a nonlinear fashion. Such nonlinear models can overcome the problems of unlimited growth/decay, are potentially valid beyond the range of observed data, often provide real-world interpretations of the parameters of change, and they are often more parsimonious than linear models with the multiple powers of a_t necessary to adequately model complex change (van Geert, 1991; Browne & du Toit, 1991; Pinheiro & Bates, 2000).

A specific nonlinear change model is generally more straightforward than is an application of Equation 1 itself. The strength of Equation 1 is in its generality. However, because it is so general, applications of Equation 1 may seem overwhelming. It should be realized, however, that a specific nonlinear model can be written in multiple levels as was done in the case of the straight-line change model. Thus, for a particular nonlinear model, the fixed effects can be written at Level 1, with the unique effects modeled as the outcome variable in the Level 2 equation(s), which in turn can themselves be modeled in the Level 3 equation(s), et cetera.

As an example, the asymptotic regression growth curve (which is formally defined and delineated in a section beginning on page 36) is defined as

$$y_{it} = \alpha_i + \zeta_i \exp(-\gamma_i a_{it}) + \varepsilon_{it}, \tag{12}$$

where α_i, ζ_i, and γ_i are the parameters of the Level 1 model for individual i. These coefficients of change can themselves be modeled as outcome variables in Level 2 of the model:

$$\alpha_i = \beta_{01} + \beta_{11} x_i + \upsilon_{1i}, \tag{13}$$

$$\zeta_i = \beta_{02} + \beta_{12} x_i + \upsilon_{2i}, \tag{14}$$

and

$$\gamma_i = \beta_{03} + \beta_{13} x_i + \upsilon_{3i}, \tag{15}$$

where x_i in this case is some time-invariant predictor (e.g., experimental condition, group membership, gender, etc.), β_{kp} is the kth fixed effect parameter for the pth individual change coefficient, and υ_{ki} is the unique effect for the kth fixed effect parameter for individual i. The full model is then given as

$$y_{it} = (\beta_{01} + \beta_{11} x_i + \upsilon_{1i}) + (\beta_{02} + \beta_{12} x_i + \upsilon_{2i}) \exp(-(\beta_{03} + \beta_{13} x_i + \upsilon_{3i}) a_{it}) + \varepsilon_{it}. \tag{16}$$

Equation 16 is the full asymptotic regression growth curve and is the analog to the full straight-line change model of Equation 11. As can be seen from Equation 16, an individual's trajectory is a function of fixed components constant across all individuals and unique components specific to the particular individual.

Although the MLM, which is an intuitively appealing method of modeling change over time, is consistent to some degree with the idea of having both nomothetic (fixed effects) and idiographic (unique effects) components, and is based on sound statistical theory, another method of modeling longitudinal data was developed largely independent of the MLM. In particular, the LGC model of change is closely related to the MLM. The next section outlines the LGC with the section after that discussing the similarities and differences with the MLM.

The Latent Growth Curve Approach to the Analysis of Change

Structural equation modeling is a general technique that quantifies the relationship between measured and latent variables, generally using only the first and second moments of the observed data. SEM developed as a generalization of factor analysis, where SEM generalized factor analysis by allowing direct relationships between latent variables to be an explicit part of the model. Early models of SEM can be found in Jöreskog (1970), which formed the basis of what was to become the linear structural relations (LISREL) model and later the computer program of the same name (Jöreskog, 1973), Bentler and Weeks (1980), known as the Bentler-Weeks model which generalized the LISREL model by allowing the measurement model to itself be represented as a higher ordered factor, and McArdle and McDonald (1984), who showed that only three model matrices were required to represent a broad class of linear structural models. The present work makes use of the modern LISREL model and notation (Jöreskog & Sörbom, 1996).

The full LISREL model consists of two submodels. Specifically, a measurement model and a structural model are combined to yield the full LISREL model. The measurement model specifies the way in which the manifest variables are functions of the latent variables. The structural model specifies direct and indirect relationships among the latent variables. The full model then consists of observed and latent variables, where the model implied covariance matrix can be compared to the observed covariance matrix, so that the adequacy of the model can be tested and the verisimilitude of theories evaluated.

The measurement model for the observed endogenous variables for the jth group is given as

$$\mathbf{y}_j = \boldsymbol{\tau}_{\mathrm{y}_j} + \boldsymbol{\Lambda}_{\mathrm{y}_j}\boldsymbol{\eta}_j + \boldsymbol{\varepsilon}_j, \tag{17}$$

where $\mathbf{y}_j$ is the T length vector of observed outcome variables for the jth group, $\boldsymbol{\tau}_{\mathrm{y}_j}$ is a vector of endogenous measurement intercepts for the jth group, $\boldsymbol{\Lambda}_{\mathrm{y}_j}$ is a T by L matrix of factor loadings linking the L latent variables in $\boldsymbol{\eta}_j$ for group j to $\mathbf{y}_j$, and $\boldsymbol{\varepsilon}_j$ is a T length vector of measurement errors in $\mathbf{y}_j$ for the n_j individuals in group j. The T by T covariance matrix for $\boldsymbol{\varepsilon}_j$ is designated $\boldsymbol{\Theta}_{\varepsilon_j}$.

The measurement model for the observed exogenous variables is given as

$$\mathbf{x}_j = \boldsymbol{\tau}_{\mathrm{x}_j} + \boldsymbol{\Lambda}_{\mathrm{x}_j}\boldsymbol{\xi}_j + \boldsymbol{\delta}_j, \tag{18}$$

where $\mathbf{x}_j$ is an M length vector of observed predictor variables for the jth group, $\boldsymbol{\tau}_{\mathrm{x}_j}$ is a vector of exogenous measurement intercepts for the jth group, $\boldsymbol{\Lambda}_{\mathrm{x}_j}$ is an M by n_j matrix of factor loadings linking $\boldsymbol{\xi}_j$ to $\mathbf{x}_j$, and $\boldsymbol{\delta}_j$ is a vector of measurement errors in $\mathbf{x}_j$. The M by M covariance matrix for $\boldsymbol{\delta}_j$ is designated $\boldsymbol{\Theta}_{\delta_j}$ and the n_j by n_j covariance matrix for $\boldsymbol{\xi}_j$ is designated $\boldsymbol{\Phi}_j$.

The structural part of the LISREL model is defined as

$$\boldsymbol{\eta}_j = \boldsymbol{\alpha}_j + \mathbf{B}_j\boldsymbol{\eta}_j + \boldsymbol{\Gamma}_j\boldsymbol{\xi}_j + \boldsymbol{\zeta}_j, \tag{19}$$

where $\boldsymbol{\alpha}_j$ is an L length vector of intercepts for the jth group, $\mathbf{B}_j$ is an L by L matrix linking the endogenous latent variables to one another for the jth group, $\boldsymbol{\Gamma}_j$ is the matrix of coefficients linking the exogenous latent variables to the endogenous latent variables for the jth group, $\boldsymbol{\xi}_j$ is an n_j length vector of latent exogenous variables for the jth group, and $\boldsymbol{\zeta}_j$ is the vector of length L of the unique effects associated with the jth group.

The expected value of the vector of endogenous manifest variables is obtained by taking the expectation of Equation 17:

$$\mathrm{E}\left[\mu_{y_j}\right] = \boldsymbol{\tau}_{y_j} + \boldsymbol{\Lambda}_{y_j}(\mathbf{I} - \mathbf{B}_j)^{-1}(\boldsymbol{\alpha}_j + \boldsymbol{\Gamma}_j\boldsymbol{\kappa}_j), \quad (20)$$

where $\boldsymbol{\kappa}_j = \mathrm{E}[\boldsymbol{\xi}_j]$. The expected value of the vector of endogenous measured variables is obtained by taking the expectation of Equation 18:

$$\mathrm{E}\left[\mu_x\right] = \boldsymbol{\tau}_{x_j} + \boldsymbol{\Lambda}_{x_j}\boldsymbol{\kappa}_j. \quad (21)$$

The expectation of the L latent variables in $\boldsymbol{\eta}_j$ is obtained by taking the expectation of Equation 19:

$$\mathrm{E}[\boldsymbol{\eta}_j] = (\mathbf{I} - \mathbf{B}_j)^{-1}(\boldsymbol{\alpha}_j + \boldsymbol{\Gamma}_j\boldsymbol{\kappa}_j). \quad (22)$$

The general LISREL model was presented here in order to provide a framework for LGC analysis. LGC analysis was originally proposed in order to "*study individual change as a function of time*" (McArdle & Epstein, 1987, p. 110) and explicitly acknowledges that "individual time paths are the proper focus for the analysis of change" (Rogosa et al., 1982, p. 722).[8] The LGC model has idiosyncracies that differentiate it from general applications of SEM. In particular, whereas $\boldsymbol{\Lambda}_{y_j}$ is generally estimated in standard applications of SEM, $\boldsymbol{\Lambda}_{y_j}$ is fixed in LGC analysis so that a structure is formally imposed upon the path coefficients. Specifically, the values of $\boldsymbol{\Lambda}_{y_j}$ are chosen such that the latent variable(s) of $\boldsymbol{\eta}_j$ are linked to $\mathbf{y}_j$ as a function of the measurement occasions and the particular model of change linear in its parameters (e.g., linear, quadratic, cubic, etc.) specified.

[8]It is widely cited in the analysis of change literature that a talk given by Meredith and Tisak at the 1984 meeting of the Psychometric Society provided the impetus for what became known as the LGC model. Apparently, Meredith and Tisak (1990) is the published version of the widely cited 1984 talk (William Meredith, personal communication, January 28, 2004).

Because the LISREL model for LGC analysis is so general, an example of a path diagram for a model of change is provided. The example combines the reticular action model (RAM) path diagram representation (McArdle & McDonald, 1984; McArdle & Boker, 1990) with LISREL notation (Jöreskog & Sörbom, 1996). In the RAM representation, latent variables are represented by circles, whereas manifest variables are represented by squares. Single-headed arrows represent a direct relationship between two variables in the direction the arrow points. Double-headed arrows represent the covariation between two variables. In the special case where a double-headed arrow "leaves" and "reenters" the same variable, the value represents the variance, a special case of the covariance when the variable covaries with itself.

Specifically, Figure 1 illustrates a path diagram for a straight-line change model with five measurement occasions in a fully balanced designed, as well as a time invariant predictor for a single population. The intercept of the straight-line change model is represented by μ_{η_1} whereas the slope is represented by μ_{η_2}. Notice that the value of the paths linking η_1 to the y_t variables are all unity and that the values of the paths linking η_2 to the y_t variables is fixed at the value of time when the particular y_t variable was measured. The reason the path coefficients are specified as unity for η_1 and a_t for η_2, is because y_t is a function of both an intercept and a slope.

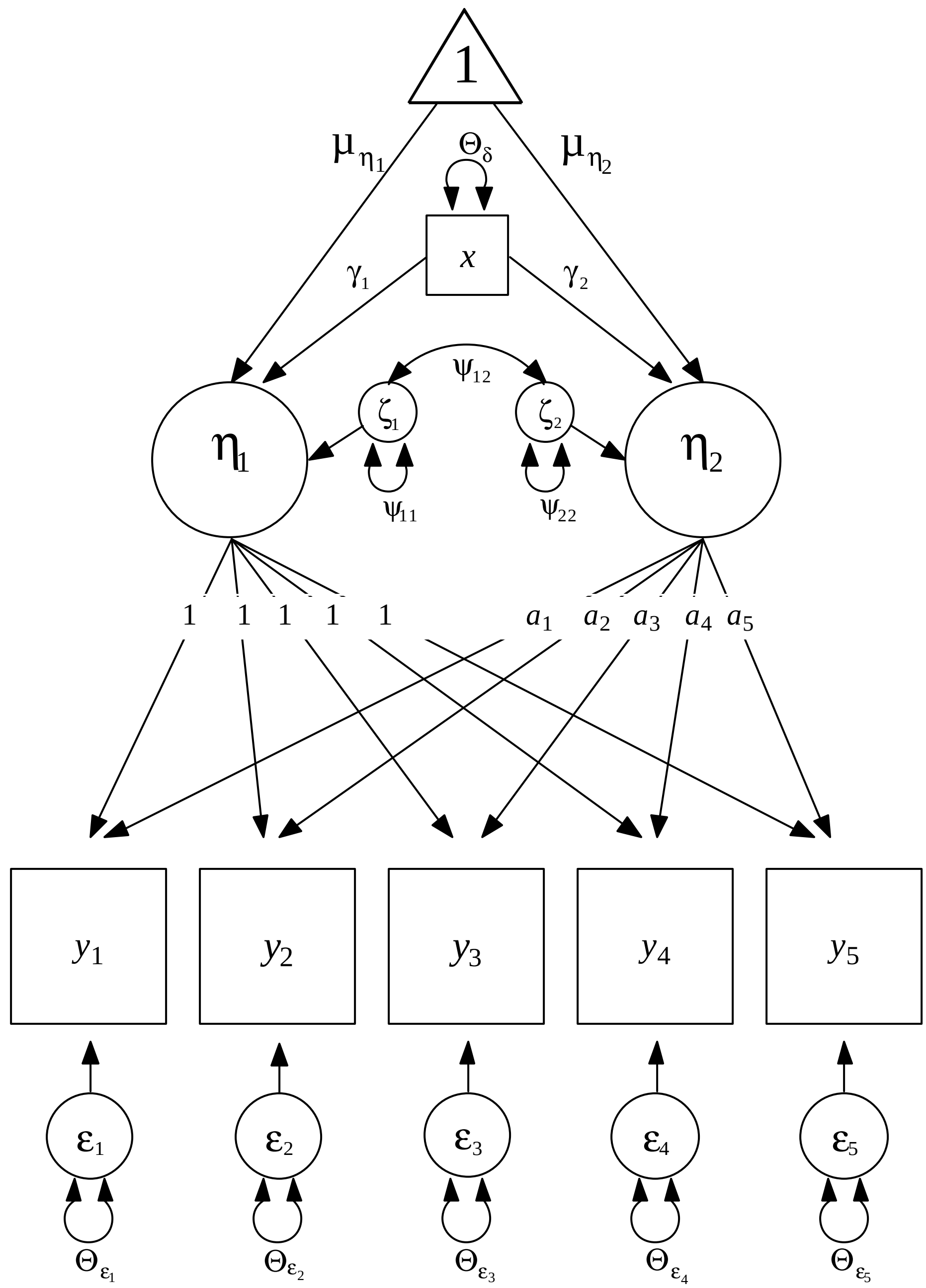

Figure 1: Path diagram illustrating a straight-line change model for five occasions of measurement with a time invariant predictor variable in the latent growth curve framework.

It can be seen by writing out the LISREL measurement model for **y** that each y_t is equal to $\eta_1 + a_t\eta_2 + \varepsilon_t$. More generally, applying Equation 20 to the straight-line change model of Figure 1:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 1 & a_1 \\ 1 & a_2 \\ 1 & a_3 \\ 1 & a_4 \\ 1 & a_5 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \end{bmatrix},$$

where $\boldsymbol{\tau}_{y_j}$ is generally a null vector in the context of LGC analysis and is not represented here. Thus, the reason that the path coefficients in $\boldsymbol{\Lambda}_\text{y}$ are fixed to 1s and a_ts is because y_t is modeled as being a function of an intercept (η_1) and a_t multiplied by a slope (η_2).

Because a time invariant predictor is included in the hypothetical model of Figure 1, the intercept and slope are a function of a fixed effect (i.e., μ_{η_1} for the intercept and μ_{η_2} for the slope), a unique effect (i.e., ζ_1 for the intercept and ζ_2 for the slope), and the effect of the time invariant predictor variable (i.e., $\gamma_1 x$ for the intercept and $\gamma_2 x$ for the slope). More generally, by applying Equation 21 and Equation 22, it can be seen that the intercept and slope are equal to the following:

$$\begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} \mu_{\eta_1} \\ \mu_{\eta_2} \end{bmatrix} + \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} x + \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix},$$

where $\boldsymbol{\alpha}$ contains the conditional (on x) factor means (i.e., fixed effects), **B** for the particular model is null like $\boldsymbol{\tau}_y$ and $\boldsymbol{\tau}_x$, where they generally are in the context of LGC analysis.

Had the LGC of Figure 1 been a quadratic model of change, thus having an intercept, linear, and quadratic change parameter, another latent variable representing the quadratic effect would have been included in the model. Specifically η_3 would represent a quadratic effect and the path coefficients linking η_3 to the y_ts would have been a_t^2 (in addition to the 1s and a_ts of the straight-line change model). Generalization to other powers of time, necessarily implying a model linear in its parameters, is straightforward in the LGC context.

Although SEM is a very general technique encompassing many “standard” statistical models, in large part the theory and application of SEM is limited to models linear in their parameters. Only in special applications can the SEM model be extended to nonlinear models. An example of such an extension of factor analysis to a nonlinear model is given by the normal-ogive item response function (e.g., Christoffersson, 1975; see also McDonald, 1999, chapter 12), where the probability of correctly responding to some test item is a latent dependent variable. Yalcin and Amemiya (2001) propose a general nonlinear factor model, yet it uses a quadratic approximation to the particular nonlinear model of interest. Yalcin and Amemiya state that it is “necessary to use a certain approximation to the nonlinear function” and that the linear approximation is inappropriate because of nonlinearity bias that would be present in the parameter estimates (2001, p. 284). They extended the linear approximation to a quadratic approximation so as to better approximate the particular nonlinear function. At the present time the model of Yalcin and Amemiya does not seem to be available in any SEM computer software package, thus making the model impractical for applied researchers. Another nonlinear SEM model (more general than the model proposed by Yalcin & Amemiya, 2001) was proposed by Lee and Zhu (2002), where the nonlinearity is in terms of polynomial and moderator effects. Although promising, even the model of Lee and Zhu does not allow the estimation of models nonlinear in their parameters (benefits, characteristics, and examples of models nonlinear in their parameters are described momentarily beginning on page 34). Even though nonlinear factor models exist in some special cases, and certain nonlinear constraints can be imposed on certain parameters (du Toit & Cudeck, 2001), no general nonlinear system for measurement models or structural models exists at the present time. Thus, SEM—and by implication confirmatory factor analysis and LGCs—is based almost exclusively on a set of linear model equations.

At another level, McArdle and Hamagami (2001; see also McArdle & Nesselroade, 1994) developed a linear model that has properties similar to that of certain nonlinear models. Specifically this linear model, termed latent difference score analysis, allows asymptotic values to be modeled much like an asymptotic regression (negative exponential) growth model (see a discussion of the asymptotic regression model beginning on page 36). Latent difference score analysis is qualitatively different than most LGCs because the dependent variable is the difference score ($D_{t+1,t} = y_{t+1} - y_t$) rather than the dependent variable y_t itself. In general, however, SEMs and factor analytic models are based on linear systems of equations and thus do not readily extend to the general nonlinear case.

The Relationship Between Multilevel Models and Latent Growth Curves

At this point the MLM approach and the LGC approach to the analysis of change have been described. An obvious question that arises is how these two models of change differ. Although both methods attempt to describe change, they do so in different ways. The MLM approach to the analysis of change was presented by way of a very general equation. So too was the LGC. However, the general MLM presented in Equation 1 describes both linear and nonlinear functional forms of change, whereas the general LGC is a special case of the SEM model presented in Equations 17, 18, and 19, and is necessarily linear in its parameters. Thus, in terms of functional forms capable of being modeled by the two methods, the MLM approach is much more general.

Comparisons of the MLM and the LGC have been ongoing for at least 11 years. The first work that outlined the similarities and differences between the MLM and the LGC was Willett and Sayer (1994). In their article, Willett and Sayer showed that in certain circumstances the two methods provided exactly the same answers. Specifically it was shown for an unconditional straight-line change model, where all individuals were measured at the same occasions of measurement (i.e., a vector of time values is common to all individuals), with errors distributed independently, and with equal variance, the two models yielded identical results.

Because in the LGC context the values of a_t are specified in the $\mathbf{\Lambda}_\mathrm{y}$ matrix, it might appear that individuals are required to have the same measurement occasions. In many instances in longitudinal research, equivalent measurement occasions are not likely to occur across participants. Raudenbush argues that the MLM "allows lots of flexibility in data structure for a limited class of models, whereas the SEM [i.e., LGC] approach . . . offers a broad class of models for a limited set of data structures" (2001b, p. 40). The generality of the LGC model that Raudenbush refers to is in terms of the error structure, not functional form of change. The reason that the LGC is said to be for limited data structures is because of the $\mathbf{\Lambda}_\mathrm{y}$ matrix and the lack of an *i* subscript on it. However, as McArdle and Hamagami (1991) showed, differences in measurement occasions can be represented as missing data. That is, even though individuals may have different measurement occasions, the individual measurement occasions could still be identified in the $\mathbf{\Lambda}_\mathrm{y}$ matrix.

McArdle and Hamagami (1991) discuss their method of treating different occasions of measurement in terms of multiple groups, where different "groups" of individuals have the same pattern of missingness. In such an approach data are treated as though they came from a balanced design with missing data. Conceptualizing missing data in such a manner allows one to define a multiple group LGC, where individuals within each group have the same pattern of missingness, and then to specify invariance across the groups. Such a method uses data from all of the "groups" to estimate the parameters of the particular model of change.

McArdle and Hamagami's (1991) method was extended by Mehta and West (2000) to an "individual data vector approach" (p. 32), where each individual could potentially have their own unique set of measurement occasions. Conceptually this is equivalent to adding an i subscript onto $\mathbf{\Lambda}_\mathrm{y}$, so as to explicitly acknowledge that individuals may have been measured at different times. When comparing the differences between MLMs and LGCs in terms of the flexibility of the data structure, Bauer (2003) and Curran (2003) both summarize the issue of unbalanced data as being a data management problem. Thus, in large part, the criticism of LGC analysis not being able to handle unbalanced or individual specific data have been largely overcome (But see Raudenbush, 2001b, for his thoughts on why differences still persist between the approaches.).

Given that general data structures can be adequately handled by both the LGC and the MLM approaches, what is really of interest is the relationship between parameter estimates. Bauer (2003) and Curran (2003) provide detailed descriptions of the relationship between parameter estimates from the MLM and the LGC (see also Willett and Sayer, 1994). Because the general SEM model is a model linear in its parameters, the comparison given below is necessarily restricted to linear MLMs.

The LGC model specified by the path diagram of Figure 1, where a straight-line change model conditional on x is fit to five occasions of measurement, and the MLM, specifically Equations 8 through 10, will be used to illustrate the isomorphism of the estimates from the MLM and the LGC approaches. The equations for the MLM and the LGC are given below, with values from the MLM in the top row directly above the corresponding values from the LGC in the bottom row:

$$\begin{array}{ccccccccccccccc} y_{it} & = & \beta_{00} & + & \beta_{01}x_i & + & u_{0i} & + & a_{it}(\beta_{10} & + & \beta_{11}x_i & + & u_{1i}) & + & \varepsilon_{it} \\ y_t & = & \mu_{\eta_1} & + & \gamma_1 x & + & \zeta_1 & + & \lambda_t(\mu_{\eta_2} & + & \gamma_2 x_i & + & \zeta_2) & + & \varepsilon_t. \end{array}$$

Because the values from the MLM are directly above their LGC analogs, it can be seen that a one-to-one relationship exists between the two models when the functional form of change is linear in its parameters.[9] Provided that the structure of the errors is specified in an equivalent manner and full information maximum likelihood is used, the two models yield identical parameter estimates.

Curran makes the following proposal: "*any two-level linear multilevel model can be estimated as a structural equation model given that this is essentially a data management problem*" (p. 557). Evidence exists, however, that in theory any linear MLM has a SEM counterpart that is statistically equivalent.[10] However, not all SEMs can be duplicated in the MLM framework. For example, SEM is generally much more flexible in how the error structure can be specified, as well as being capable of incorporating a measurement model for predictor variables (i.e., the *x*s) and/or the dependent variables (i.e., the *y*s).

One major limitation of the LGC is the fact that it is almost exclusively for linear models. As Bauer explains "many multilevel *nonlinear* models cannot be parameterized as SEMs, since structural equation modeling is predicated on a linear system of equations" (p. 162, 2003). Along these same lines, du Toit and Cudeck (2001) state that the "chief impediment" of modeling nonlinear SEM models is that software design does not allow arbitrary nonlinear functions to be represented in the design matrix (2001, p. 268). Du Toit and Cudeck go on to discuss a semi-nonlinear SEM, where unique effects exists only for the linear part of the model. Specifically if a nonlinear component is of interest, in order to carry out the model in the LISREL computer program, the particular nonlinear component must be implemented by imposing polynomial constraints on the elements of the design matrix corresponding to the nonlinear component (p. 268). If the nonlinear

[9]As an aside, notice that the MLM is subscripted with an *i* for the individual specific values. This *i* is not present in the discussion of the LGC. For some reason within the SEM framework individual specific values are not generally subscripted with an *i* to denote individual scores, variables, or values. Perhaps this stems from SEM tradition where the structure of the overall model is of interest, not an individual's scores (Recall that the standard SEM model is generally a variable-centered approach.)? Regardless, as is the norm in the LGC context, *i*s identifying individual specific values have not been included in the model's equation.

[10]Even though this may be the case in theory, practically speaking it can be very cumbersome to specify some simple MLMs as SEMs, which illustrates the data management problem to which Bauer (2003) and Curran (2003) refer.

component cannot be specified as a constrained polynomial, then the LISREL computer program cannot fit the particular nonlinear model (du Toit & Cudeck, 2001). Other SEM computer programs allow nonlinear constraints in a similar fashion, and thus have similar restrictions. In general, however, nonlinear unique effects are of interest, not just nonlinear fixed effects. Forcing the random component(s) to be fixed makes the assumption that the whole population has the same value for the nonlinear component(s), thus limiting the ability to model interindividual differences in change. While such an assumption may at times be reasonable, in general it illustrates a serious limitation of the SEM approach to nonlinear models.

Although there has been some interest in nonlinear SEMs and nonlinear factor analytic models, in large part estimating such nonlinear models is not generally practical. Such a limitation is what some say has led to some "statisticians' lack of interest in factor analysis" (Yalcin & Amemiya, 2001, p. 275). Muthén (2002) acknowledges the same type of concerns as do Yalcin and Amemiya (2001). Specifically Yalcin and Amemiya argue that linear models are often unrealistic and do not fit the data well without increasing the number of factors beyond the level explainable by the substantive theory (a similar argument will be set forth beginning on page 34).[11]

To summarize the present section, MLMs linear in their parameters can generally be thought of as a special case of a LGC model, where the MLM is generally more straight-forward to specify, but nevertheless theoretically equivalent to the LGC. However, the general MLM is nonlinear in its parameters with the general linear MLM being a special case. Because of the importance and versatility of nonlinear models of

[11]The other reasons that Yalcin and Amemiya give for statisticians' lack of interest in factor analysis are the issue of identification ambiguity and the heavy reliance on normality (2001, p. 275). Muthén (2002) notes the same type of concerns as Yalcin and Amemiya. In his article, Muthén aimed at "inspiring a better integration of psychometric modeling ideas into mainstream statistics" (p. 81). The purpose of Muthén's article was to illustrate to the "mainstream" statistics community that latent variable models are widely used in statistics only under different names and in different forms (p. 82).

change, yet the inability to carry out general nonlinear models in the LGC context, the present state of knowledge mandates that the present work revolve around MLMs for nonlinear models of change rather than LGCs.

Any functional form of change linear in its parameters can be specified as a special case of the general linear MLM presented in Equation 5. An important special case of the general linear MLM is the polynomial change model. A polynomial change model is a model of change where nonnegative integer powers of a_t are multiplied by an associated coefficient that quantifies the degree to which the nonnegative integer power of a_t contributes to the dependent variable. Actually, Equation 6 is a polynomial change model of degree K, because $k = 0, 1, 2, \ldots, K$. Equation 6 can be generalized beyond the polynomial change model yet still be a linear model. This occurs when k is not a nonnegative integer (e.g., $k = 1/2$, $k = -1/3$, $k = -1$, etc.).

In the behavioral sciences, change models of the form given by Equation 6 are the most widely used for modeling change. However, such models are often unrealistic, especially when sufficient measurement occasions are obtained and/or they occur over a relatively long time span in an effort to capture a complete process. The reason linear models typically fail in the context of change models is because unlimited growth or unlimited decay is unlikely to occur for most behavioral phenomena (van Geert, 1991; Browne & du Toit, 1991), and because the model will generally provide a poor description of the process beyond the range of the observed data. If enough polynomial terms are added to a change model, regardless of the true functional form governing the process, the observed data can be well approximated with a polynomial change model.[12] However,

[12]Well approximated in the sense that the sum of squared deviations between the model implied values and the observed values will approaches zero as the number of polynomials included in the model approaches T. Thus, by adding enough polynomial powers of time the model implied trajectory will

extrapolation or forecasting to any degree can be devastating, because linear models may quickly diverge from reasonable values of the phenomenon of interest. Furthermore, a serious problem that occurs by incorporating multiple powers of a_t into the change model is that not only is the true behavior of the phenomena of interest being modeled, so too is the error.

Although we may know a priori that a model will never be entirely correct (e.g, Browne & Cudeck, 1993), what is important is that the model be useful (Box, 1979a, 1979b). Thus, if higher ordered polynomial models (say third order or higher) are fit to complicated patterns of change, the question of interest is “are they really useful?” If indeed such a higher ordered polynomial change model is useful, then making use of such a model is not problematic. The usefulness of a model, however, is not only a function of the degree of fit between observed and predicted scores, but rather a model’s usefulness is also related to its substantive interpretation. If a model is largely uninterpretable, then it is generally not very useful (Cudeck & du Toit, 2002). Because of the limitation and problems inherent with the straight-line change model, it seems that such a model should be used with caution. However, the straight-line change models is apparently the “model of choice” for behavioral science researchers (e.g., Cudeck, 1996; Mehta & West, 2000).

Figure 2 provides an example of some of the problems inherent with linear models when change is complicated. Figure 2 illustrates a deterministic Gompertz growth trajectory (a discussion of the Gompertz growth curve begins on page 41), the solid “S-shaped” trajectory, modeled by a first through ninth order polynomial change model. The equation for the particular Gompertz growth model is $y_t = 5\exp(-\exp(7 - 1.75a_t))$, where $a_t \in [2, 8]$.

As can be seen in Figure 2, as the order of the polynomial change model is increased, the polynomial change model more closely resembles the Gompertz growth trajectory.

approach the observed trajectory. In fact, a polynomial of order $T - 1$ will fit T occasions of measurement perfectly.

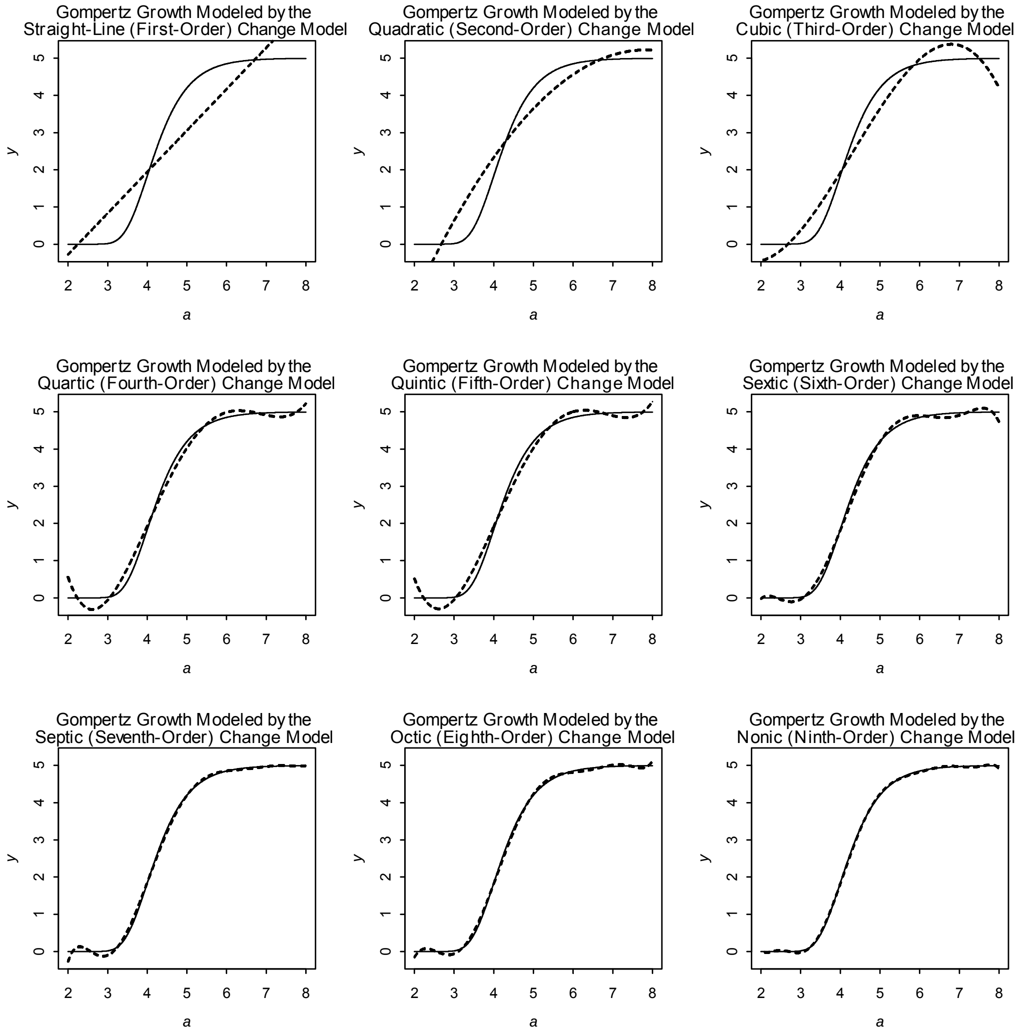

Figure 2: Comparison of Gompertz growth [$y_t = 5\exp(-\exp(7 - 1.75a_t))$] (solid line) modeled by polynomial change models from first through ninth order (dashed line).

Because the Gompertz growth model has two asymptotic values, it requires a complex polynomial change model with several change parameters to adequately model the Gompertz growth curve when $a_t \in [2, 8]$. A question that naturally arises is, “how useful are the higher ordered polynomial models?” Unless the model is intended as a prediction equation when $a_t \in [2, 8]$, the model does not seem very useful. The reason for the lack of usefulness is in the difficulty in linking the parameters of the polynomially change model to some substantive theory or real-world interpretation (unlike the parameters of the Gompertz change curve which do have real-world interpretations).

Another problem with applying linear models to complicated change processes relates to extrapolation beyond the values of time measured (i.e., forecasting). Although extrapolation is not generally a methodologically advisable technique, at times it may be necessary. A serious problem with models linear in their parameters is that they potentially exhibit wild behavior beyond the range of the observed data. For example, taking the same Gompertz growth curve used in Figure 2 but expanding the values on the abscissa to $a_t \in [0, 10]$, Figure 3 illustrates a tenth degree polynomial fit to the Gompertz growth curve when the data used to estimate the parameters of the tenth degree model were based on the data when $a_t \in [2, 8]$. As can be seen, using the tenth degree polynomial change model to predict values just beyond the range of the data collected led to predicted values that quickly diverged from their true values. Of course, not all linear models when applied to nonlinear curves diverge from "reasonable" values so quickly, but Figure 3 is characteristic of what will necessarily happen at some point as the values of a_t continue to increase or decrease beyond the range of the data collected.

Although there are potential problems when making use of linear models for complicated processes of change, the appeal of linear models is understandable. In the framework of the straight-line change model, only two parameters—the intercept and the slope—are used to describe a trajectory. Oftentimes the intercept is not directly interpretable and in some situations it is completely arbitrary. Thus, the slope is generally the parameter of most interest when describing a trajectory that has been modeled by the straight-line change model. Because parsimony is a goal of science, it seems as though making use of the straight-line change model would be good practice. However, a model should not necessarily be as simple as possible, but rather should ideally be no more complicated than necessary.[13] Because of the shortcomings of the straight-line change

[13]This is a by-product of a quote by Albert Einstein: "everything should be made as simple as possible, but not simpler" (Einstein, 1977, p. 42). Einstein's quote can be thought of as another way of conceptualizing Occam's razor (or the principal of parsimony), in that the goal of science should be to develop explanations that do not require unnecessary assumptions.

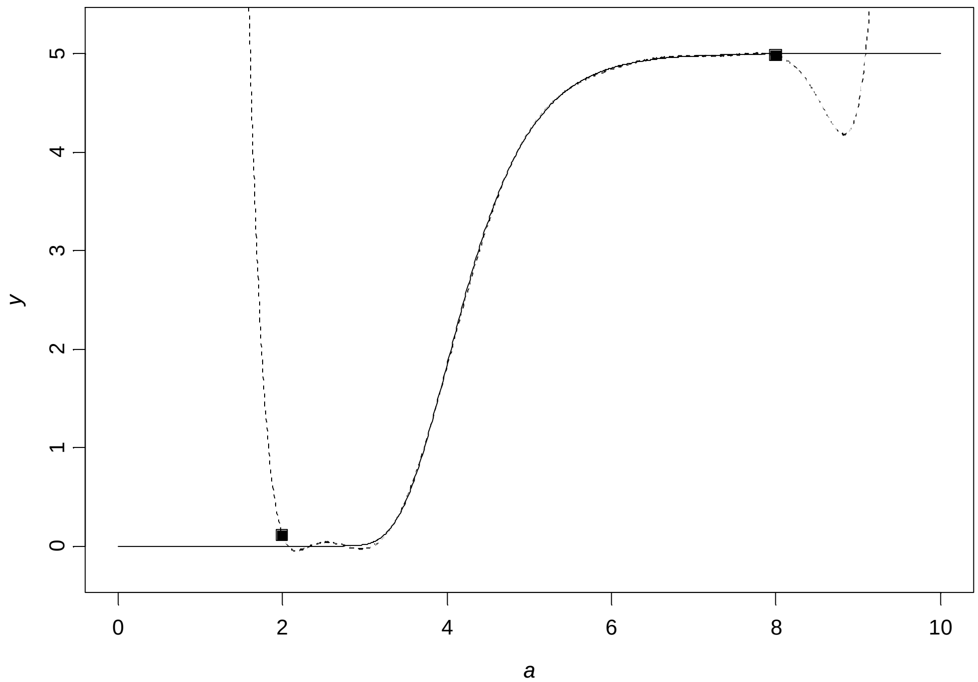

Figure 3: Gompertz growth (solid line) modeled by the decic (tenth-order) growth model (dashed line) for $a_t \in [0, 10]$ when parameter estimates for decic growth model were obtained when $a_t \in [2, 8]$ (range denoted by solid squares) and thus extrapolation occurs when $a_t < 2$ and $a_t > 8$.

model, the straight-line change model seems generally inadequate for much behavioral science research.[14,15]

[14]Obviously if the phenomenon of interest truly has a linear relationship with a_t, then a straight-line change model is appropriate. If the overall phenomenon of interest does not have a straight-line relationship with a_t, but over the a_t interval of interest a linear relationship is exhibited (i.e., there is local linearity), a straight-line change model may also be appropriate. Furthermore, the number of change parameters that can be used to model a trajectory is limited by the number of timepoints collected. In particular, with unique effects included for all fixed components, it is necessary that $T \geq P + 1$ for at least some individuals (Raudenbush & Bryk, 2002, p. 176). Thus, in the special case where $T = 3$, the most complicated polynomial change model that can be fit when each fixed effect has a corresponding unique effect is the straight-line change model.

[15]Perhaps one reason why the straight-line change model is so well accepted by applied researchers is because in illustrative methodological works, the straight-line change model is so often the first and/or only model of change discussed. This makes sense for pedagogical purposes, however, oftentimes the authors of such illustrative works do not discuss the implications of "always" using the straight-line change model. Perhaps methodologists overuse the straight-line change model to the point where it seems as if it is the only change model needed?

NONLINEAR MODELS AND THE ANALYSIS OF CHANGE

As previously stated, nonlinear models oftentimes provide a more natural description of behavioral phenomena than do models linear in their parameters. By explicitly acknowledging and then modeling asymptotic values, natural and limiting behavior can be modeled and inferences can be made. However, without making use of nonlinear models, asymptotic values cannot generally be explicitly modeled. Thus, with regards to asymptotic values, linear models are largely inadequate in their ability to model real world phenomena. Although linear models may adequately model asymptotic values within the range of observed data, linear models cannot generally continue to hold much beyond the range of the collected data (recall Figures 2 and 3).

While in theory an infinite number of nonlinear models exist, four nonlinear models are described in the next four sections. The models discussed are the asymptotic regression, logistic, Gompertz, and the Richards growth models. As will be pointed out momentarily, the logistic, Gompertz, and Richards growth models are slight generalizations of what is generally described in the literature. Specifically a parameter has been added that allows for the intercept to be an explicit part of the model, rather than being a function of other parameters in the model. Although this parameter can be constrained to be zero, and thus the curves would be isomorphic with the curves generally reported in the literature, allowing this parameter to be estimated in the model may be especially important in the behaviorial sciences. The nonlinear models to be presented momentarily have historically been used in applications where the intercept is zero or near-zero with positive change. In the behavioral sciences the intercept is oftentimes arbitrary. Because of the oftentimes arbitrary nature of the intercept in behavioral science research, the flexibility of modeling the intercept was thought to be potentially important, and thus the additional parameter was added. The meaning of each of the model parameters is detailed in sections where the model is introduced.

A great deal of work in the botanical, pharmacokinetic, and the human and animal biological sciences has been devoted to the derivation and application of various nonlinear models. In the botanical sciences, nonlinear models are often used to model the height of a tree, the area of a leaf, or crop yield, all as a function of time or age of the plant (Hunt, 1982; Causton & Venus, 1981). A crop geneticist may be interested in the genotypic variation of crops as a function of growing environment in order to breed the optimal crop variety for a particular environment (Yin, Goudriaan, Lantinga, Vos, & Spiertz, 2003). A pharmacokinetist may be interested in the saturation level of a particular medication or toxin in the bloodstream as a function of time since exposure (Davidian & Giltinan, 1995). An animal biologist may wish to describe the size or weight of some animal, the overall mass of some organ (e.g., heart, lung, etc.), or ruminal degradation of substrate (López, France, Dhanoa, Mould, & Dijkstra, 1999), all as a function of time or age. In all of these examples linear models would not make sense, since organismic change is known to be limited and generally rises (or falls) to some asymptotic value as the full potential of the process is realized. As Figure 3 illustrates, even if a linear model closely mimics the observed data within some constrained range, any prediction beyond the observed data will oftentimes sharply diverge from any limiting, and thus realistic, values.

Given the importance of nonlinear models, it is not surprising that their use permeates many scientific disciplines. However, at the present time nonlinear change models are not often used in the behaviorial sciences. The seemingly systematic avoidance of nonlinear models has probably stifled important discoveries and has likely led to inadequate descriptions of various aspects of behavior. Unlimited or unconstrained models of change applied in the behavioral sciences are for the most part known a priori to be largely inadequate if the phenomenon of interest occurs over a large enough span of measurement occasions. The following subsections detail four different change curves that are thought to be potentially useful for modeling various phenomena in the behavioral sciences, and potentially overcome some of the shortcomings inherent in linear models.

The Asymptotic Regression Growth Curve

Stevens acknowledged in 1951 that a large number of regression problems were unsuitable for polynomial regression with linear models, the most frequently encountered problem being when the dependent variable approaches an asymptotic value (pp. 247–248). Stevens suggested that to deal with asymptotic values the deterministic part of a change curve of the form

$$y_t = \alpha + \zeta\rho^{a_t} \tag{23}$$

could be used, where α is the asymptote as $a_t \to \infty$, ζ represents the negative of the total change of y, and ρ $(0 < \rho < 1)$ represents the deviation of y from the asymptotic value for each unit change in a_t (Stevens, 1951, p. 248). By taking the negative of the logarithm of ρ, Equation 23 can be reparameterized, such that it is written explicitly in terms of the anti-logarithm,

$$y_t = \alpha + \zeta \exp(-\gamma a_t), \tag{24}$$

where γ $(0 < \gamma < \infty)$ defines the curvature of the change curve and where α and ζ have the same meaning as in Equation 23.[16] A positive value for ζ implies asymptotic decay, whereas a negative value implies asymptotic growth. The intercept of the asymptotic

[16] Any reference made to logarithms implies the natural logarithm, where the base of the logarithm is set equal to the natural constant $e = \exp(1) \approx 2.71828$. Likewise, any reference to the anti-logarithm implies the natural anti-logarithm, e. Logarithms and anti-logarithms will be abbreviated as log and anti-log, respectively.

regression growth model is defined as

$$\phi = \alpha + \zeta. \tag{25}$$

Figure 4 illustrates six asymptotic regression growth curves with different combinations of parameter values. As can be seen in Figure 4, the intercept of the curves is equal to ϕ ($\phi = 0$ for curves I through III and $\phi = 1$ for curves IV through VI). The curves begin to approach their asymptote α ($\alpha = 1$ for curves I through III and $\alpha = 0$ for curves IV through VI) as a_t increases. Notice that the only difference between curves I, II, and III is the value of γ, implying that the three curves differ only in their curvature parameters. Notice also that the only difference between curves IV, V, and VI is the value of γ, thus illustrating different curvatures. The curvature can be seen to affect the rate at which the total change from ϕ to α is realized.

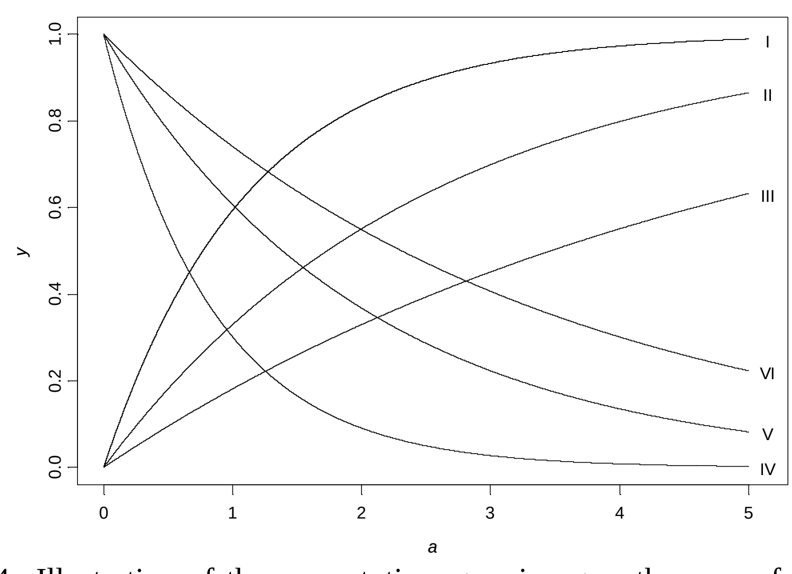

Figure 4: Illustration of the asymptotic regression growth curve of Equation 24. The parameters defining each of the curves are given as follows: *Curve I*: $\alpha = 1$, $\gamma = .9$, & $\zeta = -1$; *Curve II*: $\alpha = 1$, $\gamma = .4$, & $\zeta = -1$; *Curve III*: $\alpha = 1$, $\gamma = .2$, & $\zeta = -1$; *Curve IV*: $\alpha = 0$, $\gamma = 1.2$, & $\zeta = 1$; *Curve V*: $\alpha = 0$, $\gamma = .5$, & $\zeta = 1$; *Curve VI*: $\alpha = 0$, $\gamma = .3$, & $\zeta = 1$.

The Logistic Growth Curve

The logistic growth curve is a special case of a sigmoidal curve, with an upper and lower asymptote and a symmetric point-of-inflection. The deterministic part of the four parameter logistic curve can be written as

$$y_t = \frac{\alpha}{1 + \exp(-\gamma(a_t - \beta))} + \zeta, \tag{26}$$

where $\alpha + \zeta$ is the asymptote as $a_t \to \infty$ when γ is positive or as $a_t \to -\infty$ when γ is negative, β defines the point-of-inflection on the abscissa, γ defines the curvature, and where ζ affects the value of the intercept and is the asymptote as $a_t \to -\infty$ when γ is positive or as $a_t \to \infty$ when γ is negative (chapter 4 and pp. 167–169 of Ratkowsky, 1983; Yin et al., 2003).[17]

Actually, the logistic curve of Equation 26 is a generalization of that typically given in the literature. The logistic growth curve is generally defined with three parameters. However, a fourth parameter, ζ, has been added so that the value of the intercept can itself be modeled, rather than being a function of the other parameters of the model. When ζ is not specified in the model and γ is positive, the asymptote as $a_t \to -\infty$ is zero. Similarly, when ζ is not specified in the model and γ is negative, the asymptote as $a_t \to \infty$ is zero. These properties are potentially unrealistic, especially if a_t were to be centered. The intercept in the standard logistic model without ζ is

$$\frac{\alpha}{1 + \exp(\gamma\beta)},$$

and is thus a function of the other parameters in the model. Modeling the intercept explicitly is important because it is a potentially interesting quantity. Furthermore, because the initial value of a_t is often arbitrary in behavioral science research, allowing

[17] The logistic growth curve has also been called the Verhulst (1838) equation after its developer (as cited in Yin et al., 2003), the Verhulst-Pearl equation after Pearl who used the equation Verhulst developed to model population growth in the United States in 1920 (as cited in Tsoularis & Wallace, 2002), and the autocatalytic function of Robertson (1923; as cited in Richards, 1959).

for a flexible intercept is advantageous. The intercept when ζ is included in the model is

$$\phi = \frac{\alpha}{1 + \exp(\gamma\beta)} + \zeta. \tag{27}$$

The intercept of the modified logistic growth curve is essentially shifted by a value of ζ from what it otherwise would have been if the standard logistic growth curve were used.[18] Furthermore, the modified logistic is more flexible because the lower asymptote (on the ordinate) is ζ rather than the trivial case of zero. Note that the generalized logistic growth model can be set equal to the "standard" logistic growth model by fixing ζ to zero.

As previously mentioned, the logistic growth curve has a symmetric point-of-inflection. The point-of-inflection occurs at the point

$$a^* = \beta \tag{28}$$

and

$$y^* = \frac{\alpha}{2} + \zeta, \tag{29}$$

where a^* and y^* are respectively the values on the abscissa and ordinate. When ζ is zero the point-of-inflection occurs at 50% of the total change. A point-of-inflection is where the curvature of the trajectory changes from concave up to concave down (or vice-versa) and in sigmoidal models is the point where the rate or change (i.e., the derivative of y with respect to a) is maximal or minimal (depending on the direction of the trajectory). Formally, the point-of-inflection is defined where the second derivative of a twice differentiable function with respect to a_t is zero (Finney, Weir, & Giordano, 2001, section 3.3).

[18]The word essentially is used here because the change curve is not literally shifted by a value of ζ, but is shifted by a value related to ζ. The reason the curve is not literally shifted by a value of ζ is because the parameters are not generally independent of one another. In such a scenario, where the parameters are not independent, the α, β, and γ values of the three parameter logistic growth curve are not equal to the α, β, and γ values of the four parameter logistic growth curve, respectively. Thus, because adding ζ potentially affects what the α, β and γ values would be from the three parameter logistic growth model, the intercept is not literally shifted by a value of ζ.

Figure 5 illustrates six different logistic growth curves. Notice that three of the curves are increasing (implying $\gamma > 0$) and that the other three curves are decreasing (implying $\gamma < 0$). Notice also that the three increasing curves share the same point-of-inflection on the abscissa and that the three decreasing curves share the same point-of-inflection. This occurs because $a^* = \beta = 2$ for the increasing curves and $a^* = \beta = 3$ for the decreasing curves. Because α and ζ were held constant for each of the six curves, $y^* = .5\alpha + \zeta = .61$ and is the same for all curves. Thus, the three increasing curves cross at the point $(a^* = 2, y^* = .61)$, whereas all three of the decreasing curves all cross at the point $(a^* = 3, y^* = .61)$.

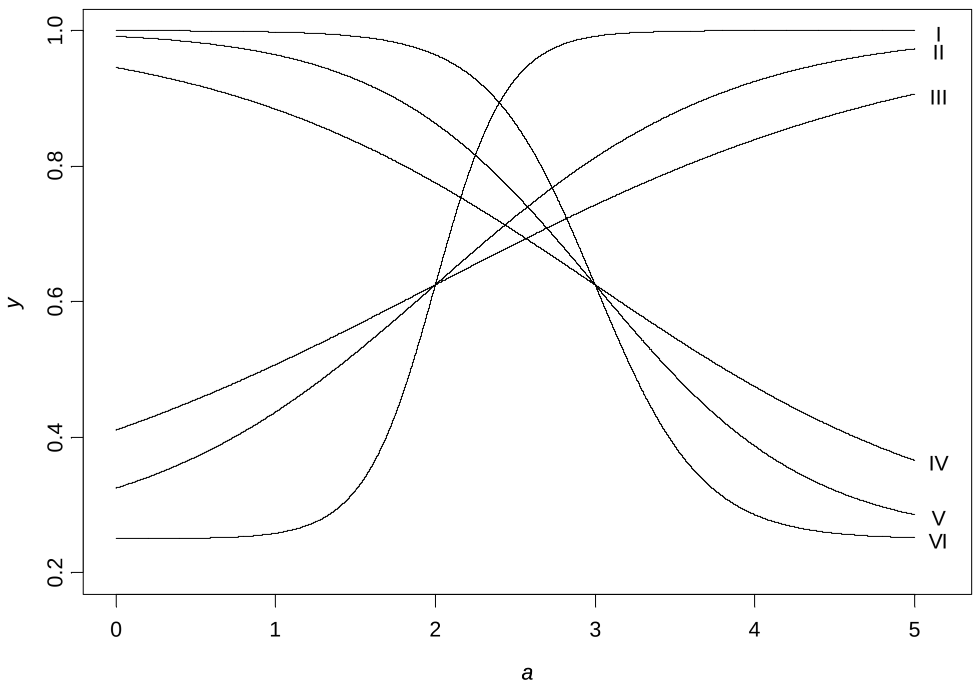

Figure 5: Illustration of the four parameter logistic growth curve of Equation 26. In each of the six curves $\alpha = .75$ and $\zeta = .25$. The β and γ values for each of the six curves are given as *Curve I*: $\beta = 2$ & $\gamma = 4.5$; *Curve II*: $\beta = 2$ & $\gamma = 1.1$; *Curve III*: $\beta = 2$ & $\gamma = .65$; *Curve IV*: $\beta = 3$ & $\gamma = -.85$; *Curve V*: $\beta = 3$ & $\gamma = -1.5$; *Curve VI*: $\beta = 3$ & $\gamma = -3$.

The Gompertz Growth Curve

Like the logistic growth curve, the general shape of the Gompertz growth curve is sigmoidal. Unlike the logistic growth curve, however, the Gompertz is not symmetric about its point-of-inflection. The deterministic part of the four parameter Gompertz growth curve is defined as

$$y_t = \alpha \exp\left(-\exp(-\gamma(a_t - \beta))\right) + \zeta, \tag{30}$$

where $\alpha + \zeta$ is the asymptote as $a_t \to \infty$ when γ is positive or as $a_t \to -\infty$ when γ is negative, β defines the point-of-inflection on the abscissa, γ defines the curvature, and ζ affects the value of the intercept and is the asymptote as $a_t \to -\infty$ when γ is positive or as $a_t \to \infty$ when γ is negative (chapter 4 and pp. 163–167 of Ratkowsky, 1983; Winsor, 1932; Yin et al., 2003).

Actually, the Gompertz growth curve of Equation 30 is a generalization of that typically given in the literature. The Gompertz growth curve is generally defined with three parameters. However, like the logistic growth curve, a fourth parameter, ζ, has been added so that the value of the intercept can itself be modeled, rather than being a function of the other parameters of the model. When ζ is not specified and γ is positive, the asymptote as $a_t \to -\infty$ is zero. Similarly, when ζ is not specified and γ is negative, the asymptote as $a_t \to \infty$ is zero. The intercept of the standard Gompertz growth curve without ζ is

$$\alpha \exp(-\exp(\gamma\beta)),$$

and is thus a function of the other parameters in the model. Modeling the intercept explicitly is important because it is a potentially interesting quantity. Furthermore, because the initial value of a_t is often arbitrary in behavioral science research, allowing for a flexible intercept is advantageous. The intercept when ζ is included in the model is

$$\phi = \alpha \exp(-\exp(\gamma\beta)) + \zeta. \tag{31}$$

The intercept of the modified Gompertz growth curve is essentially shifted by a value of ζ from what it otherwise would have been if the standard Gompertz growth curve were used.[19] Furthermore, the modified Gompertz is more flexible because the lower asymptote (on the ordinate) is ζ rather than the trivial case of zero. As previously mentioned, the Gompertz growth curve, unlike the logistic growth curve, does not have a symmetric point-of-inflection. The point-of-inflection for the Gompertz growth model occurs at the point

$$a^* = \beta \tag{32}$$

and

$$y^* = \frac{\alpha}{\exp(1)} + \zeta, \tag{33}$$

where a^* and y^* are respectively the values on the abscissa and ordinate. When ζ is zero, the point-of-inflection occurs at approximately 36.7879% of the total change.

Figure 6 illustrates six different Gompertz growth curves. Notice that three of the curves are increasing (implying $\gamma > 0$) and that the other three curves are decreasing (implying $\gamma < 0$). Notice also that the three increasing curves share the same point-of-inflection and that the three decreasing curves share the same point-of-inflection. This occurs because $a^* = \beta = 2$ for the increasing curves and $a^* = \beta = 3$ for the decreasing curves. Because α and ζ were held constant for each of the six curves, $y^* = .5259$ is the same for all curves. Thus, the three increasing curves all cross at the point $(a^* = 2, y^* = .5259)$, whereas all three of the decreasing curves all cross at the point $(a^* = 3, y^* = .5259)$

[19]As was the case for the logistic growth curve, the word essentially is used here because the change curve is not literally shifted by a value of ζ, but is shifted by a value related to ζ.

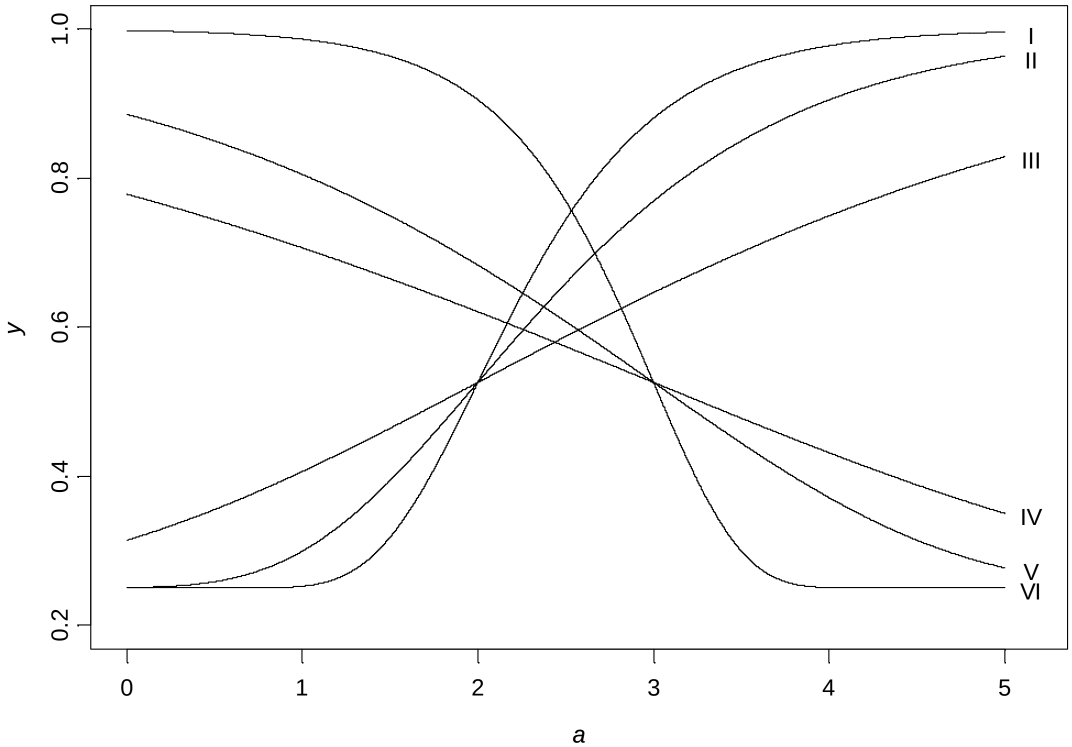

Figure 6: Illustration of the four parameter Gompertz growth curve of Equation 30. In each of the six curves $\alpha = .75$ and $\zeta = .25$. The β and γ values for each of the six curves are given as *Curve I*: $\beta = 2$ & $\gamma = 1.75$; *Curve II*: $\beta = 2$ & $\gamma = 1$; *Curve III*: $\beta = 2$ & $\gamma = .45$; *Curve IV*: $\beta = 3$ & $\gamma = -.35$; *Curve V*: $\beta = 3$ & $\gamma = -.6$; *Curve VI*: $\beta = 3$ & $\gamma = -2$.

The Richards Growth Curve

Von Bertalanffy (1941) developed a change curve with restrictions related to the theoretical expectations of anabolism and catabolism of animal metabolic rate as they related to an animal's weight (as cited in Richards, 1959). Not being satisfied with a fixed point-of-inflection due to von Bertalanffy's imposed restrictions on the change curve, Richards generalized the von Bertalanffy growth model by removing the fixed point-of-inflection, which led to a more general model of growth (Richards, 1959). The deterministic component of the five parameter Richards growth curve is defined as

$$y_t = \frac{\alpha}{(1 + \delta \exp(-\gamma(a_t - \beta)))^{1/\delta}} + \zeta, \tag{34}$$

where $\alpha + \zeta$ is the asymptote as $a_t \to \infty$ when γ is positive or as $a_t \to -\infty$ when γ is negative, β is the point-of-inflection on the abscissa, γ defines the curvature, δ affects the point-of-inflection on the ordinate, and where ζ affects the value of the intercept and is the asymptote as $a_t \to -\infty$ when γ is positive or as $a_t \to \infty$ when γ is negative (Richards, 1959; Yin et al., 2003).

Like the logistic and the Gompertz growth curves, the Richards growth curve presented in Equation 34 is actually a generalization of that typically given in the literature. Generally the Richards growth curve is presented with four parameters. However, ζ has been added so that the intercept can itself be modeled, rather than being a function of the other parameters of the model. When ζ is not specified and γ is positive, the asymptote as $a_t \to -\infty$ is zero. Similarly, when ζ is not specified and γ is negative, the asymptote as $a_t \to \infty$ is zero. The intercept in the standard Richards model without ζ is

$$\frac{\alpha}{(1 + \delta \exp(\gamma\beta))^{1/\delta}},$$

and is thus a function of the other parameters in the model. Adding ζ to the Richards growth curve allows the intercept itself to be explicitly modeled. Modeling the intercept explicitly is important because it is a potentially interesting quantity. Furthermore, because the initial value of a_t is often arbitrary in behavioral science research, allowing for a flexible intercept is advantageous. The intercept when ζ is included in the model is

$$\phi = \frac{\alpha}{(1 + \delta \exp(\gamma\beta))^{1/\delta}} + \zeta. \tag{35}$$

The intercept of the modified Richards growth curve is essentially shifted by a value of ζ from what it otherwise would have been if the standard Richards growth curve were used.[20] Furthermore, the modified Richards growth curve is more flexible because the lower asymptote (on the ordinate) is ζ rather than the trivial case of zero. Unlike the logistic and the Gompertz (as well as the von Bertalanffy) growth curves, the point-of-inflection for the Richards growth curve is flexible, meaning that it is not prespecified to occur at some fixed percentage of total change. This added flexibility allows the point-of-inflection to be defined at the point

$$a^* = \beta \tag{36}$$

and

$$y^* = \alpha(1 + \delta)^{-1/\delta} + \zeta. \tag{37}$$

As can be seen from comparing the Richards growth curve with the logistic growth curve, in the special case where $\delta = 1$, the Richards curve becomes the logistic growth curve of Equation 26. Not as obvious is the fact that taking the limit as $\delta \rightarrow 0$, the Richards growth curve is equal to the Gompertz growth curve of Equation 30. That is,

$$\lim_{\delta \rightarrow 0} \frac{\alpha}{(1 + \delta \exp(-\gamma(a_t - \beta)))^{1/\delta}} + \zeta = \alpha \exp(-\exp(-\gamma(a_t - \beta))+) + \zeta. \tag{38}$$

Thus, special cases of the Richards growth curve are the logistic and the Gompertz growth models (as well as the von Bertalanffy model, which is not discussed here). Isomorphism between the Richards growth curve and those that it subsumes as a special case depends on the value of δ. Because the Richards growth model actually defines a family of growth curves, the "Richards family of curves" subsumes the logistic, Gompertz, and the von Bertalanffy growth curves as special cases.

[20] As was the case with the logistic and Gompertz growth curves, the word essentially is used here because the change curve is not literally shifted by a value of ζ, but is shifted by a value related to ζ.

Figure 7 illustrates six different Richards growth curves. Notice that $\delta = 1$ in Curve I. Thus, Curve I is a logistic curve. In fact, Curve I is identical to Curve I from Figure 5. Notice also that $\delta = .00001$ and is close to a value of zero for Curve II. Curve II is approximately equal to a Gompertz growth curve. In fact, Curve II is essentially identical to Curve II of the Gompertz growth example given in Figure 6. Notice also that three of the curves are increasing (implying $\gamma > 0$) and the second three curves are decreasing (implying $\gamma < 0$). Perhaps not obvious is the fact that, although $\beta = 2$ for curves I through III and $\beta = 3$ for curves IV thorough VI, the point-of-inflection is different for each of the curves. Although $a^* = \beta$, y^* depends on δ. The fact that the upper and lower asymptotes, the curvature, and the point-of-inflection can be modified so readily makes the Richards growth curve very general. Coupled with the fact that the parameters are readily interpretable, there seems to be many potential uses for the Richards growth curve in behaviorial science research.

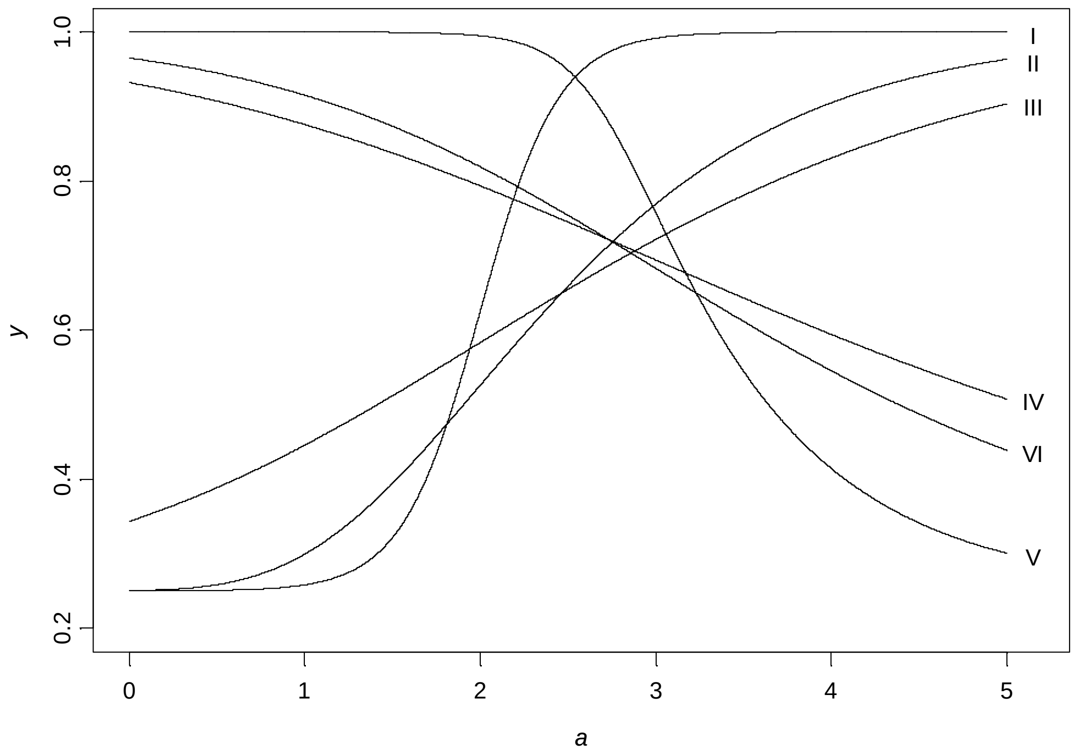

Figure 7: Illustration of the five parameter Richards growth curve of Equation 34. In each of the six curves $\alpha = .75$ and $\zeta = .25$. The β, δ and γ values for each of the six curves are given as *Curve I*: $\beta = 2$, $\delta = 1$, & $\gamma = 4.5$; Curve II: $\beta = 2$, $\delta = .00001$ & $\gamma = 1$; *Curve III*: $\beta = 2$, $\delta = .5$, & $\gamma = .65$; *Curve IV*: $\beta = 3$, $\delta = 2.25$ & $\gamma = -.75$; *Curve V*: $\beta = 3$, $\delta = 4.25$ & $\gamma = -5$; *Curve VI*: $\beta = 3$, $\gamma = 2$ & $\gamma = -1$.

At this point several modern approaches for modeling change have been presented and the benefits of nonlinear change models have been described. Four potentially useful functional forms of change were delineated. These functional forms of change are thought to be especially useful in the behavioral sciences where behavior is often constrained in some way (van Geert, 1991). At this point in the work all statistical models have been based on a homogeneous population or a heterogeneous population with known sources of heterogeneity. An interesting and important problem in science is the classification of entities with similar characteristics. The next section provides an overview of the general topic of mixtures of distributions and models that describe those mixed distributions. The section is necessary because it provides a context for the GMM and ultimately the LCDC model.

THE FINITE MIXTURE MODEL

Using data collected by Wendall (1892, 1893) on the ratio of forehead to body length of 1,000 crabs from Naples, Karl Pearson laid the ground work for what later became known as the finite mixture model (FMM; Pearson, 1894). Because of the nonsymmetric distribution of the ratio of forehead to body length of the crabs, Wendall was interested in knowing if two species of crab existed. Taking his question and data to Pearson, Pearson realized that the asymmetric frequency curve could be broken up into two component probability curves, specifically into two normal curves (Pearson, 1894, p. 79).[21] Pearson used the "method of moments" to estimate the properties of the two component distributions in a model that became known as the FMM. Interestingly, using the Naples crab data, it was recently shown by McLachlan and Peel (2001) that modern methods of finite mixture modeling, where parameter estimation is carried out via maximum likelihood methods, gives a fit very similar to that obtained by Pearson with his method of moments (p. 2).

As can be inferred from the description of Pearson's work on "the dissection of asymmetrical frequency curves" (1894, p. 71), the goal of the FMM is to accurately represent an observed frequency distribution with a finite number of component distributions. The composite of the component distributions is then the statistical model

[21]It is interesting to note that Weldon's articles (1892, 1893) were some of the first to use the statistical methods of Sir Francis Galton on animals other than man (as cited in Pearson, 1906, pp. 16–17). Regarding Weldon's articles, Pearson (1906) said that "these two papers are epoch-making in the history of science, afterwards called biometry" (p. 17).

for the observed data. These component distributions are what is mixed together in order to form the overall composite distribution.

Most often it is assumed that the component distributions are each normally distributed, although this is not necessary. However, if the component distributions are each normally distributed and differ nontrivially from one another, the composite distribution is necessarily nonnormal. That is, when the component distributions are thought to be normally distributed, a necessary condition of the FMM is that the composite distribution be nonnormal. Although nonnormality of the composite distribution is a necessary condition, it is not a sufficient condition to identify a mixture of component distributions. That is, the "composite" distribution need not be a mixed distribution, it could simply be a realization from a homogeneous population that has some nonnormal distribution. Regarding these differences, Pearson acknowledged that he had "been unable to find any general condition among the moments, which would be impossible for a skew curve and possible for a compound [i.e., mixture], and so indicate compoundness" (p. 394). Although Pearson did not despair of such a method being found (1895, p. 394), as of yet no such solution exists, and it is likely that one never will. Thus, in situations where a nonnormal distribution exists, theory must dictate whether or not to regard the nonnormal distribution as a mixture of heterogeneous populations or as a nonnormal homogeneous distribution.

The basic definition and formulation of the FMM given here is summarized from Everitt and Hand (1981), McLachlan and Basford (1988), and McLachlan and Peel (2001). Let $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N]'$ be a random sample of N individuals for P variables, where $\mathbf{x}_i$ represents a P dimensional random vector with probability density function $\mathrm{d}(\mathbf{x}_i)$. The N by P matrix $\mathbf{X}$ is thus the complete set of random variables. In the finite mixture modeling framework, it is supposed that $\mathrm{d}(\mathbf{x}_i)$ can be written as

$$\mathrm{d}(\mathbf{x}_i) = \sum_{g=1}^{G} \pi_g \mathrm{d}_g(\mathbf{x}_i), \tag{39}$$

where π_g is the proportion of cases in the gth component distribution ($g = 1, \ldots, G$) and d_g represents the density for the gth class. Because π_g represents the proportion that the gth component distribution contributes to the total distribution, it must be true that

$$0 \leq \pi_g \leq 1$$

and

$$\sum_{g=1}^{G} \pi_g = 1.$$

Thus, $\mathbf{X}$ is a mixture of G distributions, where it is generally unknown which individual data vector (i.e., $\mathbf{x}_i$) belongs to which component distribution. Note that G is considered fixed here, although in many applications the number of components is inferred from the data itself (McLachlan & Peel, 2001).

In applications of the FMM, the component densities are specified as belonging to a particular type of distribution. The probability density function of $\mathrm{d}(\mathbf{x}_i)$ can thus be specified in terms of the necessary parameters of the particular type of distribution as

$$\mathrm{d}(\mathbf{x}_i; \boldsymbol{\Psi}) = \sum_{g=1}^{G} \pi_g \mathrm{d}_g(\mathbf{x}_i; \boldsymbol{\Theta}), \tag{40}$$

where $\boldsymbol{\Psi}$ is a vector containing all of the parameters in the mixture model and can be written as $\boldsymbol{\Psi} = [\pi_1, \ldots, \pi_G, \boldsymbol{\Theta}]'$, where $\boldsymbol{\Theta} = [\boldsymbol{\theta}_1', \ldots, \boldsymbol{\theta}_G']$, and is the vector of unknown parameters necessary to identify each of the G component distributions. When the component distributions are presumed to belong to the same parametric family, then Equation 40 can be reduced to

$$\mathrm{d}(\mathbf{x}_i; \boldsymbol{\Psi}) = \sum_{g=1}^{G} \pi_g \mathrm{d}(\mathbf{x}_i; \boldsymbol{\Theta}). \tag{41}$$

Notice that in Equation 41 no subscript is needed for the probability density function of the gth component, as all of the components are presumed to follow the same distributional form.

Oftentimes the component distributions are hypothesized to each be multivariate normally distributed. In such a case $\boldsymbol{\Psi}$ can be written as $\boldsymbol{\Psi} = \left[\pi_1, \ldots, \pi_G, \boldsymbol{\mu}_1', \ldots, \boldsymbol{\mu}_G', \text{vech}(\boldsymbol{\Sigma})_1', \ldots, \text{vech}(\boldsymbol{\Sigma})_G'\right]'$, where $\boldsymbol{\mu}_g$ is the P length mean vector of the gth component distribution, $\boldsymbol{\Sigma}_g$ is the covariance matrix of the gth component, and vech$(\cdot)$ is a function that stacks the $p(p+1)/2$ main diagonal and lower triangular elements of a symmetric matrix. Thus, Equation 41 can be rewritten as

$$\text{d}(\mathbf{x}_i; \boldsymbol{\Psi}) = \sum_{g=1}^{G} \pi_g \phi_p(\mathbf{x}_i; \boldsymbol{\mu}_g, \boldsymbol{\Sigma}_g), \tag{42}$$

where ϕ represents a P-variate normal density function, with the univariate normal density function being a special case. Because the right hand side of Equation 42 contains the multivariate normal density function multiplied by π_g, it follows that

$$\pi_g \phi_p(\mathbf{x}_i; \boldsymbol{\mu}_g, \boldsymbol{\Sigma}_g) = \pi_g \left[\frac{1}{(2\pi)^{\frac{p}{2}} |\boldsymbol{\Sigma}_g|^{\frac{1}{2}}} \exp\left(-\frac{1}{2} (\mathbf{x}_i - \boldsymbol{\mu}_g)' \boldsymbol{\Sigma}_g^{-1} (\mathbf{x}_i - \boldsymbol{\mu}_g) \right) \right]. \tag{43}$$

Often what is of interest is the posterior probability that a certain entity belongs to class g, given $\mathbf{x}_i$ and $\boldsymbol{\Psi}$. The probability that the entity having observations $\mathbf{x}_i$ belongs to the gth class given the parameters of the classes can be obtained as follows:

$$\tau_g(\mathbf{x}_i; \boldsymbol{\Psi}) = \text{Prob}(i\text{th entity} \in \text{Class}_g | \mathbf{x}_i; \boldsymbol{\Psi}) \tag{44a}$$

$$= \frac{\pi_g \text{d}_g(\mathbf{x}_i; \boldsymbol{\Psi})}{\sum_{g=1}^{G} \pi_g \text{d}_g(\mathbf{x}_i; \boldsymbol{\Psi})}, \tag{44b}$$

where $\tau_g(\mathbf{x}_i; \boldsymbol{\Psi})$ is the probability that the ith entity belongs to class g given the data vector $\mathbf{x}_i$ and parameter vector $\boldsymbol{\Psi}$. When it is assumed that each of the component distributions is normally distributed, Equation 44b can be rewritten as

$$\tau_{ig} = \frac{\pi_g \phi_p(\mathbf{x}_i; \mathbf{\Psi})}{\sum_{g=1}^{G} \pi_g \phi_p(\mathbf{x}_i; \mathbf{\Psi})}, \tag{45}$$

where $\tau_g(\mathbf{x}_i; \mathbf{\Psi})$ will be written as τ_{ig} for simplicity.

In general there are two reasons to make use of FMMs: (a) to identify distinct classes of individuals in a heterogeneous population and (b) to approximate a particular distribution with multiple component distributions. The applied researcher is almost always concerned with the first purpose, specifically to identify homogeneous classes of individuals within a heterogeneous population. The following subsection outlines some of the important aspects of identifying such homogeneous classes and the classification of individuals in those classes.

Classification Based on Mixtures

The issue of classification arises when a desire exists to separate a set of observations into G nonoverlapping classes. It has been said that classification is a fundamental process of science necessary in our quest for understanding nature (Hampel, 2002, p. 7; Sokal, 1977, p. l). McLachlan and Basford (1988) approach the issue of clustering from a model based perspective. The model based approach to clustering implies a statistical model exists for the means, covariances, and probability density function, for each of the G component distributions. The FMM can be used for classification purposes by making use of the posterior probability of class membership. Specifically, the finite mixture approach to classification assigns the entity with data matrix $\mathbf{x}_i$ to the class where the posterior probability of membership, $\hat{\tau}_{ig}$, is highest (McLachlan & Basford, 1988). That is, each entity is assigned to the class where they are most likely to belong. Such an allocation method for discrimination and classification is the optimal rule in terms of the overall error rate (McLachlan & Basford, 1988, pp. 11, 45-46) and is sometimes called the Bayes rule (Anderson, 1984, chapter 6).[22]

The effectiveness of the FMM approach to classification, where assignment to class is based on the highest posterior probability and is the optimal method of classification, is an important concept to understand. When the FMM is carried out and the maximum $\hat{\tau}$ for each of the individuals is close to the upper limit of unity, "it suggests that the mixture approach can cluster the sample at hand into G distinct groups with a high degree of certainty" (McLachlan & Basford, 1988, p. 123, original notation changed to reflect present system). However, if the maximum $\hat{\tau}$ for each of the individuals is not very large, relatively speaking, and is thus much less than unity, "it indicates that the components of the fitted mixture model are too close together for the sample to be clustered with any [degree of] certainty" (McLachlan & Basford, 1988, p. 123). Thus, depending on how far apart the G P dimensional distributions are in multidimensional space, the accuracy of the classifications will vary.

The present discussion of the FMM is a prelude to its incorporation, along with applications of MLMs with nonlinear change curves (each of which have been previously presented), into a general method of modeling change in heterogeneous populations. Because the methods to be proposed in future sections combines several distinct areas of applied statistics, it was important for each of the methods to be adequately addressed in order for the work to be coherent and comprehensive.

[22]It should be noted that other "clustering" procedures exist, such as K-means and hierarchical clustering. However, these and other commonly used methods of classification are not model based, but rather are based on a certain set of "algorithms and criteria in the belief that intuitively reasonable criteria should produce good results over a wide range of possible [yet unstated] models" (Hawkins, Muller, & ten Krooden, 1982, p. 353). From this viewpoint Hawkins et al. (1982) emphasize their support for the FMM approach to classification.

THE GROWTH MIXTURE MODEL

Sixteen years ago Bengt Muthén gave a presidential address to the Psychometric Society where he emphasized the importance of explicitly considering population heterogeneity in latent variable models (Muthén, 1989). The address argued that "data are frequently analyzed as if they were obtained from a single population" and that "it is often unlikely that all individuals in our sample have the same set of parameter values" (p. 558). Muthén was able to show that in certain situations when heterogeneity was ignored in the context of latent variable modeling that "strongly distorted results [could potentially] occur" (p. 557). Since this time Muthén (Muthén, 2001a; Muthén, 2002; Muthén, 2001b) and Muthén and colleagues (Muthén & Shedden, 1999; Muthén & Muthén, 2000) have developed a general latent variable modeling framework, where a unified approach to structural equation modeling is taken that combines continuous and categorical observed variables with continuous and categorical latent variables. Special cases of this general latent variable model are confirmatory factor analysis, structural equation modeling, latent class analysis, latent transition analysis, finite mixture modeling, factor mixture models, GMM, and mixture structural equation modeling to name a few special cases. Most importantly, however, the general latent variable modeling framework opens up the possibility of combining continuous and categorical data analytic procedures into new procedures that are capable of modeling both continuous and categorical latent variables, thus allowing statistical models to be carried out that have previously been impossible or that had not yet been developed.[23]

[23]It is important to note that while Muthén has been developing the general latent variable modeling framework, he has also been involved in the development of software that will carry out his general

Of particular interest to the present work is Muthén's GMM, which is a special case of the general latent variable modeling framework. Recall from the previous discussion regarding the MLM and the LGC frameworks that individual differences in change are captured by unique effects. These unique effects are continuous latent variables generally assumed to follow some multivariate normal distribution. When quantitative differences in parameters of change exist among individuals, categorical latent variables can be included, so as to make the change model more realistic by conditioning unique effects on class membership and the class specific parameters (Muthén, 2001a, p. 14). That is, when heterogeneity in change exists within some population, multiple group models may be used so that individuals within each class are represented by a set of class specific parameters for some functional form of change linear in its parameters, rather than a single set of parameters that are supposed to represent the overall heterogeneous population. This added flexibility translates into group specific models, with parameters potentially more consistent with reality than parameters that represent all quantitatively different groups.

When classification variables are known, specifying such a multiple group factor model is straight-forward (chapter 8 of Jöreskog & Sörbom, 1979; Sörbom, 1974). However, theory may dictate that multiple classes of individuals exist within some population, yet the classes are unobservable latent variables (e.g., see also the applied works of Burchinal & Appelbaum, 1991; Croudace, Jarvelin, Wadsworth, & Jones, 2003; Oxford et al., 2003; Moffitt, 1993; Muthén, Khoo, Francis, & Boscardin, 2002; Schulenberg, O'Malley, Bachman, Wadsworth, & Johnston, 1996b; Tubman, Vicary,

model. The Mplus computer program, of which LISCOMP was a predecessor, is currently the only commercially available program capable of carrying out the full general latent variable modeling framework he proposes. Because the theory of the general latent variable modeling framework and the Mplus program have developed simultaneously, sometimes it is difficult to distinguish the methods proposed by Muthén and his colleagues from the capabilities of the Mplus computer program. In fact, Muthén often refers to the "Mplus framework" rather than the general latent variable model. Because of the simultaneous development, limitations of the general latent variable model framework and limitations of the Mplus software are inextricably linked and cannot be easily be teased apart.

von Eye, & Lerner, 1990). For this reason the GMM was developed to overcome the shortcomings of standard models for longitudinal data when latent classes exist.

The GMM is thus made more realistic by incorporating a probabilistic component of group membership into the model, where group specific parameter estimates are weighted by the likelihood of the individual being a member of the particular unknown latent class (Muthén, 2001a; Muthén & Shedden, 1999). Specifically the GMM provides estimates of class probability, estimates of the value of the fixed effects for each class, and estimates of the variability around the fixed effects within each class (Muthén, 2001a, p. 16). From the estimates of class probabilities, individuals can be, and indeed typically are, classified into the class in which they are most likely to belong.

The GMM can be thought of as a generalization of the FMM applied to factor models. The GMM can also be thought of as a generalization of the LGC applied to populations with latent classes. Muthén's GMM is literally a combination of the features from the LGC and the FMM. This generalized model is very powerful and flexible, and is itself a special case of Muthén's general latent variable model. Although the general latent variable framework is a generalization of the LISREL model and its system of equations, the general latent variable model is largely based on the same set of assumptions. Perhaps the biggest limitation of the general latent variable model framework is that the system of equations are limited to linear models and generalized linear models (such as those discussed by McCullagh & Nelder, 1989).

Muthén describes the generalized latent variable model in terms of four frameworks (Muthén, 2002). Framework A is the continuous latent variable framework and is consistent with mainstream SEM. This framework is analogous to the LISREL model and is thus based on a set of linear equations. Thus, special cases of Framework A are the factor analysis model, LGC, and the general linear MLM. Framework A uses continuous observed variables as indicators and predictors of continuous latent variables, and allows for multiple group models when group status is known.

Framework B uses observed categorical indicator variables and continuous predictor variables to model latent (ordered or unordered) categorical variables. The latent categorical variables represent latent class membership and require the assumption of conditional independence. Conditional independence—also known as the axiom of local independence—holds when responses are independent of one another conditional on class membership and predictor variables (Goodman, 2002; Lazarfeld & Henry, 1968). Thus, Framework B is utilized when observed and latent variables are categorical in nature (possibly with continuous predictor variables of the categorical latent variables), unlike the case of continuous observed and continuous latent variables characteristic of Framework A. Framework B subsumes latent class analysis (Bartholomew, 1987; Clogg, 1995; Goodman, 1974), latent class analysis with predictors (Bandeen-Roche, Miglioretti, Zeger, & Rathouz, 1997; Formann, 1992; Heijden, Dressens, & Bockenholt, 1996), latent transition analysis (Graham, Collins, Wugalter, Chung, & Hansen, 1991; Reboussin, Reboussin, Liang, & Anthony, 1998), and logistic regression mixture analysis (Follmann & Lambert, 1989). Framework B is largely based on the multinomial logistic regression model or, when classes are ordered, the ordered polytomous logistic regression model.

Framework C is characterized by adding latent class indicator variables to Framework A. Allowing for latent categorical variables in the standard SEM model is analogous to a multiple group analysis, the difference being that group membership is unobserved and is itself estimated. Framework C differs from Framework B because the latent categorical variable of class membership predicts latent continuous variables in Framework C, whereas the latent categorical variable in Framework B explains the relationship among the observed categorical variables. It is important to realize that Framework C allows a unique set of parameters for each latent class, potentially with variability around the fixed effects, whereas within class constraints are imposed in Framework B because of the conditional independence assumption, which does not allow within-class variability. Special cases of Framework C are the SEM mixture model (Arminger & Stein, 1997; Arminger, Stein, & Wittenberg, 1998; Dolan & van der Maas, 1998), the factor mixture model, the GMM, the FMM, and, when within class variability is not allowed, the latent class growth analysis (Nagin, 1999; Nagin & Tremblay, 2001) .

Framework D represents the generalized latent variable model that subsumes all of the other frameworks as special cases. Such a general conceptualization of latent variable modeling provides a large number of statistical models that can be described as special cases of the generalized latent variable model. Framework D provides the opportunity to carryout complier-average causal effect models (Angrist, Imbens, & Rubin, 1996; Frangakis & Baker, 2001) and allows for the combination of latent class analysis, say on some outside variable, with the GMM (Muthén & Muthén, 2000; Muthén, 2002). Furthermore, this general framework opens up the possibility of studying models that have not previously been considered and allows questions to be addressed that until recently were not feasible. As Muthén states, Framework D is an illustration of the "whole [model] being more than the sum of its parts" (Muthén, 2002, p. 108).

Of particular interest in the present work is Framework C and the GMM. Muthén's GMM is a generalization of the LGC (a discussion of which took place beginning on page 16) and Nagin's latent class growth analysis (a discussion of which begins on page 64). Conceptually these two models of change are combined so that latent classes of individuals, who follow some linear model of change, can be modeled over time where the parameters are conditional on class. That is, each of a finite number of latent classes are assumed to exist and the parameters of the MLM for each supposed class are estimated by weighting the individuals' scores proportional to the probability of the individual being a member of that particular class.[24] These classes, however, must themselves be estimated, because class membership is an unknown latent variable.

Explicit formulas are not generally available for the parameter estimates and standard errors of mixture models (McLachlan & Peel, 2001; Everitt & Hand, 1981). It seems that because no closed form solution exists, a vast literature on estimation methodology for mixture models has developed (Titterington, 1996). Because of the lack of analytic solvability of mixture models, parameter estimates and standard errors are solved via iterative techniques that attempt to find the set of estimates that maximize the likelihood function, most commonly with maximum likelihood using the expectation-maximization (EM) algorithm (Dempster, Laird, & Rubin, 1977). Because the observed data does not generally contain information on class membership, the EM algorithm as applied to mixture problems treats the observed data as incomplete, where class membership is unobserved and considered a missing value (McLachlan & Peel, 2001, section 2.8).

Like most applications of the FMM, at the present time the Mplus computer program uses maximum likelihood estimation to maximize the likelihood function and one of three maximum likelihood approaches to estimate standard errors (Muthén & Muthén, 1998, technical appendix 8; see also McLachlan & Peel, 2001, section 2.15 for information on methods of estimating standard errors in mixture models). The three methods of estimation for standard errors are maximum likelihood, maximum likelihood with robust standard errors, and maximum likelihood with standard errors approximated by first-order derivatives (Muthén & Muthén, 1998, technical appendix 8).

Under the strong law of large numbers (Stuart & Ord, 1994, section 8.46), which states that as $N \rightarrow \infty$, $|N^{-1} \sum_{i=1}^{N} x - \mu_x| = 0$ with extremely high probability, where x is some random variable, maximum likelihood estimates have desirable properties. In particular, under the strong law of large numbers, maximum likelihood estimates are

[24] Although the parameters may be class specific, invariance on any of the parameters can be imposed, such that multiple groups may have one or more parameter estimates constrained to a common value.

consistent ($\hat{\theta} \rightarrow \theta$ as $N \rightarrow \infty$), asymptotically normal (as $N \rightarrow \infty$ the asymptotic distribution of the estimate is normally distributed), and are efficient (the estimate has minimal variance of all asymptotically consistent and asymptotically normal estimators) (Stuart, Ord, & Arnold, 1999, chapters 17 & 18). One fact sometimes left unmentioned in discussions of maximum likelihood estimation, however, is that in order for these optimal properties of maximum likelihood estimators to hold, the statistical model must be correctly specified (White, 1982). Because the likelihood function is itself a function of a probability density function, the observed data are thus assumed to follow whichever distribution was used to calculate the likelihood function (Hogg & Craig, 1978, chapter 6, Stuart et al., 1999, chapter 18). In most applications of maximum likelihood estimation, residuals are assumed to follow a multivariate normal distribution. To the extent that this is not true, the desirable properties of maximum likelihood estimates may be negated by inappropriate parameter estimates (White, 1982).

Limitations of the Growth Mixture Model

Even though the generalized latent variable model is general and flexible in one sense, it is restrictive in another sense. Earlier in this section it was shown that Muthén's GMM is a special case of the generalized latent variable model (Muthén, 2001a, 2001b, 2002). A major limitation of the generalized latent variable model, and thus the GMM, is that it can only be used for models linear in their parameters or that can be made to be linear in their parameters by transformation via a link function in the spirit of generalized linear models (e.g., McCullagh & Nelder, 1989). Because of the necessary reliance on linearity, an entire class of models, namely those nonlinear in their parameters, cannot be fit by the GMM or anywhere within the general latent variable modeling framework. This limitation is detrimental when interest is in understanding a process of change that is assumed to follow some nonlinear functional form in a heterogeneous population.

Because of the requirement that models be linear, or made linear by transformation, the strengths and benefits of using nonlinear models in the behavioral sciences cannot be realized within the GMM context (e.g., Cudeck, 1996; van Geert, 1991; Browne & du Toit, 1991; Burchinal & Appelbaum, 1991). Thus, explicitly acknowledging heterogeneity in the population by allowing latent classes to be estimated with class specific fixed effects is not possible with the GMM when the desire is to describe change with a model nonlinear in its parameters.

To help solidify current limitations of the GMM, suppose that some phenomenon of interest is hypothesized to follow a particular nonlinear functional form of change. Further suppose that theory states the population of interest is heterogeneous with G latent classes, where each class has a unique set of unknown parameter values with variability around the unknown class specific parameters. Given the present state of development of the GMM, it is currently not possible to fit such a model. Although the question seems to describe a situation where the GMM would be appropriate, the GMM is of no use because the change curve of interest is nonlinear. As will be delineated momentarily (beginning on page 70), Pauler and Laird's fully Bayesian mixture model for nonlinear MLMs provides an option for answering such a research question. However, because the results depend in part on a multiparameter prior distribution specified by the researcher, and is thus not fully objective, the fully Bayesian model of heterogeneous change is potentially troubling and the LCDC model proposes a fundamentally different method for addressing what is essentially the same question.

The benefits of using nonlinear models for describing individual trajectories in the behavioral sciences have been illustrated (Browne & du Toit, 1991; Cudeck, 1996; van Geert, 1991). The use of mixture modeling technology has been set forth as a potentially powerful technique that should be considered by researchers if heterogeneous populations are hypothesized (McLachlan & Peel, 2001; Muthén, 2002; Yung, 1997). However, the use of nonlinear models in the context of heterogeneous populations has not often been considered (cf. Pauler & Laird, 2000). Because these two active research areas are seemingly thought of as being mutually exclusive within the behaviorial sciences, the major goal of the present work is to combine these two topics into a unified model of change that allows heterogeneous populations to be modeled following some nonlinear change curve with a different set of unknown class specific parameter values.

ALTERNATIVES AND PREDECESSORS TO THE GROWTH MIXTURE MODEL

Although the GMM has recently received a considerable amount of attention in the methodological literature of the behavioral science, other models with similar rationales exist. In particular, the latent class growth model, the grade of membership model, the cluster analytic model, and the fully Bayesian finite mixture model of nonlinear growth, all offer methods for modeling change in heterogeneous populations. Each of these models will be delineated in a model specific section, where similarities and differences as compared to the GMM will be described.

Latent Class Growth Model

Nagin and colleagues (D'Unger, Land, McCall, & Nagin, 1998; Jones, Nagin, & Roeder, 2001; Land & Nagin, 1996; Nagin, 1999; Nagin & Tremblay, 2001) developed a model of longitudinal change that was meant to "complement two well-established methods for analyzing change" (Jones et al., 2001, p. 374), namely the MLM and the LGC. Nagin's methods are based on latent class models with the intention of "identifying distinctive clusters of individual trajectories within the population" (Nagin, 1999, p. 139) and have been termed latent class growth analysis (e.g., Muthén, 2002, 2001b). Nagin's latent class growth model is, as he describes it, presaged in Rindskopf's (1990) work on nonparametric methods for the analysis of repeated measures of dichotomous response data. The point of Rindskopf's work was to identify distinct groups of dichotomous response sequences across time for some heterogeneous population. The work of Nagin and colleagues, however, was primarily developed to identify distinct groups of individuals over time on some quantitative variable of interest. In this sense Nagin and his colleagues have extended Rindskopf's methods for studying change beyond that of a dichotomous variable to one that allows the estimation of class membership and inferences for the optimal number of classes.

Like Muthén's GMM, Nagin's latent class growth analysis is based on a linear change model that explicitly acknowledges heterogeneity in the population of interest. However, there are fundamental differences between Muthén's GMM and Nagin's latent class growth analysis. The biggest difference is that the GMM allows variability within each class, whereas the latent class growth analysis does not allow for individual variability within class. Thus, unique effects do not exist in the context of latent class growth analysis, whereas functions of these unique effects are of explicit interest in the GMM context.

The latent class growth analysis was developed after MLMs and LGCs became popular for longitudinal data analysis. Although the ideas of the GMM were in place before introduction of the latent class growth analysis (e.g., Muthén, 1989), the GMM was developed after the latent class growth analysis. The GMM combines the idea of individual differences from the MLM and the LGC with the idea of heterogeneous groups from the latent class growth analysis. In this way the GMM is more general and flexible than the MLM and LGC because it allows unknown heterogeneous groups to be modeled, and is more flexible than the latent class growth model because it does not constrain within group variability to zero. Special cases of the GMM are the linear MLM and the LGC, when only one group is specified, and the latent class growth model, when latent classes exist and no variability is allowed within class (Muthén, 2002).

Grade of Membership

Another model that in some ways can be thought of as a competitor to general latent class analytic methods is the grade of membership model. The grade of membership model attempts to grade individuals on their likeness to various membership classes. The grade of membership model is based on fuzzy set theory, where a “fuzzy set” is one not assumed to be a “crisp set.” A crisp set adheres to the condition of all or none, meaning that an entity either belongs to some set or that the entity does not belong to the set. A fuzzy set is one where an entity potentially belongs to multiple sets with varying degrees of membership in each of the sets (Manton, Woodbury, & Tolley, 1994).

In the grade of membership context, fuzzy set theory extends the range of a class, group, or condition indicator variable from the set {0,1} to the interval [0,1] (Manton, Woodbury, Stallard, & Corder, 1992). That is, rather than making a dichotomous decision of whether an entity is or is not contained in a particular set or class, a value between zero and unity is estimated that represents the grade of the individuals’ membership in each of the classes. Across all classes for an individual the sum of the grades is unity. However, it is important to realize that the grade of membership (some value $\in [0, 1]$) is not a probability. Whereas a probability refers to the relative likelihood of some event being realized, grade of membership refers to the degree of partial inclusion (Woodbury, Clive, & Arther Garson, 1978). To solidify this potentially confusing point, Woodbury et al. provide the following example: “an hermaphrodite may have characteristics which, upon observation, imply a grade of membership of 0.75 in the set of all females, and a grade of membership of 0.25 in the set of all males. However, it is meaningless, again upon observation, to speak of this individual’s probability of being female or male” (1978, pp. 280–281). In a clinical context, the grade of membership can be used to quantify an individual’s condition into each of several classes or conditions and can also be used to define disease type or stages of a disease (Corder et al., 2000).

The grade of membership model makes use of discrete (Woodbury et al., 1978; Manton et al., 1992, p. 277) or categorical responses data (Manton et al., 1994, pp. 1–2).[25] Even though under some data structures the grade of membership model can be extended from cross sectional research to encompass longitudinal and event history data (Manton et al., 1994, chapter 5), the grade of membership model will not be further considered. Justification for this comes from the fact that continuous data is what is generally collected and of interest in studies of longitudinal research in the behavioral sciences. Furthermore, interest does not generally lie in the grade of membership an individual receives because of having characteristics from multiple classes, but rather in the probability of belonging to a particular class.

Cluster Analytic

Dumenci and Windle (2001) discuss the use of cluster analysis to identify distinct subpopulations in a presumed heterogeneous sample (these subpopulations are the classes that Muthén and his colleagues and Nagin and his colleagues discuss in their respective work). Noting the importance of accounting for interindividual differences in change, Dumenci and Windle acknowledge the increased interest in the substantive literature on subtype patterns of change in heterogeneous populations. The concerns of interindividual differences in change and the existence of distinct classes of individuals closely parallel those commonly given in discussions of the GMM. Thus, the GMM and the cluster

[25]Manton et al. state the for continuous response data to be used, the data must be recoded into discrete intervals in order to generate a categorical variable. They go on to state that "it can be shown that, for most standard types of continuous random variables, the number of categories L_j [i.e., number of response categories] needed to represent a given variable j [i.e., some continuous variable] seldom needs to be larger than 15" (1994, p. 2). Manton et al. (1994) cite Terrell and Scott (1985) in their rationale of artificially categorizing a continuous variable. However, Terrell and Scott (1985) do not discuss artificial categorizing continuous data. Rather they discuss the optimal number of bins to use in histogram formation using continuous data. They show that under one set of conditions $(2n)^{1/3}$ is the optimal number of bins, whereas under another set of conditions $(\frac{147n}{2})^{1/5}$ is the optimal number of bins. There does not seem to be any support in the Terrell and Scott (1985) work to justify artificially categorizing a continuous random variable into 15 categories.

analytic model proposed by Dumenci and Windle are largely meant to address the same set of problems, albeit they do so in different ways.

The methods of Dumenci and Windle are similar in spirit to those proposed 18 years before the publication of their work by McCall, Appelbaum, and Hogarty (1973). McCall et al. used cluster analysis in an attempt to "derive clusters that were homogeneous for pattern of IQ within cluster but heterogeneous between clusters" (1973, p. 47). Specifically McCall et al. used four descriptive components of each individual's trajectory (each being based on 17 measurement occasions) by making use of the Eckart-Young procedure (Eckart & Young, 1936). These four components were then submitted to a cluster analysis in an effort to classify individuals into a homogeneous pattern of change. Their results showed that the data exhibited five clusters of similar patterns of change, and one collection of "isolates," where patterns of change for these individuals were atypical. These clusters were then compared in a multivariate analysis of variance context to see whether differences in orthogonal polynomial trends (e.g., linear, quadratic, cubic, and quartic) existed. The conclusion reached was that "the clustering did produce groups having significantly different patterns of IQ change over age" (McCall et al., 1973, p. 48).

Along the same lines as McCall et al. (1973), Dumenci and Windle state, given class membership is unknown yet heterogeneity is hypothesized to exist, "classification of growth trajectories into a small number of homogeneous groups (e.g., increasing, decreasing) would be desirable" (2001, p. 503). In large part, the idea of classifying individuals into homogeneous groups was the goal of McCall et al. (1973), the GMM, the latent class growth model, and to some degree the grade of membership model. A major difference between the cluster analytic model of Dumenci and Windle (2001), McCall et al. (1973), and Pauler and Larid's (2000) model (discussed momentarily) is that these three methods are based on crisp sets, whereas the GMM, the latent class growth model, and to some extent the grade of membership model are based on fuzzy sets. Thus, inconsistency in methods of classification among models of change that attempt to identify latent homogeneous classes in a heterogeneous population exists, because three models discussed classify individuals into crisp sets while three other models discussed classify individuals into fuzzy sets.[26]

A difference between the Dumenci and Windle cluster analytic approach and the approaches previously outlined, is that the cluster analytic approach does not impose the number of latent classes a priori, whereas the previous methods are all conditional on the number of classes specified. The cluster analytic method proceeds by first defining similarity/association measures (e.g., Mahalanobis distance, Pearson correlation coefficient, Euclidean distance, etc.). Because a multiple number of cluster solutions can be obtained for each similarity measure and clustering technique selected, Dumenci and Windle suggest that "the solution that satisfies certain theoretical expectations is then selected to represent the heterogeneity among members of the study population" (2001, p. 504). Thus, Dumenci and Windle's (2001) cluster analytic approach provides a somewhat arbitrary solution because the selection criteria offered by Dumenci and Windle are not objective; the researcher literally picks and chooses, from a potentially large set, the model that most satisfies his or her theory. Although they go on to state that replication and external validity studies are required to support cluster types (p. 504), for any given study that makes use of the cluster analytic method, to some extent the researcher selects the set of results that "satisfy certain theoretical expectations" and thus is most consistent with the researcher's theoretical framework or ideology. In this way the procedure is analogous to an exploratory factor analysis rather than an confirmatory factor analysis,

[26]The terms "crisp set" and "fuzzy set" are used more liberally than Manton et al. (1994) use the terms. Whereas Manton et al. did not associate sets with probabilistic values, rather they associated them with grades of membership, the present work uses the terms to denote whether parameter estimates are based on probabilistic crisp sets (where an individual contributes to the parameters of one and only one class) or fuzzy sets (where an individual potentially contributes to the parameters of multiple classes).

which would be the analog of all of the other methods for modeling changed discussed in the present work. The model selection criteria of Dumenci and Windle (2001) do not conform to any sort of severe test (Mayo, 1996), nor are they in any way a test of a highly precise model (Bamber & van Santen, 2000).

The main impetus for the Dumenci and Windle work was to bring attention to the fact that "estimation of population-level change parameters [in a MLM or LGC context] does not necessarily capture subgroup heterogeneity that may reflect qualitatively distinct subtype patterns or subgroup heterogeneity" (p. 502), and to evaluate the effectiveness of the cluster analytic approach to class identification in a longitudinal context via Monte Carlo simulation techniques. The goal of their simulation work was to investigate the adequacy of the cluster analytic method in terms of correctly separating mixtures of homogeneous trajectories of change in a heterogeneous population as a function of trajectory shape, reliability, sample size, and measures of similarity. An example of what Dumenci and Windle were trying to determine (taken from a condition they simulated) was as follows: suppose that 2,000 trajectories were generated, half of which followed a linear functional form and the other half followed a quadratic functional form of change, for a two cluster solution how effective is cluster analysis at classifying the trajectories into the appropriate class? The results of the simulation study were largely inconclusive and further research was suggested.

Pauler and Laird's Fully Bayesian Mixture Model for Nonlinear Multilevel Models

In the context of pharmacokinetics compliance/noncompliance research, where interest was on the identification of individuals who did or did not follow a self-administered drug regiment, Pauler and Laird (2000) introduced a fully Bayesian mixture model for nonlinear MLMs. Pauler and Laird's model "allows estimates of component membership probabilities and random effect distributions for longitudinal data arising from multiple subpopulations" (2000, p. 464). Pauler and Laird's model is conceptually similar to Muthén's GMM, however Pauler and Laird's model was developed to deal with nonlinear relationships that occur over time and that are to be modeled by a nonlinear change model.

Besides one being for linear and the other nonlinear models, a fundamental difference between Pauler and Laird's model and Muthén's model exists. Pauler and Laird's model is based on a fully Bayesian methodology, whereas Muthen's model is consistent with the frequentist approach to statistical inference (for a discussion of the differences between the Bayesian and frequentist schools of thought from a statistical standpoint, see Pawitan, 2001; for a discussion from a philosophy of science point of view, see Mayo, 1996). Pauler and Laird's model requires one to specify, a priori, properties of the parameters based on a priori or "subjective probabilities."[27] As Howard, Maxwell, and Fleming explain, "the heart of Bayesian approaches lies in researchers precisely stating their beliefs (called *a priori* probabilities) about the outcome of the study prior to conducting the study" (2000, p. 321). These priors are accompanied by measures of variability that together define a prior distribution that measures the researcher's degree of belief about the parameter values. After collecting data, the researcher combines information from the specified prior distribution with the observed data in order to create a posterior distribution. This posterior distribution is thus a function of the researchers beliefs and empirical evidence, and is what inferential decisions (e.g., statistical significance test, confidence intervals, etc.) are based.

[27] Although, theoretically speaking, it should be possible to specify a noninformative prior distribution (essentially a prior distribution with some mean and infinite variance) for Pauler and Laird's model, Pauler and Laird do not discuss such a possibility in their article. Although they do mention the possibility of having "reasonably vague priors" (p. 467), it may be the case that not specifying a prior distribution for such a rich model leads to convergence problems. In responding to a question regarding whether or not a prior distribution had to be specified, Donna Pauler Ankerst stated that "prior distributions are necessary to tighten some components of the model (flat priors [i.e., noninformative prior distribution] or a ML [maximum likelihood] approach would not converge)" (personal communication, March 22, 2004). Thus, at the present time, it does not seem as though Pauler and Laird's model has been formally evaluated, or even developed, in the context of a noninformative prior distribution.

An argument set forth by the Bayesian school of thought is that the frequentist interpretation of a probability value is confusing and that it represents an uninteresting hypothesis. In main stream statistics, which is based on the frequentist approach, a probability value for a test of statistical significance is conditional on the null hypothesis being true. Specifically, a probability value in the context of a test of statistical significance can be interpreted as follows: if the null hypothesis is true, the probability of obtaining data this extreme or more extreme is **P**, where **P** is some probability value. In the Bayesian conceptualization of probability, the null hypothesis is conditional on the observed data, which fundamentally alters the meaning of a frequentist probability value. Thus, a Bayesian probability value in the context of a test of statistical significance can be interpreted as follows: given the observed data, the probability of the hypothesis is **P**. As Berger and Berry discuss, "nothing is free" and "the elegantly simple Bayesian conclusion requires additional input," because in order to obtain the probability of the hypothesis given the observed data, a prior distribution must be specified (1988, p. 162). Although less elegant, it is more accurate to state the meaning of a Bayesian probability as: given the prior distribution specified and the observed data, the probability of the hypothesis is **P**. This is more correct because, in addition to the model and observed data, the posterior probability is a function of the prior distribution specified.

Because of the subjectivity inherent in defining a prior distribution, serious problems persist within the Bayesian framework. Because "strict objectivity is one of the crucial factors separating scientific thinking from wishful thinking" (Efron, 1986, p. 3), alternatives to Bayesian methods are important. As Mayo shows, "the Bayesian rule of support is unreliable in the sense that it allows support to accrue to hypotheses, with high probability, even if the hypotheses are false" (1996). Obviously, supporting false hypotheses, especially with "high probability," is not desirable, yet false hypotheses potentially receive a great deal of support as a function of the prior distribution. Another fundamental difference between the frequentist and the Bayesian conceptualizations of statistical inference is that frequentists regard a parameter as a fixed constant, whereas Bayesians regard a parameter as having a distribution with some mean and some variance. Such a distinction illustrates why the two approaches to statistical inference are so fundamentally different.

As the prior distribution changes from less certain to more certain, the posterior distribution relies less and less on the observed data. In the extreme case where a researcher is “100 percent certain” that his or her beliefs are consistent with characteristics of the population, the observed data—regardless of sample size—is given zero weight. Conversely, if a researcher is “zero percent certain” that his or her beliefs are consistent with characteristics of the population, the prior specifications are given zero weight. When no weight is given to prior specifications, the priors are said to be noninformative. Although there is a wide variety of noninformative priors, often the choice suggested is the one that matches the corresponding frequentist result (Datta & Ghosh, 1995, p. 1357). However, specifying a Bayesian analysis so that the results are equivalent with the frequentist analysis in some ways bypasses the supposed benefits of the Bayesian method so often touted by its supporters. As Mayo argues, “you cannot be just a little bit Bayesian” (1996, chapter 9), and thus for the most part, one needs to adopt either the Bayesian approach or the frequentist approach. However, for those who try to capitalize on the advantages of both methods by using noninformative priors, “in almost all cases of interest [the noninformative prior distribution] is improper, so that it gives infinite ‘probability’ to the whole parameter space” (Dawid, Stone, & Zidek, 1973, p. 189). Improper priors can lead to paradoxical results and results that have “unBayesian” properties in multiparameter problems (Dawid et al., 1973, p. 189).

In terms of parameter estimation and/or predictions, Bayesians oftentimes need to integrate over high-dimensional probability distributions. In particular, Bayesians need to integrate over the posterior distribution of model parameters given the data, whereas frequentists need to integrate over the distribution of observables given the parameters (Gilks, Richardson, & Spiegelhalter, 1996, p. 1). Markov chain Monte Carlo methods are used in order to estimate an integral (i.e., parameter expectation) by way of simulating Markov chains. Monte Carlo integration samples, from a posterior distribution, in order to approximate parameter expectations. Rather than obtaining analytic derivations for parameter estimates, or obtaining a likelihood function that needs to be maximized iteratively in order to obtain optimal parameter estimates, given the model, prior distribution, and observed data, Monte Carlo methods produce samples from a posterior distribution and arrive at estimates of the parameters of the posterior distribution (Jackman, 2000). Even though the method is not optimal in the sense of having an elegant analytic solution, Markov chain Monte Carlo methods are effective because they lead to solutions for parameters of the posterior distribution that may otherwise not be estimable. Because of the heuristic power of Markov chain Monte Carlo methods, models in the Bayesian framework once considered "too hard" are now potentially doable by using Markov chain Monte Carlo methods (Jackman, 2000, p. 375).[28] Pauler and Laird's mixture model for nonlinear MLMs uses Markov chain Monte Carlo methods for obtaining samples from the posterior distribution of parameters and is composed of two steps: (a) sampling parameters in component densities conditional on individual memberships and (b) sampling indicators of class membership for each individual in order to determine membership (2000, p. 468). The first step estimates class

[28]Most applications of Markov chain Monte Carlo methods rely on the work of Metropolis, Rosenbluth, Rosenbluth, Teller, and Teller (1953) and Hastings (1970). After Hastings (1970) generalized the work of Metropolis et al. (1953), the general procedure for Markov chain Monte Carlo methods became known as the Metropolis-Hastings algorithm. The Gibbs sampler (Geman & Geman, 1984) is another algorithm commonly used to carry out Markov chain Monte Carlo estimation. It can be shown that the Gibbs sampler is a special case of the Metropolis-Hastings algorithm (Gilks et al., 1996).

specific fixed effects based only on the unique effects for the individuals currently assigned to the particular class. Thus, estimates of within class fixed effects are based only on individuals most likely to belong to the particular class. This is in contrast to Muthén's GMM—and indeed most applications of mixture modeling—as all individuals contribute to the fixed effects of all groups, with the amount of contribution being conditional on their likelihood of belonging to the particular class. Because Pauler and Laird's model does not require the functional form of change to be constant across class, the second step of the procedure requires the reversible jump procedure (Green, 1995). The reversible jump procedure is used for constructing Markov chains that "jump" between parameter subspaces of differing dimensionality (Green, 1995, p. 711). Although not stated in their work, it seems to be the case that fundamentally different models (i.e., when one model is not a special case of another model) would not be estimable if all individuals contributed to all classes. From this standpoint it seems to make sense that individuals only contribute to the parameter estimates from the class that they are judged most likely to belong.

In Pauler and Laird's model "proper subjective priors are necessary for all population parameters in the component densities" (p. 467). Because it is often difficult to accurately estimate a single parameter a priori, accurately specifying multiple parameters a priori seems especially difficult. Oftentimes novel studies are conducted in an analysis of change framework with the intent of estimating a set of unknown population parameters. In such situations there may be no previous information to rely on when specifying the prior distribution. Because specifying a multiparameter prior distribution is generally so difficult, yet the idea of allowing multiple latent classes to each be modeled by a different set of class specific parameters from some nonlinear change model so appealing, the major purpose of the present work is to develop a general nonlinear change model, where latent classes each have their own unique set of class specific parameters and where a prior distribution is not specified. Although the conceptual framework for such a model is straightforward, developing methods of estimation is quite difficult. In the upcoming sections the LCDC model is formally developed, five methods of estimation are proposed and then evaluated, and recommendations for carrying out the LCDC model are given.

THE LATENT CLASSIFICATION DIFFERENTIAL CHANGE MODEL

Because of the deficiency of statistical methods currently available for modeling longitudinal data when interest lies in latent classes that each have their own unique set of parameters from a nonlinear change model, a new model of longitudinal data analysis is proposed. The proposed model will be denoted the latent classification differential change (LCDC) model. The model is a *latent classification* model because unknown classes of individuals are assumed to exist within a heterogeneous population. It is a *differential change* model because it is assumed that the different classes of individuals follow a unique set of unknown parameter values and thus change differentially over time. The model explicitly acknowledges the fact that different classes of individuals potentially have their own unique set of parameter values and allows for within class variability around the class specific fixed effects.

Conceptually the LCDC model is an extension of the GMM. The LCDC model is an extension of the GMM because the LCDC model allows nonlinear change models to be carried out in heterogeneous populations when class membership is unknown. Rather than being an extension or a special case of Pauler and Laird's Fully Bayesian Mixture Model for Nonlinear MLMs, per se, the LCDC model is a competitor of Pauler and Laird's model. Estimation of the two models is fundamentally different, with Pauler and Laird's model being developed within a fully Bayesian framework necessarily requiring a priori parameter specification and the use of Markov chain Monte Carlo methods to obtain estimates. Although the two models address a conceptually similar question, they do so in very different ways.

The LCDC model is predicated on a heterogeneous population, where the parameters of change differ across latent classes. These interindividual differences in change are in part a function of class membership and in part a function of individual uniqueness. In this sense, the LCDC fulfills the ideals of integrating the idiographic and nomothetic conceptualizations of behavior into a single unified model. The core LCDC model is nonlinear in its parameters, where a special case is a model linear in its parameters. An assumption of the LCDC model is that a finite number of classes exist, each with a unique set of parameter values. Although the general model allows a unique class specific value for each model parameter, parameter invariance can be imposed for selected parameters across some or all of the classes (with some of the methods of estimation). Conceptually, the LCDC is a combination of the FMM and the nonlinear MLM. In particular, the theory of the LCDC combines both methodologies so that a finite number of classes can each be represented by a nonlinear MLM with unique class specific parameters.

The LCDC model assumes that individuals within a class have trajectories of change that are relatively homogeneous, whereas trajectories of change across class are relatively heterogeneous. Of course, because the LCDC assumes that interindividual differences in change are a function of both class membership and individual uniqueness, the degree of within group homogeneity is relative to the across group heterogeneity. This is the case because unique effects are assumed to exist around the fixed effects within each class, while different parameter values are allowed across class. At the present time the optimal method of estimation is not clear. Following the formal definition of the LCDC model, five estimation procedures are proposed as reasonable ways to obtain parameters estimates of the LCDC model. The accuracy of the methods is then explored via a comprehensive Monte Carlo simulation study.

Defining the Latent Classification Differential Change Model

Let y_{it} be the outcome variable of the ith individual ($i = 1, \ldots, N$) at the tth measurement occasion ($t = 1, \ldots, T_i$), a_{it} be the value of some nonstochastic time dependent basis at the tth measurement occasion for the ith individual, and x_{im} be the mth time invariant predictor variable ($m = 1, \ldots, M$) for the ith individual. Further let $\mathbf{y}_i = [y_{i1}, \ldots, y_{iT_i}]'$ be a vector of length T_i of the observed scores for the ith individual, $\mathbf{a}_i = [a_{i1}, \ldots, a_{iT_i}]'$ be the vector of length T_i of the time dependent basis for the ith individual, $\mathbf{x}_i = [x_{i1}, \ldots, x_{iM}]'$ be the vector of length M of time invariant predictor variables for the ith individual, and $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_N]'$ be the N by M matrix of predictor variables for all individuals. The functional form governing the trajectory of change for all individuals is denoted $f(\mathbf{a})$, where $\mathbf{a}$ is a generic representation of the time dependent basis with the function $f(\cdot)$ having P parameters. Given $f(\mathbf{a})$, let $\boldsymbol{\theta}_i = \left[\theta_{i1}, \ldots, \theta_{ip}\right]'$ be a P length vector of true change coefficients defining the trajectory of interest for the ith individual and $\boldsymbol{\Theta} = [\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_N]'$ be the N by P matrix of individual change coefficients. The vector of length N of ones and zeros identifying which of the G classes from a crisp set the ith individual is a member is denoted $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_N]'$.

Given $f(\mathbf{a})$, $\boldsymbol{\Theta}$, and $\boldsymbol{\gamma}$, let $f(\mathbf{a}_g)$ be a special case of $f(\mathbf{a})$ for the gth class with π_g $\left(\sum_{g=1}^{G} \pi_g = 1\right)$ being the proportion of the population who are members of the gth class and n_g $\left(\sum_{g=1}^{G} n_g = N\right)$ being the number of individuals in the sample who are members of the gth class. The functional form of $f(\mathbf{a}_g)$ is thus equal to or a special case of $f(\mathbf{a})$, however the P parameters of change in $f(\mathbf{a}_g)$ are specific to the gth class. Conditional on $\boldsymbol{\theta}_i$, γ_i, and $\boldsymbol{\mu}_g$, where $\boldsymbol{\mu}_g = [\mu_{1g}, \ldots, \mu_{Pg}]'$ is the gth class specific population mean vector for the P fixed effect parameters of change, $\mathrm{E}[\boldsymbol{\upsilon}_i] = \mathbf{0}$ with variance $\boldsymbol{\Sigma}_g$ and has a P dimensional multivariate normal distribution, where $\boldsymbol{\upsilon}_i = [\upsilon_{i1}, \ldots, \upsilon_{iP}]'$ are the individual specific unique effects defined as $\upsilon_{ip} = \theta_{ip} - (\mu_{gp}|\gamma_i)$.

The LCDC model implies that the coefficients of change for the individuals conform to the following probability density function:

$$\mathrm{d}(\boldsymbol{\theta}_i, \mathbf{x}_i; \boldsymbol{\Psi}) = \sum_{g=1}^{G} \pi_g \phi_P(\boldsymbol{\theta}_i, \mathbf{x}_i; \boldsymbol{\mu}_g, \boldsymbol{\Sigma}_g), \tag{46}$$

where $\boldsymbol{\theta}_i$ and $\mathbf{x}_i$ are taken together, ϕ_P represents a P dimensional multivariate normal probability density function, and $\boldsymbol{\Psi} = \left[\boldsymbol{\Psi}_1', \ldots, \boldsymbol{\Psi}_G'\right]'$, where $\boldsymbol{\Psi}_g = \left[\pi_g, \boldsymbol{\mu}_g', \mathrm{vech}(\boldsymbol{\Sigma}_g)'\right]'$ that may or may not be saturated. Equation 46, which defines the density of the heterogeneous population of change coefficients, is analogous to the general density of a mixture distribution from Equation 41. As was true with the standard MLM, predictor variables are important in helping to explain interindividual differences in parameters of change.

What is of interest in the LCDC framework is not literally the probability density function of $\boldsymbol{\theta}_i$ given $\mathbf{x}_i$, rather what is of interest are the parameters from a nonlinear MLM with multiple classes of individuals. However, class membership is unknown and must itself be estimated. Thus, the desire is to have a model similar to that given in Equation 4, but rather than having a j subscript that represents a known grouping of individuals, the LCDC model has a g subscript, which implies a latent classification of individuals:

$$\mathbf{y}_{ig} = f(\mathbf{A}_{ig}\boldsymbol{\beta}_g + \mathbf{Z}_{ig}\boldsymbol{\upsilon}_{ig}, \mathbf{x}_{ig}) + \boldsymbol{\varepsilon}_{ig}. \tag{47}$$

Thus, the LCDC model is conditional on the unknown class memberships, which themselves must be estimated. The class identification vector, $\boldsymbol{\gamma}$, can be estimated with the optimal rule as discussed beginning on pages 53 from with a FMM whose probability density is defined by Equation 46. Note the one-to-one relationship between $\boldsymbol{\theta}_i$, the ith individual's change coefficients, in Equation 46 and $\boldsymbol{\upsilon}_{ig}$, the unique effects associated with the ith individual whom is in the gth class, from Equation 47.

Each of the classes in the general LCDC model of Equation 47 potentially has a unique set of fixed effects, $\boldsymbol{\beta}_g$, a unique covariance matrix of errors, $\boldsymbol{\Sigma}_{\varepsilon_g}$, and has a unique covariance matrix for the unique effects, $\boldsymbol{\Sigma}_{\upsilon_g}$. Without imposing constraints on the model parameters, either by fixing parameters to specified values or imposing parameter invariance, there are potentially a large number of parameters that can be estimated. Momentarily supposing each of the individuals is measured at the same T measurement occasions, there are potentially GP fixed effect parameters, $G(T(T+1))2^{-1}$ error covariance parameters, and $G(P(P+1))2^{-1}$ unique covariance parameters. Thus, with no parameter constraints, there are a total of $G[p+(T(T+1)+p(p+1))2^{-1}]$ parameters defining the LCDC model. Of course, in order for the model not to be under-identified, parameter constraints must be imposed. Thus, not all possible parameters can be free to vary, as doing so yields an inestimable model where there are more equations than unknowns (see Loehlin, 1998, for a discussion of these issues in a different, yet related, context).[29] The potential flexibility of the model necessarily implies more parameters than either a standard MLM or a standard FMM. Given the difficulties that often exist for estimation of complicated FMMs and especially for nonlinear MLMs, it is no surprise that estimation of the LCDC model can be difficult. The next section discusses how likelihood estimation theoretically applies to the parameters of the LCDC model.

[29]Of course, oftentimes it is assumed the structure of the error variance is $\sigma^2_\varepsilon \mathbf{I}_T$, where σ^2 is some value for the error variance and $\mathbf{I}_T$ is a T by T identity matrix. If all classes are assumed to share the same error covariance structure, a constant variance across time, and no covariation between the error at the t and t' measurement occasions ($t \neq t'$), the number of parameters of the LCDC model can be reduced by $T[G(T+1)-2]2^{-1}$, which leaves $G[P+P(P+1)]2^{-1}+T$ total parameters.

Parameters of the LCDC Model

Because the LCDC model combines ideas from the FMM with ideas from the general nonlinear MLM, the likelihood function for the LCDC model can be written as

$$\mathcal{L}(\mathbf{Y}, \mathbf{X}; \boldsymbol{\Omega}) = \prod_{i=1}^{N} \mathrm{d}(\mathbf{y}_i, \mathbf{x}_i; \boldsymbol{\Omega}) \tag{48}$$

where $\mathcal{L}$ represent the likelihood function and $\boldsymbol{\Omega}$ is the R length vector of parameter values defining the particular LCDC model. The parameter vector $\boldsymbol{\Omega}$ can be written in a general form:

$$\boldsymbol{\Omega} = \left[\pi_1, \ldots, \pi_{G-1}, \boldsymbol{\beta}_1', \ldots, \boldsymbol{\beta}_G', \mathrm{vech}(\boldsymbol{\Sigma}_{\varepsilon_1})', \ldots, \mathrm{vech}(\boldsymbol{\Sigma}_{\varepsilon_G})', \mathrm{vech}(\boldsymbol{\Sigma}_{\upsilon_1})', \ldots, \mathrm{vech}(\boldsymbol{\Sigma}_{\upsilon_G})'\right]', \tag{49}$$

where a unique $\boldsymbol{\beta}_g$ is the vector of fixed effects for the gth class, $\boldsymbol{\Sigma}_{\varepsilon_g}$ is the covariance matrix of errors for the gth class, and $\boldsymbol{\Sigma}_{\upsilon_g}$ is the covariance matrix for unique effects for the gth class. Appendix A provides a brief overview of maximum likelihood estimation and defines the notation involving likelihood estimation used in the present section.

An obvious question that arises is how the likelihood function of Equation 48 relates to the probability density function defined in Equation 46. The vector of group specific population means contained in $\boldsymbol{\mu}_g$ from Equation 46 is analogous to the fixed effect change parameters contained in $\boldsymbol{\beta}_g'$ from Equation 49. Furthermore, the class specific population covariances contained in $\boldsymbol{\Sigma}_g$ from Equation 46 are analogous to the class specific covariances contained in $\boldsymbol{\Sigma}_{\upsilon_g}$ from Equation 49. Knowing this, Equation 46 can be rewritten so that it is consistent with the notation used for the likelihood function of the LCDC model defined in Equation 48:

$$\mathrm{d}(\boldsymbol{\theta}_i, \mathbf{x}_i; \boldsymbol{\Psi}) = \sum_{g=1}^{G} \pi_g \phi_P(\boldsymbol{\theta}_i, \mathbf{x}_i; \boldsymbol{\beta}_g, \boldsymbol{\Sigma}_{\upsilon_g}). \tag{50}$$

METHODS OF PARAMETER ESTIMATION FOR THE LCDC MODEL

In large samples when all assumptions are met and the model is correctly specified, as previously discussed, parameter estimation by full information maximum likelihood provides many benefits and is generally the optimal method from a statistical point of view. However, for many complicated models, especially those with nonlinear unique effects (Pinheiro & Bates, 2000, section 7.2.1), the likelihood function may be difficult or impossible to write analytically, given the current state of knowledge and the limitations of statistical theory. If the likelihood function cannot be analytically derived, an approximate likelihood function can be used. Rather than maximizing the likelihood function, a reasonable way to proceed is to make use of an approximate likelihood function. As Davidian and Giltinan describe in the context of nonlinear MLMs, "the analytical intractability of likelihood inference has motivated many approaches based on approximations" for nonlinear MLMs (2003, p. 403). Thus, given the current limitations of likelihood estimation and inference, carrying out nonlinear MLMs is almost always based on approximate procedures. As a result, parameter estimates based on approximate likelihood functions are themselves necessarily approximate. Appendix B provides a brief overview of some of the difficulties that must be dealt with in order to estimate the parameters of a nonlinear MLM.

Given the approximate nature of parameter estimates from nonlinear MLMs, the estimation of the parameters of the LCDC model is necessarily constrained. Evidence of this is understandable given that a special case of the LCDC model is a nonlinear MLM when $G = 1$. In such a situation, the "mixture" is actually a single homogeneous population and the LCDC model is reduced to a MLM. Because the present work does not attempt to improve or propose estimation procedures for the general nonlinear MLM, but rather relies on estimation procedures commonly used for carrying out the model, a known shortcoming of the LCDC model (like nonlinear MLMs in general) is that estimation of the parameters is based on estimation methods that do not have the optimal properties of full information maximum likelihood. A major goal of the work is to evaluate several estimation procedures in an effort to determine if the parameter estimates of the LCDC model can be accurately recovered with the proposed estimation methods.

Five methods are presented that are thought to be reasonable estimation procedures for the parameters of the LCDC model. Theoretically, the ideal estimation procedure for the LCDC model would be one that has an exact analytic solution. Unfortunately, at the present time no such solution seems to exist. The best alternative to an analytic solution is an iterative solution where an exact likelihood function exists and can be maximized. Unfortunately, at the present time it seems no exact likelihood function exists for the LCDC model (or the nonlinear MLM in general, see Appendix B). A major goal of the work will be to determine the properties and evaluate the effectiveness of the five proposed estimation procedures. The proposed methods of estimation for the parameters of the LCDC model are described in the following five subsections.

Estimation Method 1

The first proposed method for estimating the class specific parameters is perhaps the most intuitive and straightforward. Method 1 is a three-stage method, where the first-stage of the model is the estimation of the P parameters of $f(\mathbf{a})$ for each of the N individuals. Estimation of each individual's $\boldsymbol{\theta}$ can be done by any acceptable estimation procedure, but since nonlinear least squares using the Gauss-Newton procedure is the most widely recommended estimation method for individual specific nonlinear models (Ratkowsky, 1983; Bates & Watts, 1988), it will be the method used in Stage 1. Given the N $\hat{\boldsymbol{\theta}}_i$s and thus $\widehat{\boldsymbol{\Theta}}$ from Stage 1, estimation of $\boldsymbol{\gamma}$ occurs at Stage 2. Stage 2 involves the fitting of a FMM with a fixed number of classes, where individuals are classified into a crisp set of G classes based on the highest posterior probability of class membership (i.e., the optimal method, as discussed in the *Classification Based on Mixtures* section which began on page 53). Stage 3 involves the fitting of G separate MLMs, where each class has a unique set of parameter estimates. These G MLMs have parameter estimates that are based only on the individuals who were judged most likely to be members of the particular class.

Part of the success of this method is based on the distance between the G P dimensional multivariate distributions and thus the success of the classifications. In particular, if the G distributions representing each of the classes are, relevantly speaking, far apart from one another, few misclassifications will occur and this method will recover the class specific parameters when large sample sizes represent each class. If the individuals can be correctly classified into the appropriate class, then each of the MLMs will be based on the appropriate set of individuals and thus no contaminated distributions will distort the parameter estimates. As is true with the application of any mixture model, as the overlap between distributions becomes more pronounced, accurate classification of individuals is difficult, and crisp classes of individuals will likely have a nontrivial proportion of individuals who are misclassified. Thus, as the misclassification rate approaches zero, the accuracy of Method 1 increases to the point that the LCDC model is equivalent to the accuracy of the nonlinear MLM.

Recall that the cluster analytic techniques of Dumenci and Windle (2001), McCall et al. (1973), and Pauler and Laird (2000) are based on crisp sets and thus restrict the contribution of each individual to only the fixed effects of the class that the individual is most likely to belong. Thus, the crisp set classification procedure of Method 1 is consistent with three previously proposed models of heterogeneous change. Further justification for Method 1 being a reasonable procedure is based on the theory of multi-stage estimation. Multi-stage estimation procedures are at times carried out when the likelihood function of the particular model is not analytically derivable and thus when full information maximum likelihood is not possible, or, given the current state of knowledge, has not yet been derived (e.g. Stukel & Demidenko, 1997; Chatterjee, 2004). Multi-stage estimation procedures are also advantageous when there is a concern that the model may be misspecified (e.g., Bollen, 2001) or when a desire exists to evaluate the model at different levels individually (even when a likelihood function is known) rather than evaluating the full model simultaneously (e.g., Yuan & Bentler, in press). Full information maximum likelihood assumes that the model is correctly specified. When models are misspecified, "specification error in one part of the system can spread bias to other parts of the model" (Bollen, 1996, p. 109). Even though in the ideal situation full information maximum likelihood estimation is known to be optimal, when the likelihood function cannot be derived or when the model is in some way misspecified, multi-stage estimation procedures, although themselves potentially biased, can still outperform maximum likelihood methods. Notice that in Method 1 because each class is fit without regard to the other classes, different functional forms of change as well as different covariance structures for the unique effects and errors can be specified to be the same or different across some or all of the classes. However, because each of the classes are treated in isolation, cross-class constraints must be imposed post hoc.

Estimation Method 2

The second proposed method is similar to the first method, however, rather than separating the individuals into G classes and performing a different MLM for each class in isolation at Stage 3, Method 2 is carried out by way of a multiple group MLM. In particular, given $\hat{\boldsymbol{\gamma}}$, obtained with a crisp set in the same way as was done in Method 1, an N by $G - 1$ matrix of class identifiers would be constructed and used as an identifier of class membership. Each class would then be treated in a multiple group MLM.

Method 2 is a three-stage procedure like Method 1, and thus the advantages of multi-stage estimation procedures also apply to Method 2. Rather than estimating class specific parameters without regard to individuals estimated to be from other classes, Method 2 combines limited information from all classes when estimating parameters from each class. Method 2 also allows parameter invariance (such as cross-class constraints) to be imposed for any parameters of interest. Parameter invariance across class is straightforward with Method 2, but difficult with some of the other methods. In fact, other Methods of cross-class constraint require that such constraints be imposed with post hoc weighting procedures (details of which are discussed on page 147 in the *Results* section) that seem less than optimal. Method 2 also allows the possibility of testing an interaction between the estimated class membership and predictor variables (e.g., Is the asymptotic value for Class 1 larger than the asymptotic value in Class 2?), which adds a potentially interesting dimension describing change.

Notice that in Method 2 each class is fit in the context of a multiple group design which requires that the functional form of change and covariance structures for the unique effects and errors to generally be kept constant across all of the classes. Information from Stage 1 and Stage 2 is only used to classify the individuals into the G crisp sets. After the individuals have been classified, Method 2 treats the data as though it came from a multiple group design where the grouping factors are known (because they are assumed known after Stage 1 and Stage 2). Because all classes are fit in a multiple group MLM, the standard errors are based on N individuals (rather than n_g for each of the classes as in Method 1). The estimated standard errors are analogous to the pooled error term in a general linear model context when homogeneity of variance is assumed. In a simple analysis of variance, for example, when the within group standard error is assumed constant across all groups, the model error term is based on the total sample size and thus the degrees of freedom for the error term is $N - J$, where J is the number of independent groups. Basing the error term on all individuals in the analysis leads to more precise and more powerful inferential tests in exactly the same way it does in a multiple group MLM. Thus, when the model and covariance structure is truly the same across class, Method 2 is a more powerful and precise method because more information is used in the inferential tests.

Estimation Method 3

Estimation of $\mathbf{\Theta}$ and $\boldsymbol{\gamma}$ is required prior to estimation of the parameters of the MLM in Method 1 and Method 2. Because $\widehat{\mathbf{\Theta}}$ and $\hat{\boldsymbol{\gamma}}$ are not likely to be independent of one another, information contained in $\widehat{\mathbf{\Theta}}$ can provide information about $\hat{\boldsymbol{\gamma}}$, and vice versa. A full information maximum likelihood approach uses information from all levels of an analysis in the estimation of all parameters (which is why bias in one part of the model affects the parameter estimates at all other parts of the model). Thus, all available information is used in the estimation of all parameter estimates in a full information maximum likelihood estimation procedure. Method 3 attempts to capitalize on the ideas of full information maximum likelihood without literally being based on full information maximum likelihood. These ideas are carried out by using $\widehat{\mathbf{\Theta}}$ in the estimation of $\hat{\boldsymbol{\gamma}}$, and vice versa.

Method 3 begins by estimating the individual change coefficients given in $\widehat{\mathbf{\Theta}}$. The N by P matrix of initial estimates, obtained in the same way as Method 1 and Method 2, is denoted $\widehat{\mathbf{\Theta}}_{(0)}$, where the subscript in parentheses implies the number of iterations necessary to obtain that set of estimates, with a zero subscript representing the initial conditions. Given $\widehat{\mathbf{\Theta}}_{(0)}$, a FMM is carried out in order to obtain $\hat{\boldsymbol{\gamma}}_{(0)}$. A MLM in accordance with Method 1 or Method 2 is then carried out. After the MLM is carried out, new estimates of individual change coefficients that are known as empirical Bayes or James-Stein estimates are available and $\widehat{\Theta}_{(0)}$ is updated to $\widehat{\Theta}_{(1)}$ based on these new estimates. The empirical Bayes estimates in $\widehat{\Theta}_{(1)}$ are based in part on the individuals' scores and in part on the fixed effects of the particular class the individual is most likely to belong as a function of the reliability of the individuals scores. These updated individual change coefficients are now used as input for the FMM in order to obtain $\hat{\boldsymbol{\gamma}}_{(1)}$. The MLM is then carried out given $\hat{\boldsymbol{\gamma}}_{(1)}$ in order to obtain $\widehat{\Theta}_{(2)}$, and so forth. This iterative process continues until some criterion value converges to some essentially unchanging value after successive iterations (e.g., the change in the likelihood function or the mean square error after successive iterations is $\leq .001$). To summarize the difference between Method 3 and Methods 1 and 2, after the MLM has been fit at Stage 3, the updated individual change coefficients (i.e., the empirical Bayes estimates) are used as input in an updated FMM. The new classifications (if any) are used to reestimate the fixed effects and the empirical Bayes estimates so that they can again be used as input into the FMM. This iterative process continues until the convergence criterion is met.

This method seems likely to improve the estimation of parameters obtained in both Method 1 and in Method 2 by reducing the number of individuals that are misclassified. The reason for the suspected improvement is because of the iterative nature of using the FMM and the MLM in tandem. Although both the FMM and the MLM are iterative in nature, Methods 1 and 2 do not combine the iterative process of the FMM and the MLM in an omnibus estimation procedure that attempts to use information from across levels to improve parameter estimates at each level, as does Method 3. The combination of the FMM and the MLM into an overall back and forth iterative scheme is more consistent with the theory of full information maximum likelihood estimation than either Method 1 or Method 2 because of the idea of "shared information". Because Method 3 can be based on the estimation procedures of Method 1 or Method 2, Method 3 will be denoted Method

3.1 or Method 3.2, depending on whether estimation using the rationale of Method 1 or Method 2 is used, respectively. Thus, "Method 3" actually consists of two different methods.

Estimation Method 4

In order to estimate the class specific parameters, a standard application of the FMM could be applied to the individual change coefficients, $\widehat{\boldsymbol{\Theta}}$, which were estimated at the first stage of the analysis (e.g., using the Gauss-Newton procedure). This first stage of Method 4 is thus the same as the first stage in the three previous methods. The difference between Method 4 and the previous methods is that the FMM itself is used to estimate the class specific parameters, rather than using a nonlinear MLM after individuals have been classified into crisp sets based on their posterior probabilities of class membership. The rationale for this approach is apparent when it is realized that the FMM provides estimates and hypothesis tests of the class specific means and covariances for the P variables assumed to be mixed. Because what is of interest is the estimation of the means and covariances of the individuals' change coefficients, a FMM of $\widehat{\boldsymbol{\Theta}}$ will provide estimates of the parameters of interest for the LCDC model because a mean vector of the P parameters of change is obtained as is the covariance of the parameter estimations. The mean vector represents the fixed effects whereas the covariance of the parameters of change represents the covariance of the unique effects.

A major difference between Method 4 and the previous methods is that Method 4 does not classify individuals into a crisp set. Rather, all individuals potentially contribute to the class specific parameter estimates from all classes. In Method 4, if an individual has a high probability of belonging to the gth class, then the individual's $\boldsymbol{\theta}_i$ is heavily weighted in the estimation of the parameter estimates of the gth class. However, in Method 4 if an individual's posterior probability of belonging to a particular class is small, the individual's scores receive little weight in the estimation of the parameter estimates from that particular class. This is because the mean vector and the covariance matrices are weighted by a function of the probability that the individual belongs to the particular class. The rationale of Method 4 is thus consistent with the GMM, the latent class growth model, and to some extent the grade of membership model, in that all individuals potentially influence all parameter estimates. Method 4 seems to be especially useful when a crisp set is difficult to obtain because overlap among the classes leads to probabilities of class membership that are not close to unity or zero. Note that the FMM allows for covariance structures to be class specific or constrained across the classes.

Summary of Estimation Methods

Because some of the details of the estimation methods differ only in subtle ways, a summary of each of the estimation methods is provided. The summary provided is not meant to supplant the description provided for each of the methods of estimation, but rather it is meant to supplement the description provided. The finer details of the estimation methods are not delineated in the summary section. It is therefore important to read the appropriate section in the text before selecting any of the estimation methods for use in applied settings:

Summary of Method 1

Stage 1: Estimate the P parameters of $f(\mathbf{a})$ for each of the N individuals using the Gauss-Newton estimation procedure (or other appropriate estimation procedure), obtain $\widehat{\boldsymbol{\Theta}}$.

Stage 2: Fit a FMM with G classes using $\widehat{\boldsymbol{\Theta}}$ as input and then classify the N individuals into G crisp sets based on the the highest posterior probability, obtain $\hat{\boldsymbol{\gamma}}$.

Stage 3: Using the original data and $\hat{\boldsymbol{\gamma}}$ to identify class membership, fit G separate MLMs in isolation.

Summary of Method 2

Stage 1: Same as Method 1.

Stage 2: Same as Method 2.

Stage 3: Using the original data and $\hat{\boldsymbol{\gamma}}$ to identify class membership, fit a G-group MLM.

Summary of Method 3.1

Stage 1: Same as Methods 1 and 2.

Stage 2: Same as Methods 1 and 2.

Stage 3: Same as Method 1.

Stage 4: Using the empirical Bayes estimates obtained after G MLMs have been fit in isolation, refit the FMM (Stage 2) using $\widehat{\boldsymbol{\Theta}}_{(1)}$ in an effort to improve the classification success. Iterate between Stage 2, Stage 3, and Stage 4 until the some convergence criterion is met.

Summary of Method 3.2

Stage 1: Same as Methods 1, 2, and 3.1.

Stage 2: Same as Methods 1, 2, and 3.1.

Stage 3: Same as Method 2.

Stage 4: Using the empirical Bayes estimates obtained after a G-group MLM has been fit, refit the FMM (Stage 2) using $\widehat{\boldsymbol{\Theta}}_{(1)}$ in an effort to improve the classification success. Iterate between Stage 2, Stage 3, and Stage 4 until the some convergence criterion is met.

Summary of Method 4

Stage 1: Same as Methods 1, 2, 3.1, and 3.2.

Stage 2: Fit the FMM as is done in Methods 1, 2, 3.1, and 3.2; use the parameter estimates obtained in the FMM (which uses fuzzy sets) to estimate the parameters of the LCDC model.

METHODS

The idea of allowing latent classes to be estimated within some heterogeneous population, where each of the classes follows a special case of some functional form of change is consistent with several methods already in existence. The main difference in the present conceptualization of modeling heterogeneous change is that heterogeneous change governed by a nonlinear functional form is of explicit interest. The benefits of nonlinear change models and the benefits of latent classifications are generally well-known and appreciated. With the exception of the limited work of Pauler and Laird (2000), the combination of nonlinear functional forms in heterogeneous populations is thought of as being mutually exclusive. The purpose of the present work is to formally develop a model that combines nonlinear functional forms of change and methods of classification into a unified model of heterogeneous change, so that reasonable method(s) for estimating the parameters of the LCDC model can be developed.

Although the conceptual idea of allowing heterogeneous populations to be modeled by nonlinear change models may be fairly straightforward, carrying out such a model is not trivial. Because the desire is to follow the mainstream frequentist approach to statistical inference and thus not work within a fully Bayesian framework for nonlinear models in a heterogeneous population, as was done in Pauler and Laird's 2000 preliminary work, it is unclear what method of estimation provides the statistically optimal estimators in situations of large sample size. The idea of the present work is to methodically investigate the five proposed estimation methods that attempt to recover the parameters of the LCDC model of change in order to evaluate the accuracy of each of the methods.

In general, there is no closed form solution for the parameter estimates of fixed effects nonlinear models. There is also no closed form solution generally available for the parameter estimates of FMMs. Not only do closed form solutions generally not exist, the likelihood function of the nonlinear MLM is not generally exact (e.g., Davidian & Giltinan, 1995, 2003; Pinheiro & Bates, 2000). Thus, in each of these three statistical procedures, iterative methods attempt to find the optimal set of parameter estimates based on some convergence criteria, typically by maximizing the likelihood (or approximate) function. The combination of these three statistical procedures into a single unified model of heterogeneous change for nonlinear functional forms is thus especially difficult in terms of deriving analytic properties of the model, estimating parameters, and forming confidence intervals around the population parameters. Thus, only limited analytic properties of the proposed estimation methods are analytically tractable given the current state of knowledge. Because of the intractability of the analytic properties of the proposed estimation methods for the LCDC model, a Monte Carlo simulation study is necessary to evaluate the properties of the estimation procedures. The next section details the simulation study conducted to evaluate the effectiveness of the estimation procedures and to determine which of the estimation procedures might be recommended to carry out applications of the LCDC model in what situations.

Simulation Study

A problem inherent with simulation studies is that only a limited number of conditions can be investigated. Whereas a purely analytic solution encompasses all cases within some domain, it is sometimes difficult to generalize the results obtained in a simulation studies. Nevertheless, simulation studies often address questions not readily solvable by analytic methods alone. If only models with well defined analytic properties were thought to be interesting or useful, difficult and potentially beneficial models would go undeveloped because such models might not offer an analytic solution because the model stretches beyond the current state of knowledge of statistical inference. Thus, a major reason for conducting simulation studies is that they can address problems that are not directly solvable with analytic methods or where analytics do not exist. The strict reliance on analytic methods potentially leads to missed opportunities in the development and investigation of models whose parameters cannot readily be obtained with the current state of existing analytic methods. While at some point analytic methods may overshadow the results of a Monte Carlo simulation study, the pursuit of analytic results may be motivated by the results of a simulation study.

Because a simulation study must be limited in nature and thus cannot evaluate all potentially interesting models, it is thought that a thorough investigation of a single functional form of change for multiple conditions will be most valuable. Promising results in accurately recovering parameters of the LCDC model for the particular functional form of change chosen across numerous conditions by one or more estimation methods would lend evidence to the fact that the method is effective and would presumably generalize to other nonlinear functional forms of change. The present study investigates a logistic growth curve. Rather than using the logistic growth curve presented in Equation 26, a reparameterized version of the three parameter model (i.e., when ζ is set to zero) will be used for computational reasons. Specifically, the logistic growth curve used in the simulation study will be defined as follows:

$$y_t = \frac{\alpha}{1 + \exp(-\frac{(a_t - \beta)}{\gamma})}. \tag{51}$$

Notice that the only difference between the reparameterized logistic growth curve of Equation 51 and the original three parameter logistic growth curve of Equation 26 is that the γs are inverses of one another. Thus, α and β have the same interpretation as before, but now γ represents the distance on the abscissa between the point of inflection and (approximately) .73α (see Appendix C.7 of Pinheiro & Bates, 2000). Thus, rather than "only" being a curvature parameter, γ from Equation 51 has its own substantive interpretation. Another reason that it is advantageous to use the parameterization of Equation 51 is that starting values for such curves are straightforward to determine. Details of the starting values are given in the *Selection of Starting Values* section beginning on page 112.

The reason for choosing a three parameter logistic growth curve as being representative of nonlinear change models is because of: (a) its relative popularity in the medical and biological literature, (b) given certain parameter combinations it can take on characteristics that in some ways are similar to the asymptotic regression growth curve (i.e., exhibit change to a single asymptotic value in the range of observed data), and (c) because the logistic growth curve is representative of sigmoidal models in the Richards family of growth curves, which are especially useful for modeling limited capacity data (e.g., Browne & du Toit, 1991; van Geert, 1991). There is nothing unique about the logistic growth curve to imply that the proposed methods would be more or less effective than if another nonlinear change model were used. Thus, the logistic growth curve was selected because of its representativeness to other nonlinear change curves and the widely applicable form it embodies.

Evaluation of the LCDC model will proceed by manipulation of five factors: (a) sample size (two levels), (b) proportion of individuals belonging to each class (three levels), (c) the number of measurement occasions (2 levels), (d) the magnitude of the error variance (two levels), and (e) the distance between the G P dimensional multivariate normal distributions by modifying class specific fixed effects (three levels). These factors will be manipulated in the simulation study in an effort to determine how they relate to the effectiveness of each of the proposed methods. Further elaboration of the specific conditions examined in the simulation study are given momentarily in the *Conditions Examined in the Simulation Study* section beginning on page 101.

The proposed estimation methods will be evaluated for two latent classes only. The proportion of the individuals belonging to the latent classes will be a manipulated factor with three levels. The value for π_1 will be specified as .50, .75, and .875 (recall that $\pi_2 = 1 - \pi_1$). A value of .50 seems to be a reasonable starting point in terms of the proportion of the individuals being members of each class, as this represents a case where two latent classes occur with equal probability. Because .75 is halfway between the latent classes being equally divided and having a homogeneous population, a condition where 75% of the individuals are nested in one class seems to be a reasonable condition to examine. Similarly, .875 is halfway between 75% of the individuals being in one latent class and having a homogeneous population. A condition where 87.5% of the individuals are nested in one class also seems to be consistent with the disproportionate likelihood of class membership for various behavioral conditions, where a large proportion of the individuals are classified in one class (e.g., "normal") and a relatively small proportion of the population classified in another class (e.g., "problematic"). The selected levels of the proportion of individuals belonging to Class 1 (and Class 2 by definition) presumably encompasses reasonable values of class heterogeneity.

Throughout the simulation study the duration of time will be kept constant, specifically $\mathbf{a}_i \in [0, 100]$. This does not limit the usefulness of the study in any way because the scaling of time is arbitrary.[30] Rather than manipulating the fixed effect parameters for both groups, the fixed effects for Class 1 will be held constant throughout the simulation study. The within group error variance will have the following structure for both of the classes: $\sigma^2_\varepsilon \mathbf{I}_T$, where $\mathbf{I}_T$ is an identity matrix of rank T. Thus, the classes

[30]In fact, because $\mathbf{a} \in [0, 100]$, it is accurate and straightforward to think of the abscissa as representing the percentage of an arbitrary length of time. Thus, if one is interested in $\mathbf{a} \in [0, 5]$, 50% completion of the study at $a = 2.5$ would correspond to $a = 50$ (i.e., 50%) on the time axis used in the simulation study.

will share the same error structure, where the errors have an expectation of zero, and conditional on the individual and the model have variance σ^2_ε with no covariation across timepoints. The within group covariance structure for the unique effects will be held constant across class, meaning that $\boldsymbol{\Sigma}_{\upsilon_1} = \boldsymbol{\Sigma}_{\upsilon_2}$. For simplicity, all individuals will be measured at the same occasions of measurement, that is, $\mathbf{a}_1 = \ldots = \mathbf{a}_N = \mathbf{a}$.

Missing data, conditional models (i.e., those with time invariant concomitant variables), and model misspecifications will not be addressed in the simulation study. Lack of model misspecification implies that the correct functional form (i.e., the three parameter logistic growth curve) will always be applied to the individual trajectories and that the number of latent classes modeled (i.e., two) is in fact the number of latent classes that exist in the population. The reason misspecified models will not be considered is because the purpose of the work is to evaluate the proposed methods in order to determine their effectiveness at recovering parameters from correctly specified models. If it is the case that one or more of the proposed methods is effective at recovering the parameters of the LCDC model, then a logical next step in a future project would be to evaluate the robustness of the estimation procedure when the model is misspecified. Another important issue that will not be addressed is how to determine the number of component distributions that should be allowed. This problem permeates itself in any type of FMM (or clustering procedure more generally) and in fact occupies considerable space in the FMM literature (Everitt & Hand, 1981; McLachlan & Peel, 2001; McLachlan & Basford, 1988). Although model misspecification is an important and interesting question, addressing misspecified models and determining the correct number of classes to include in the model is beyond the scope of the present work.

The goal of the LCDC model is to accurately recover the parameters that generated the data from some specified change model (the logistic growth curve in the present situation). What is unknown at the present time is how effective the proposed procedures are at recovering those parameters. The goal of the simulation study is thus to evaluate the effectiveness of the proposed procedures in an effort to determine which, if any, of the proposed procedures yield accurate estimates of the parameter values.

The simulations were done using the S-language in the R computer program (R, 2004). In addition to the standard functions contained in R, the NLME package (Pinheiro, Bates, DebRoy, & Sarkar, 2004) and the MCLUST package (Fraley & Raftery, 2004) were used. The NLME package is used to fit nonlinear MLMs whereas the MCLUST package is used to fit FMMs. The number of replications used in the simulation study was 1,000 for each of the specific situations examined. Conditions where the particular data set fails to be estimable with any of the methods were not considered in the results. An additional data set was generated for each of the data sets that failed to yield results.

The nls() function is used to estimate the nonlinear least squares estimates of the individual coefficients of change in the first stage of each method. The default parameters of the nls() function were changed so that the maximum number of iterations (maxiter) was 1,250 (rather than 50) and so that the minimum step factor (minFactor) in the Gauss-Newton algorithm was $5{,}120^{-1}$ (rather than $1{,}024^{-1}$). The effect of decreasing the minimum step factor potentially allows for more precise estimates to be obtained, whereas increasing the maximum number of iterations allows for the convergence criteria to be met after more computations have been conducted. A decrease in the minimum step size, so as to obtain more precise estimates, should be accompanied with an increase in the maximum number of iterations, because the jump from sets of parameter estimates is smaller. Not surprisingly, computational speed is hampered by such adjustments, because the program must “do more work” in order to obtain the potentially more precise estimates. See the nls.control() function for more information on the parameters of the nls() function (R, 2004).

The nlme function in the NLME package can be used to fit a specified nonlinear MLM to longitudinal data. The NLME package was used in the simulation study to fit the nonlinear MLMs for each of the methods except for Method 4 (which did not involve the use of a MLM). The default parameters of the nlme() function were changed in order to allow the maximum number of iterations (maxIter) to be 500 (rather than 50), the maximum number of iterations for the penalized nonlinear least squares step to be 70 (rather than 7), the number of iterations for the first part of the penalized nonlinear least squares step (niterEM) to be 100 (rather than 25), the number of iterations for general optimization of the locally linear mixed model (msMaxIter) to be 500 (rather than 50), and the minimum absolute parameter value in the approximate variance calculation (minAbsParApVar) to .025 (rather than .05).[31] As with the changes to the default parameters of the nls() function, the changes to the default parameters of the nlme() function are made so that the parameter estimates are more precise (at the cost of computational speed). See the nlmeControl() function for more information on the parameters of the nlme() function (R, 2004).

Conditions Examined in the Simulation Study

Oftentimes simulation studies attempt to find sets of parameters that bound reasonable conditions. In the present context, it would probably not be very useful to test the proposed procedures when there was essentially no differences between the class specific fixed effects and/or only a few timepoints. Doing so would probably not recover the parameters very well and the individuals would be classified into one of the G classes essentially at random. On the other hand, performing the simulation when there was a huge difference between the fixed effects with many timepoints would

[31]The author would like to thank Douglas M. Bates, coauthor of the NLME package, for help in better understanding the parameters of the nlme() function and the effects of modifying the control parameters (personal communication, July 24, 2004.)

likely lead to individuals who are almost never misclassified and thus the fixed effects would necessarily correspond closely to their population values. In order to evaluate the proposed procedures effectively, conditions where clear failure or clear success seem inevitable should generally be avoided. A difficult part of simulation studies is thus determining reasonable parameter values to use. In an effort to find bounding conditions that seem realistic, preliminary evaluation suggested several combinations of parameter values. Although "reasonable" is a relative term that is open to debate, the wide variety of conditions examined should satisfy many of the qualms that might be raised.

A fully factorial five (methods) by three (πs) by three (distances between the two classes) by two (Ns) by two (Ts) by two (σ_εs) design was used yielding 360 total examination sets (i.e., $5 \times 3 \times 3 \times 2 \times 2 \times 2 = 360$). For identification purposes, each of the situations will be identified using the following format: CxSy, where Cx represents the xth condition ($x = 1, \ldots, 24$) and Sy represents the yth scenario ($y = 1, \ldots, 3$). As an example, the first scenario for the thirteenth condition is thus represented as C13S1. A situation is a particular crossing of condition by scenario. Thus, a total of 72 situations ($24 \times 3 = 72$) exists. A condition is used to identify the crossing of scenarios and series. A scenario is used to identify the particular set of parameters other than fixed effects that were manipulated in the simulation study. A series is used to identify the particular sets of fixed effects that exist. The parameters for each situation (Condition $\times$ Scenario $\times$ Series) contained in Series 1, Series 2, and Series 3 are given in Tables 1, 2, and 3, respectively.

TABLE 1

RECORD OF CONDITIONS FOR SERIES 1 (CONDITION 1 THROUGH 8)

C	S	LC	*N*	π	α	β	γ	δ_α	δ_β	δ_γ	Δ	σ^2_ε	σ^2_α	σ^2_β	σ^2_γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	*T*
1	1	1 2	100 100	.5 .5	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	4	10	8	6	.5	.3	.4	25
1	2	1 2	150 50	.75 .25	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	4	10	8	6	.5	.3	.4	25
1	3	1 2	175 25	.875 .125	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	4	10	8	6	.5	.3	.4	25
2	1	1 2	250 250	.5 .5	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	4	10	8	6	.5	.3	.4	25
2	2	1 2	375 125	.75 .25	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	4	10	8	6	.5	.3	.4	25
2	3	1 2	438 62	.875 .125	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	4	10	8	6	.5	.3	.4	25
3	1	1 2	100 100	.5 .5	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	8	10	8	6	.5	.3	.4	25
3	2	1 2	150 50	.75 .25	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	8	10	8	6	.5	.3	.4	25
3	3	1 2	175 25	.875 .125	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	8	10	8	6	.5	.3	.4	25
4	1	1 2	250 250	.5 .5	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	8	10	8	6	.5	.3	.4	25
4	2	1 2	375 125	.75 .25	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	8	10	8	6	.5	.3	.4	25
4	3	1 2	438 62	.875 .125	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	8	10	8	6	.5	.3	.4	25
5	1	1 2	100 100	.5 .5	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	4	10	8	6	.5	.3	.4	15
5	2	1 2	150 50	.75 .25	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	4	10	8	6	.5	.3	.4	15
5	3	1 2	175 25	.875 .125	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	4	10	8	6	.5	.3	.4	15
6	1	1 2	250 250	.5 .5	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	4	10	8	6	.5	.3	.4	15
6	2	1 2	375 125	.75 .25	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	4	10	8	6	.5	.3	.4	15
6	3	1 2	438 62	.875 .125	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	4	10	8	6	.5	.3	.4	15
7	1	1 2	100 100	.5 .5	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	8	10	8	6	.5	.3	.4	15
7	2	1 2	150 50	.75 .25	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	8	10	8	6	.5	.3	.4	15
7	3	1 2	175 25	.875 .125	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	8	10	8	6	.5	.3	.4	15
8	1	1 2	250 250	.5 .5	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	8	10	8	6	.5	.3	.4	15
8	2	1 2	375 125	.75 .25	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	8	10	8	6	.5	.3	.4	15
8	3	1 2	438 62	.875 .125	100 92.885	35 29.343	15 10.713	2.25	2	1.75	2.613	8	10	8	6	.5	.3	.4	15

TABLE 2

RECORD OF CONDITIONS FOR SERIES 2 (CONDITION 9 THROUGH 16)

C	S	LC	N	π	α	β	γ	δ_α	δ_β	δ_γ	Δ	σ^2_ε	σ^2_α	σ^2_β	σ^2_γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	T
9	1	1 2	100 100	.5 .5	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	4	10	8	6	.5	.3	.4	25
9	2	1 2	150 50	.75 .25	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	4	10	8	6	.5	.3	.4	25
9	3	1 2	175 25	.875 .125	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	4	10	8	6	.5	.3	.4	25
10	1	1 2	250 250	.5 .5	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	4	10	8	6	.5	.3	.4	25
10	2	1 2	375 125	.75 .25	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	4	10	8	6	.5	.3	.4	25
10	3	1 2	438 62	.875 .125	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	4	10	8	6	.5	.3	.4	25
11	1	1 2	100 100	.5 .5	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	8	10	8	6	.5	.3	.4	25
11	2	1 2	150 50	.75 .25	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	8	10	8	6	.5	.3	.4	25
11	3	1 2	175 25	.875 .125	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	8	10	8	6	.5	.3	.4	25
12	1	1 2	250 250	.5 .5	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	8	10	8	6	.5	.3	.4	25
12	2	1 2	375 125	.75 .25	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	8	10	8	6	.5	.3	.4	25
12	3	1 2	438 62	.875 .125	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	8	10	8	6	.5	.3	.4	25
13	1	1 2	100 100	.5 .5	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	4	10	8	6	.5	.3	.4	15
13	2	1 2	150 50	.75 .25	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	4	10	8	6	.5	.3	.4	15
13	3	1 2	175 25	.875 .125	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	4	10	8	6	.5	.3	.4	15
14	1	1 2	250 250	.5 .5	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	4	10	8	6	.5	.3	.4	15
14	2	1 2	375 125	.75 .25	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	4	10	8	6	.5	.3	.4	15
14	3	1 2	438 62	.875 .125	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	4	10	8	6	.5	.3	.4	15
15	1	1 2	100 100	.5 .5	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	8	10	8	6	.5	.3	.4	15
15	2	1 2	150 50	.75 .25	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	8	10	8	6	.5	.3	.4	15
15	3	1 2	175 25	.875 .125	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	8	10	8	6	.5	.3	.4	15
16	1	1 2	250 250	.5 .5	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	8	10	8	6	.5	.3	.4	15
16	2	1 2	375 125	.75 .25	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	8	10	8	6	.5	.3	.4	15
16	3	1 2	438 62	.875 .125	100 94.466	35 30.757	15 11.938	1.75	1.5	1.25	1.976	8	10	8	6	.5	.3	.4	15

TABLE 3

RECORD OF CONDITIONS FOR SERIES 3 (CONDITION 17 THROUGH 24)

C	S	LC	*N*	π	α	β	γ	δ_α	δ_β	δ_γ	Δ	σ^2_ε	σ^2_α	σ^2_β	σ^2_γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	*T*
17	1	1 2	100 100	.5 .5	100 100	35 43.485	15 15	0	-3	0	3.635	4	10	8	6	.5	.3	.4	25
17	2	1 2	150 50	.75 .25	100 100	35 43.485	15 15	0	-3	0	3.635	4	10	8	6	.5	.3	.4	25
17	3	1 2	175 25	.875 .125	100 100	35 43.485	15 15	0	-3	0	3.635	4	10	8	6	.5	.3	.4	25
18	1	1 2	250 250	.5 .5	100 100	35 43.485	15 15	0	-3	0	3.635	4	10	8	6	.5	.3	.4	25
18	2	1 2	375 125	.75 .25	100 100	35 43.485	15 15	0	-3	0	3.635	4	10	8	6	.5	.3	.4	25
18	3	1 2	438 62	.875 .125	100 100	35 43.485	15 15	0	-3	0	3.635	4	10	8	6	.5	.3	.4	25
19	1	1 2	100 100	.5 .5	100 100	35 43.485	15 15	0	-3	0	3.635	8	10	8	6	.5	.3	.4	25
19	2	1 2	150 50	.75 .25	100 100	35 43.485	15 15	0	-3	0	3.635	8	10	8	6	.5	.3	.4	25
19	3	1 2	175 25	.875 .125	100 100	35 43.485	15 15	0	-3	0	3.635	8	10	8	6	.5	.3	.4	25
20	1	1 2	250 250	.5 .5	100 100	35 43.485	15 15	0	-3	0	3.635	8	10	8	6	.5	.3	.4	25
20	2	1 2	375 125	.75 .25	100 100	35 43.485	15 15	0	-3	0	3.635	8	10	8	6	.5	.3	.4	25
20	3	1 2	438 62	.875 .125	100 100	35 43.485	15 15	0	-3	0	3.635	8	10	8	6	.5	.3	.4	25
21	1	1 2	100 100	.5 .5	100 100	35 43.485	15 15	0	-3	0	3.635	4	10	8	6	.5	.3	.4	15
21	2	1 2	150 50	.75 .25	100 100	35 43.485	15 15	0	-3	0	3.635	4	10	8	6	.5	.3	.4	15
21	3	1 2	175 25	.875 .125	100 100	35 43.485	15 15	0	-3	0	3.635	4	10	8	6	.5	.3	.4	15
22	1	1 2	250 250	.5 .5	100 100	35 43.485	15 15	0	-3	0	3.635	4	10	8	6	.5	.3	.4	15
22	2	1 2	375 125	.75 .25	100 100	35 43.485	15 15	0	-3	0	3.635	4	10	8	6	.5	.3	.4	15
22	3	1 2	438 62	.875 .125	100 100	35 43.485	15 15	0	-3	0	3.635	4	10	8	6	.5	.3	.4	15
23	1	1 2	100 100	.5 .5	100 100	35 43.485	15 15	0	-3	0	3.635	8	10	8	6	.5	.3	.4	15
23	2	1 2	150 50	.75 .25	100 100	35 43.485	15 15	0	-3	0	3.635	8	10	8	6	.5	.3	.4	15
23	3	1 2	175 25	.875 .125	100 100	35 43.485	15 15	0	-3	0	3.635	8	10	8	6	.5	.3	.4	15
24	1	1 2	250 250	.5 .5	100 100	35 43.485	15 15	0	-3	0	3.635	8	10	8	6	.5	.3	.4	15
24	2	1 2	375 125	.75 .25	100 100	35 43.485	15 15	0	-3	0	3.635	8	10	8	6	.5	.3	.4	15
24	3	1 2	438 62	.875 .125	100 100	35 43.485	15 15	0	-3	0	3.635	8	10	8	6	.5	.3	.4	15

Given the set of 72 “reasonable” situations, it is helpful to translate some of the parameter values contained in Tables 1, 2, and 3 into graphical displays. Figure 8 shows a random sample of the trajectories from 15 individuals from Class 1 and 15 individuals from Class 2 of a representative situation from Series 1, Series 2, and Series 3. Specifically, Series 1 is represented by the parameters from C3S1, Series 2 by the parameters from C11S1, and Series 3 by the parameters from C19S1. The black lines represent those individuals from Class 1, whereas the blue lines represent individuals from Class 2.

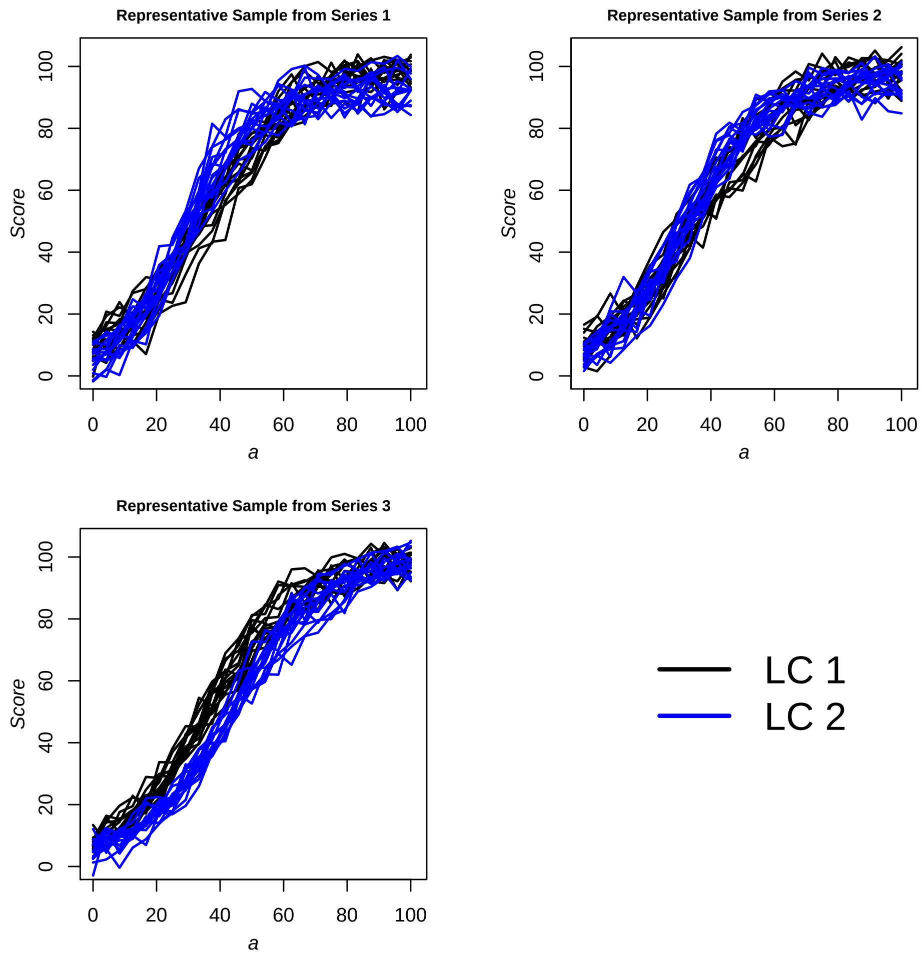

Figure 8: Illustration of 15 random trajectories from each latent class for representative samples from each of the three series (individuals from Class 1 represented with black trajectories whereas individuals from Class 2 represented with blue trajectories).

Another way to conceptualize each of the trajectory plots in Figure 8 is to plot the bivariate relationships between each of the individual change coefficients. Figures 9, 10, and 11 illustrate how the individual change coefficients relate to one another in Series 1, 2, and 3, respectively. The solid squares represent Class 1, whereas the open triangles represent Class 2.

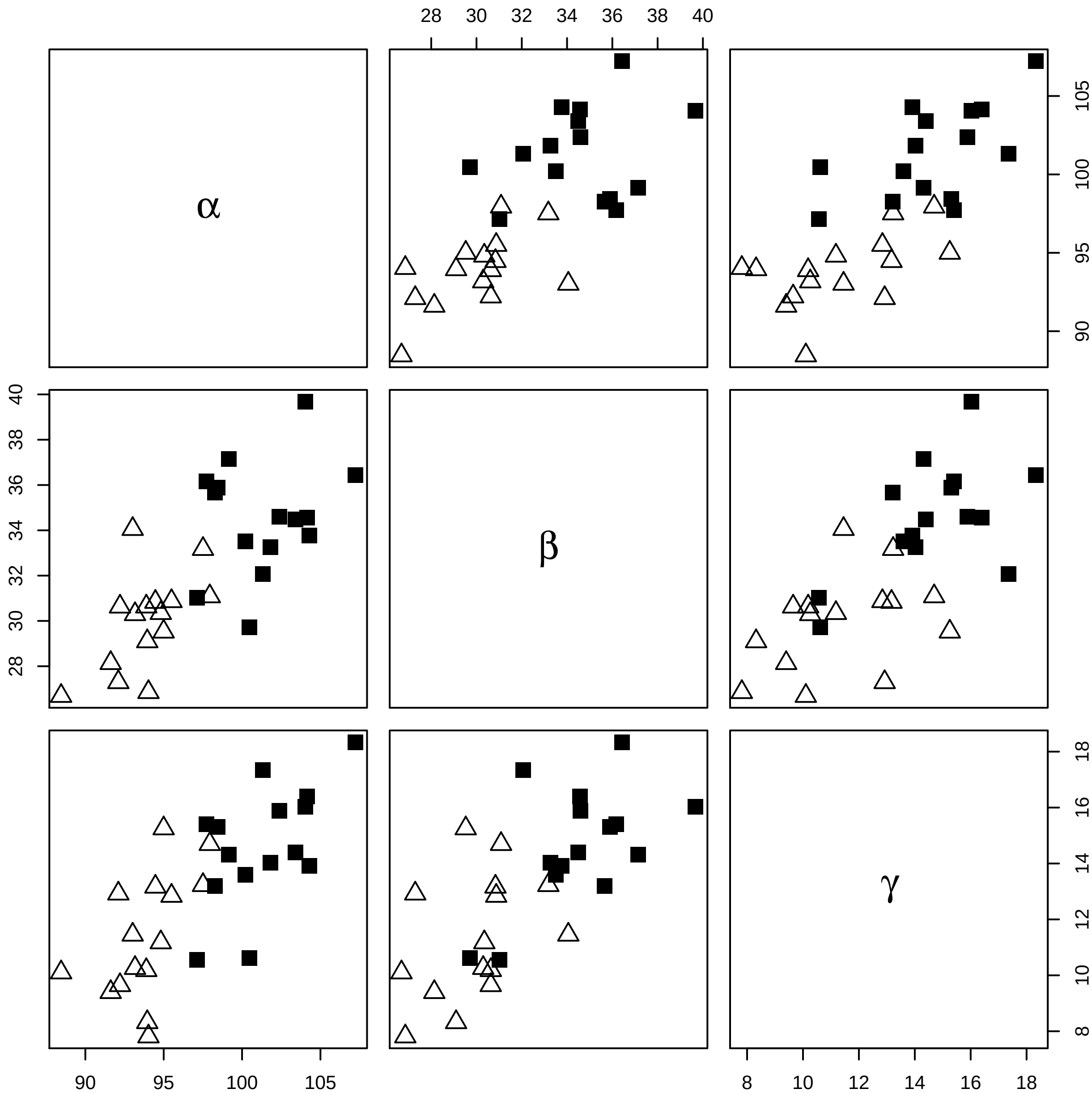

Figure 9: Illustration of 15 random sets of individual change coefficients from each class for Series 1 (The squares represent Class 1, whereas the triangles represent Class 2.).

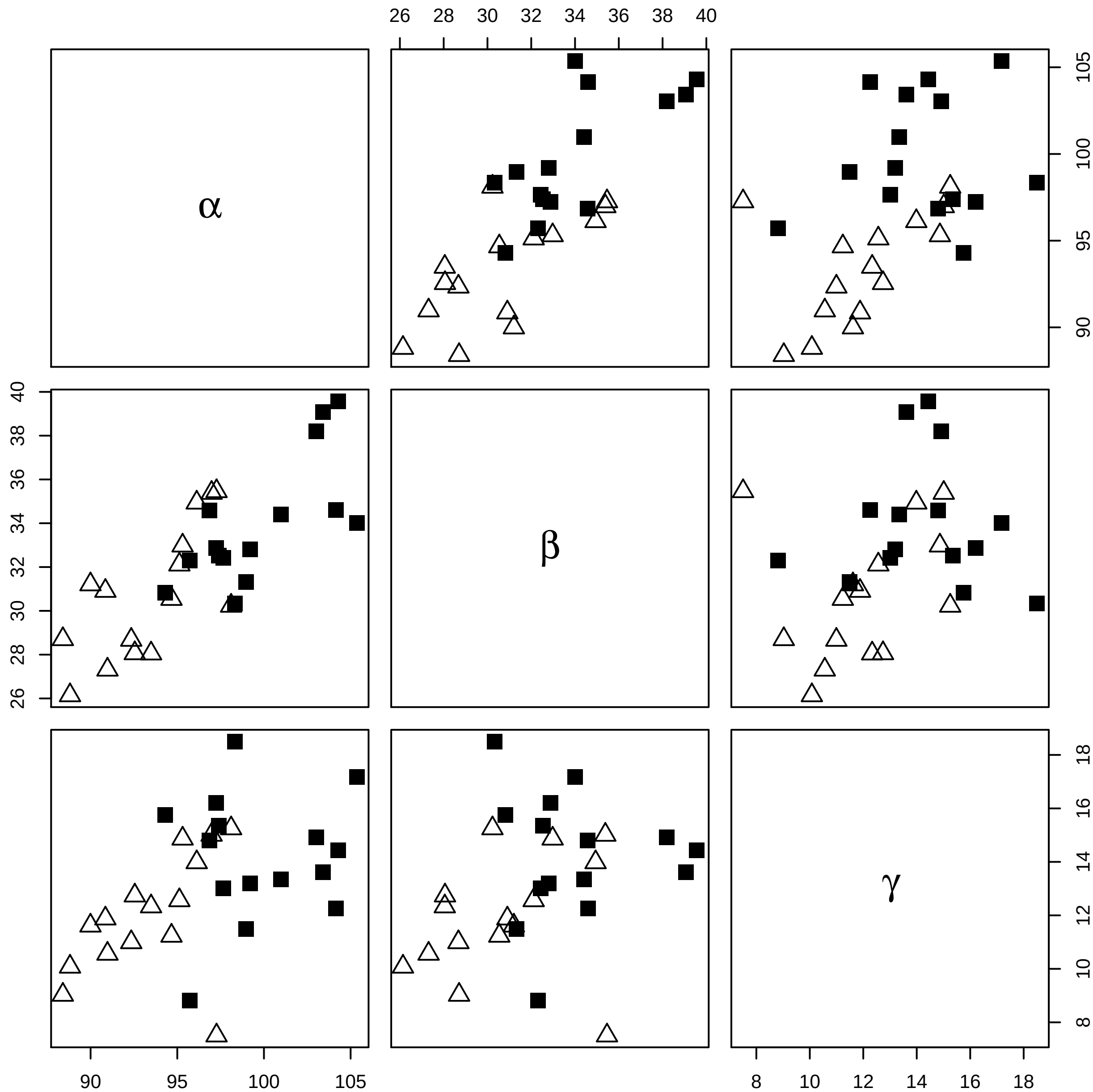

Figure 10: Illustration of 15 random sets of individual change coefficients from each class for Series 2 (The squares represent Class 1, whereas the triangles represent Class 2.).

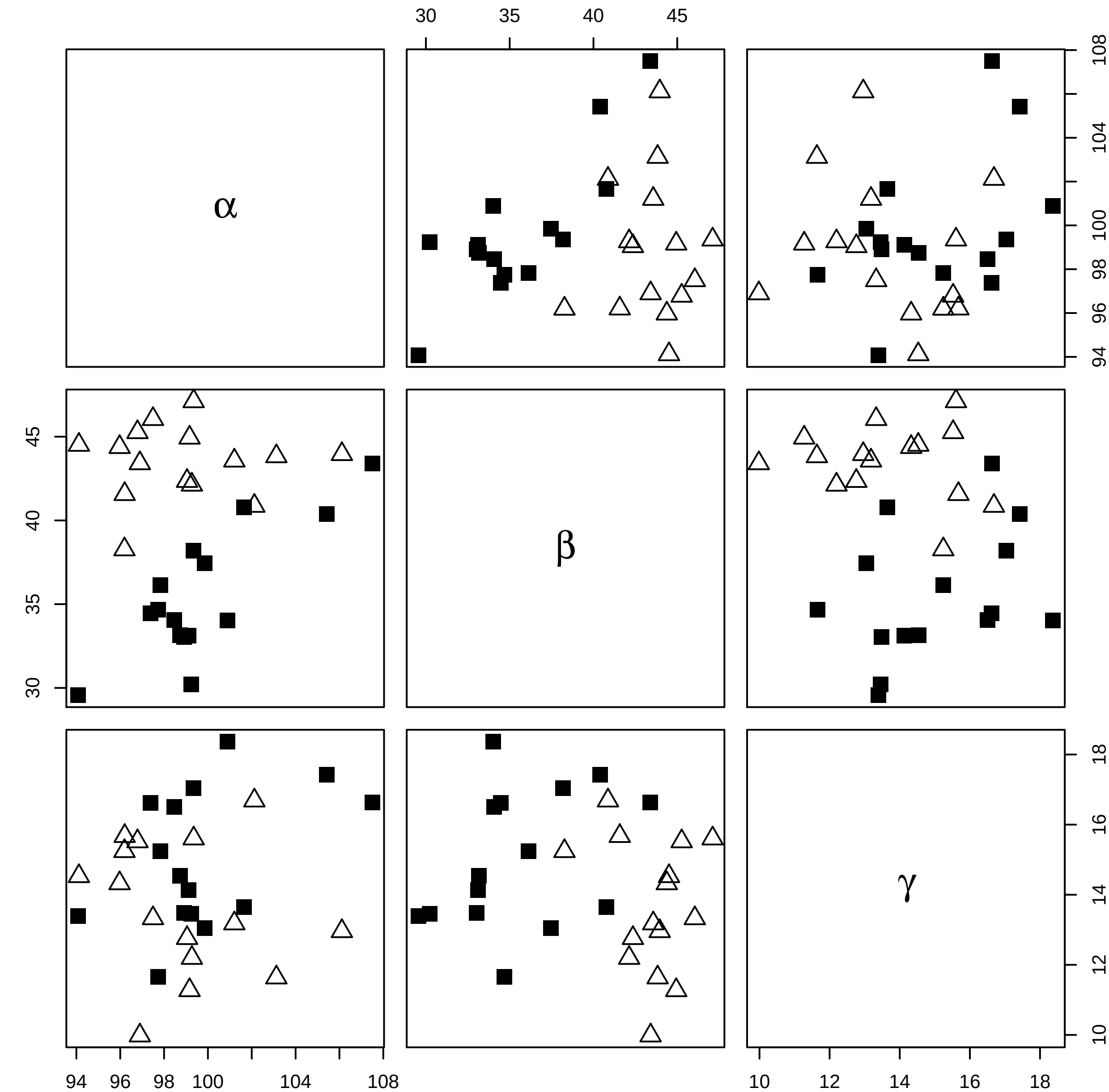

Figure 11: Illustration of 15 random sets of individual change coefficients from each class for Series 3 (The squares represent Class 1, whereas the triangles represent Class 2.).

Notice that in Figure 8 and in Figures 9, 10, and 11, the distributions of individual change coefficients are reasonably close to one another. Specifically, the multivariate standardized effect size (which is momentarily defined) is 2.613, 1.976, and 3.635, for Series 1, 2, and 3, respectively. Translating the multivariate standardized effect size (and the parameters from each of the series) into graphical displays helps to illustrate that the differences between the latent classes is not large. Thus, the methods are evaluated in conditions that do not suggest unambiguously which individual is in which class. In making the latent class differences subtle, improvement in each of the methods' effectiveness will be ensured in cases where the differences between the latent classes are more pronounced (because less misclassification will occur). This is the case because the FMM will more accurately classify individuals into their proper latent class.

Selection of Starting Values

Iterative algorithms require starting values to begin the process of convergence to (ideally) obtain the optimal set of parameter estimates. Unfortunately, iterative algorithms can fail or provide parameter estimates that are not optimal. For example, an iterative algorithm designed to find the maximum likelihood estimate may find a local minimum rather than the global minimum for the discrepancy function. Although the algorithm might have converged, because the global minimum was not found, the parameters calculated from a local minimum would not be the maximum likelihood estimates. Thus, the particular set of starting values might lead to a poor set of parameter estimates. In addition to obtaining parameter estimates that differ from the maximum likelihood estimates, starting values can lead to nonconverging algorithms or convergence may be reached at implausible values (e.g., a correlation whose absolute value is greater than one, a negative variance, etc.).

Designing simulation studies that involve iterative procedures requires a method to chose the starting values to be used to invoke the iterative algorithms. While sometimes it may seem as though "no starting values are specified," in such cases the algorithm or procedure has a default set or a default procedures that determines the starting values for the iterative algorithm. In simulation studies it is sometimes reasonable to set the starting values to the population values (e.g., Chen, Chen, & Kalbfleisch, 2004). It is also sometimes reasonable to use multiple sets of starting values (e.g., Steinley, 2003) and select the results that correspond to the largest (log)likelihood function and thus the presumed "best fit" out of several attempts. The starting values used for each of the three techniques that require starting values in the present simulation study are based on algorithms that use only the observed data. Thus, the simulation takes into consideration the limitations of applied researchers in that an applied researcher cannot specify the starting values equal to their population values. Due to the already burdensome simulation, one set of starting values will be used. Because of the iterative nature of estimating the individual change coefficients, the FMM, and the MLM, the method of choosing the starting values will be given for each of the three procedures.

The starting values for individual estimates of the logistic growth curve of Equation 51 uses the SSlogis() function. This function provides starting values for the logistic growth curve of Equation 51 using the nls() function. The starting values are obtained by first taking the logistic transformation:

$$z_t = \log\left(\frac{y_t}{1 - y_t}\right). \tag{52}$$

Given z_t (i.e., the score after logistic transformation), a simple linear regression model is fit so that estimates of ϕ_0 and ϕ_1 can respectively be used as starting values for β and γ:

$$a_t = \phi_0 + \phi_1 z_t. \tag{53}$$

Given ϕ_0 and ϕ_1, the logistic growth curve of Equation 51 is fit using a partially linear model (see Appendix C.7 of Pinheiro & Bates, 2000).

Fitting of the FMM begins by first fitting a hierarchical agglomeration model. From the hierarchical agglomeration, the classification results based on the desired number of latent classes (in this case two) was used as the starting classifications for input into an EM algorithm. Given the estimated hierarchical agglomeration classifications, the EM algorithm starting with the M-step was implemented (Fraley & Raftery, 2002; see also the user guides accompanying the MCLUST package, Fraley & Raftery, 2004).

The procedure for obtaining starting values for the nonlinear MLM is conditional on the estimation method. For Method 1, the results from the nlsList() function using the Gauss-Newton estimation method were used as input into the nlme() function. The nlsList() function calculates estimates of the individual change coefficients for all of the individuals for the specified functional form of change. As recommended by Pinheiro and Bates (2000; see also the user guides accompanying the NLME package, Pinheiro et al., 2004), the results from the nlsList() function were used directly in the nlme() function as starting values for the fixed effects and covariance structure for each of the *G* (two in the present study) nonlinear MLMs. The fixed effects obtained from Method 1 were used as the starting values for Method 2. Because Method 2 is a multiple group MLM for the *G* latent classes, there exist parameter estimates for the fixed effects from the reference class and a deviation between the values from the reference class fixed effect parameters and each of the other classes. Thus, the actual set of starting values was the fixed effects from the first class, and the difference between the fixed effect parameter estimates of the first class and the second class. The starting values for Method 3.1 and 3.2 are the same as for Method 1 and Method 2, respectively. The starting values for Method 4 are the same as the first two steps in Method 1 and Method 2.

Measures

The simulation study proposed here is meant to determine how effective the proposed estimation procedures are at recovering the parameters from the LCDC model of nonlinear heterogeneous change. The quantification of the effectiveness of the estimation procedures will be in terms of the discrepancy between the known population values and the estimated values obtained from the LCDC model. In particular, the bias and relative bias from each condition will be determined empirically from the simulation study. The bias for the *r*th parameter is operationally defined as

$$\mathrm{B}_r = \xi(\hat{\boldsymbol{\omega}}_r) - \omega_r, \tag{54}$$

whereas the relative bias for the *r*th parameter is operationally defined as

$$\mathrm{P}_r = \frac{\xi(\hat{\boldsymbol{\omega}}_r) - \omega_r}{\omega_r} = \frac{\mathrm{B}_r}{\omega_r}, \tag{55}$$

where ω_r represents the *r*th parameter contained in the LCDC parameter vector $\boldsymbol{\Omega}$ from Equation 49, $\hat{\boldsymbol{\omega}}_r$ represents a vector of estimates of ω_r obtained from the replications within a particular condition from the simulation study, and $\xi(\cdot)$ is a function of $\hat{\boldsymbol{\omega}}_r$ (e.g., the mean and/or median). As can be seen from Equations 54 and 55, a value of zero would represent a scenario where the estimated value is equal to the population value (i.e., the estimate would be unbiased). Whereas the bias quantifies the raw discrepancy between estimated and population values, the relative bias scales the bias so that the degree of discrepancy is relative to the population value of the parameter. Although these two discrepancy measures have a one-to-one relationship, they quantify the discrepancy in different ways. For example, in some cases a bias of five units may be considered trivial (e.g., when $\omega_r = 500$), whereas other times a bias of five units may be tremendously large (e.g., when $\omega_r = .1$). The relative bias would illustrate the fact that the latter scenario, where $P_r = 50$, is proportionally (in terms of the population value) much worse than the former scenario, where $P_r = .01$.

The function $\xi(\cdot)$ will be specified as both the mean and the median. That is, the mean and the median will be used in the calculation of B_r and P_r in an effort to determine the effectiveness of the proposed procedures in estimating the parameters from $\boldsymbol{\Omega}$. Thus, for each parameter from $\boldsymbol{\Omega}$ in each condition two B_rs and two P_rs will be obtained. When B_r and P_r are estimated by making use of the mean value of the estimated parameters, they will be denoted $\bar{B}_r$ and $\bar{P}_r$, respectively. When B_r and P_r are estimated by making use of the median from the estimated parameters, they will be denoted $\tilde{B}$ and $\tilde{P}$, respectively. In order to gauge the variability and precision of the parameter estimates, the standard deviation of the parameter estimates will also be collected. When the distribution of the parameter estimates is symmetric, the biases and relative biases based on the mean and median will be equal. Because of the unknown nature of the distribution of the parameter estimates, their skew and kurtosis will also be calculated.[32]

In addition to $\bar{B}_r$, $\bar{P}_r$, $\tilde{B}_r$, $\tilde{P}_r$, the variance, skew, and kurtosis for each of the estimated parameters, the classification success rate and the estimated π values will also be collected. Because the success of the FMM is not directly of interest, the classification

[32]The values of skew and kurtosis will be collected in an effort to determine characteristics of the estimates obtained from the proposed procedures, not to necessarily evaluate the effectiveness of the procedures. Because the goal of the simulation study is to determine the effectiveness of the procedures in capturing the population parameters contained in $\boldsymbol{\Omega}$, a procedure can be ruled "successful" even if the methods produce highly skewed and/or highly kurtotic sampling distributions. Nevertheless, values of skew and kurtosis have the potential to be helpful on some levels of evaluation and may provide additional insight into differences that might exist between the estimation procedures at some point in the future. Measures of skew and kurtosis will be calculated in accordance with the recommendations of Stuart and Ord (1994, chapter 3). In particular, the degree of skew is defined as $\frac{n^{-1}\sum_{i=1}^{n} w_i^3}{\left(n^{-1}\sum_{i=1}^{n} w_i^2\right)^{1.5}}$, whereas the degree of kurtosis is defined as $\frac{n^{-1}\sum_{i=1}^{n} w_i^4}{\left(n^{-1}\sum_{i=1}^{n} w_i^2\right)^{2}} - 3$, where $w_i = W_i - \bar{W}$ and n is the number of W_i values. Given the definition of skew and kurtosis used here, the skew and kurtosis of a normal distribution is zero.

success and the estimated π values will not be used directly. The purpose of collecting classification success rate and the estimated πs is for archival and completeness purposes.

A way to quantify the difference between the fixed effects is by making use of an analog of the Mahalanobis distance (Mahalanobis, 1936). In particular, a standardized multidimensional effect size can be used in the present situation in order to describe simultaneously how far apart the fixed effects of each latent class are relative to their variance:

$$\Delta = [(\boldsymbol{\omega}_1 - \boldsymbol{\omega}_2)' \boldsymbol{\Sigma}_{\boldsymbol{\omega}}^{-1} (\boldsymbol{\omega}_1 - \boldsymbol{\omega}_2)]^{1/2}, \tag{56}$$

where $\boldsymbol{\omega}_g$ is a vector of means from the gth class and $\boldsymbol{\Sigma}_{\omega}^{-1}$ is a covariance matrix of the parameters contained in $\boldsymbol{\omega}_g$ (recall that the within class variance structures will be held constant across class). A special case of Equation 56 is the standardized mean difference for a single fixed effect. In particular, for a single fixed effect, the Mahalanobis distance for a univariate effect size is defined as

$$\delta_p = \frac{|\omega_{p_1} - \omega_{p_2}|}{\sigma_{\omega_p}}, \tag{57}$$

where ω_{p_g} is the population value of the pth fixed effect for the gth class, and σ_{ω_p} is the population standard deviation for the pth fixed effect. Notice that Equation 57 is oftentimes referred to as the population Cohen's *d* in the behavioral science literature (e.g., Cohen, 1988, chapter 3) or as the population Glass's g' in the educational literature (Glass, 1976; see also Hedges, 1981). Technically, what was manipulated in the simulation study was each of the univariate effect sizes measures for each of the fixed effects. The Mahalanobis distance is used to quantify the difference between the groups given all three of the univariate effect sizes.

Although not a problem in practice, a problem that arises in the context of simulations studies involving the successfulness of a clustering/classification procedure is deciding which cluster should be labeled "Class *g*." The FMM algorithm combined with the use of the optimal rule of classification generally yields the optimal method of classification of the N entities into G crisp sets. In order to evaluate how successful any procedure is via simulation methods, the realized classes cannot be haphazardly labeled “Class 1” or “Class 2” (of course the problem becomes even more burdensome when more than two classes are of interest). That is, the model implied classifications need to be linked to the classes they are thought to represent. Basford and McLachlan (1985; see also McLachlan & Basford, 1988, section 5.1) use the following rule: an observed clustering of individuals is labeled Class g if the proportion of individuals within the observed cluster belonging to Class g is higher than in any other cluster. For example, suppose 85% of the individuals in the model implied Cluster A (one of two clusters observed after applying a FMM and the optimal rule of classification) are actually members of Class 1, whereas 75% of the individuals in the model implied Cluster B are actually members of Class 2. Cluster A would then be labeled Class 1 and Cluster B would be labeled Class 2. As another example, suppose 55% of the individuals in Cluster A are actually members of Class 1, whereas 60% of the individuals from Cluster B are actually members of Class 1. Cluster B would then be labeled Class 1 and Cluster A would be labeled Class 2. This is the same rule used by Bayne, Beauchamp, Begovich, and Kane (1980) in their simulation work on the effectiveness of various clustering procedures.

Although such a class identification rule is reasonable when determining which cluster is associated with which class, it is not the only method that can be used. For example, from two bivariate distributions with means $(0, 0)$ and $(\varphi, 0)$, where $\varphi > 0$, Kuiper and Fisher (1975) identified the cluster with the largest mean for the first variable as belonging to Class 2. An extension of the method used by Kuiper and Fisher (1975) would be to identify the cluster with the largest observed Mahalanobis effect size as belonging to the class with the largest population Mahalanobis effect size, the smallest observed Mahalanobis effect size as belonging to the class with the smallest population

Mahalanobis effect size, et cetera. Although making use of the Mahalanobis effect size seems reasonable, it cannot be more effective in terms of the proportion of individuals correctly classified than the rule used by Basford and McLachlan (1985). This is because the rule employed by Basford and McLachlan (1985) ensures that identification of class membership is based on the largest proportion of correctly classified individuals on each replication of the simulation. For these reasons and following the methods used by Basford and McLachlan (1985) and McLachlan and Basford (1988), the present simulation study will use the identification rule based on the largest proportion of individuals that belong to a cluster.

Measures of Accuracy

Accuracy in a statistical sense can be conceptualized as the square root of the mean square error (RMS). The RMS is a quantitative measure of the degree to which an estimated value represents the population value it is supposed to represent. The statistical definition of accuracy is given as

$$\sqrt{E[(\hat{\theta} - \theta)^2]} = \sqrt{E[(\hat{\theta} - E[\hat{\theta}])^2] + (E[\hat{\theta}] - \theta)^2} = \text{RMS}, \tag{58}$$

where θ is a parameter of interest and $\hat{\theta}$ is an estimate of the parameter (Rozeboom, 1966, p. 500). Notice that the first component under the second radical sign is the precision. Further, notice that the second component under the second radical sign is the squared bias. Thus, as can be seen from Equation 58, accuracy is a function of bias and precision. As a measure becomes more accurate the value of the RMS decreases. The more inaccurate a measure, the larger the RMS becomes. In situations where an estimate is unbiased, the RMS (i.e., the accuracy), is equal to the precision, which is defined as the standard deviation of the estimates.[33]

[33]Perhaps the term "accuracy" used to describe the effectiveness of the recovered parameters should be termed "inaccuracy," in that lower levels (i.e., smaller values) imply a more accurate measure. Nevertheless,

Although the accuracy of a measure is an important piece of information in determining how well an estimation procedure recovers the population values, accuracy measures of the form given in Equation 58 are necessarily univariate and can thus not be evaluated simultaneously for several parameter estimates. However, generalized accuracy measures can be used that attempt to simultaneously evaluate an estimation procedure across multiple parameter estimates. In an attempt to simultaneously evaluate the collection of fixed effects for each method and situation, generalized accuracy of the set of fixed effects was carried out with three different techniques. The first method of calculating the generalized accuracy was by taking the determinant of the discrepancy matrix. The discrepancy matrix was determined by calculating the discrepancy between the population value and the obtained value for each of the replications in each of the situations. Thus, for each of the 72 situations, a matrix with 1,000 rows and six columns (for Conditions 1–16) or a matrix of 1,000 rows and four columns (for Conditions 17–24) was calculated. The determinant of the covariance matrix of the discrepancies was calculated and used as the first generalized accuracy measure. Recall that the determinant of a covariance matrix can be conceptualized as a generalized variance (Rencher, 2000, p. 69). Because accuracy is itself the square root of the variance of the discrepancies (which is itself a function of the bias and precision), the generalized variance (i.e., the determinant) of the discrepancies between the observed and the true values is a useful measure of the degree to which the estimated fixed effects represent there corresponding population values. When the discrepancies tend to be small (meaning that the parameters were recovered effectively), the determinant of the covariance matrix of discrepancies is relatively close to zero. Conversely, when the discrepancies tend to be large (meaning that the parameters were not recovered effectively), the determinant of the covariance matrix of discrepancies is relatively large.

---

the way in which accuracy is used in the present work implies that a more accurate measure has a smaller value.

The determinant of a covariance matrix is a function of the main diagonal and the off diagonals. In the special case of a two by two covariance matrix **A**, the determinant is

$$\det(\mathbf{A}_{2,2}) = a_{11}a_{22} - a_{12}^2, \tag{59}$$

where a_{jk} is the element of the matrix **A** in the jth row in the kth column. As can be seen in Equation 59, the covariance between the variables is subtracted from the product of the variances (specifically in the case of a two by two matrix, but the idea generalized to k by k matrices). Because it is not clear how the covariance of the discrepancies should or should not affect the overall accuracy, another generalized accuracy was calculated based only on the diagonal of the covariance matrix of the discrepancies. The second method of describing the overall accuracy of the fixed effects is defined as the determinant of the variances of the discrepancies (i.e., the covariance matrix of the discrepancies with the off diagonals set to zero).

Another useful but fundamentally different approach to estimate the generalized accuracy is to first calculate the prediction equation for each of the 1,000 replications in the 72 situations for each of the five methods given the estimated fixed effect values. The discrepancies between the observed and population values at the timepoints are then determined. The variance of these discrepancies is determined in an analogous fashion as the residual variance in a regression context. The major difference is that this method of determining the "residual variance" is not based on the observed minus the predicted scores, but rather on the predicted scores minus the true scores. Determining the residual variance in this manner is another method at quantifying the generalized accuracy of the set of fixed effects. Because each of the three methods of generalized accuracy are each reasonable ways to define an overarching measure of accuracy, all three will be used in the evaluation of the results. If the pattern of results were to hold across the three generalized accuracy measures across many of the situations, strong evidence would exist for the rank ordering of the methods in terms of accuracy.

Although evaluating other aspects of the methods' effectiveness may be possible, the fixed effects are regarded as the most important set of estimates. Without accurate fixed effects, accurate standard errors and/or correlation structures are moot. If the fixed effects are themselves accurately estimated, even if the model's standard errors obtained by fitting a nonlinear MLM in the stage-wise procedures discussed are incorrect, alternative methods of estimating the model's standard errors exist (e.g., the jackknife or bootstrap correction for standard errors, the degrees of freedom can be modified, etc.) and can be used. Regardless of the other ways in which the effectiveness of the methods might be evaluated, the accurate estimation of the fixed effects is regarded as a necessary, although not sufficient, condition for a method's success when using the LCDC model of change. Nevertheless, it is the fixed effects that are of most interest in the present study and thus the methods will be evaluated based on their success at recovering the parameters of the fixed effects.

RESULTS

Determining the accuracy of the recovered fixed effects is important to determine how well each of the proposed methods worked in the various situations examined in the Monte Carlo simulation study. Although unbiasedness is a desirable property for an estimation procedure to posses, as is shown in Equation 58, accuracy is a function of both bias and precision. Thus, it is not necessary to have an unbiased estimate in order to have an accurate estimate. Recall that the preferred method is operationally defined as the one that most accurately recovers the population fixed effects of the LCDC model. However, as is discussed in the previous section, determining the "most accurate" method is not trivial. Because of the numerous conditions examined and the multiple methods of evaluating accuracy that were used in the study, the best way to simultaneously evaluate all combinations of condition, scenario, and accuracy measure was not known. Ideally, the most accurate and thus optimal method will identify itself by consistently outperforming the other methods. Should one of the methods distinguish itself from the other methods by consistently and unambiguously achieving more accurate estimates of the fixed effects, the optimal method will be straightforward to identify. If such clear-cut results emerge, a measure that simultaneously evaluates all situations and all measures of accuracy need not be developed or used as an evaluation tool. However, if the "best method" is itself not very effective at recovering the parameters of interest, it is a moot point to determine the accuracy of the methods relative to one another. Thus, a necessary first step in evaluating the methods is to examine the bias and the relative bias to see if any or all of the methods are reasonably effective at recovering the population values of the fixed effects.

Table 4 shows results of the bias and relative bias for each of the fixed effects (recall α_g is the asymptotic value for the *g*th class, β_g is the point-of-inflection for the *g*th class, and γ_g relates to the curvature of the *g*th class), as well as the bias and relative bias for the parameters of the correlation structure and the error variance(s), for all of the methods in C1S1 (recall that C*x*S*y* represents the notation for identifying the *x*th condition and the *y*th scenario). Each of the numbers in the table are based on 1,000 replications of the simulation study. Note also that values of skew and kurtosis are given in the tables. The skew and kurtosis for the observed values, the bias, and the relative bias are all the same and thus there need not be a separate skew/kurtosis value for each.[34] As previously mentioned, the results from replications that failed are not included in any of the reported results. Although widespread failures of the methods would cause concern, there were relatively few failures. Across all methods there were a total of 128 failures out of 72,128 total attempts (original attempts plus the necessary additional replications), which is approximately .18% of the total number of attempts. Table C-1 in Appendix C gives specific cells and counts where the failures occurred. Not surprisingly, the majority of the failures occurred in situations with small sample sizes in one group (e.g., Scenario 3), larger error variances (e.g., $\sigma^2_\varepsilon = 8$ rather than $\sigma^2_\varepsilon = 4$), and fewer occasions of measurement (e.g., $T = 15$ rather than $T = 25$).

[34] Although the results of the bias and relative bias for the correlational structure, as well as the values for the skew and kurtosis are included in the tables of results, only the fixed effects are used in the evaluation of the methods. While the accuracy of the recovered correlational structure is an important component of the LCDC model, for purposes of the present work only the fixed effects were evaluated. Perusal of the tables of results suggests that the accuracy with which the correlational structure was recovered closely resembled the accuracy of the fixed effects. The inclusion of the skew and kurtosis and of the results for the correlational structure are included for completeness and for archival purposes.

TABLE 4

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 1 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 1; Scenario 1; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.242	-0.156	0.226	-0.116	0.136	-0.083	-0.102	-0.064	-0.118	-0.081	-0.088	-0.088	-0.002	0.001
	$\widehat{M}$	0.256	-0.155	0.255	-0.127	0.15	-0.109	-0.104	-0.065	-0.134	-0.091	-0.087	-0.087	-0.001	-0.005
	$\widehat{\sigma}^2$	0.409	0.388	0.305	0.266	0.222	0.202	0.016	0.012	0.022	0.019	0.019	0.019	0.015	0.015
	2.5%	-1.01	-1.409	-0.866	-1.026	-0.801	-0.884	-0.339	-0.265	-0.37	-0.318	-0.344	-0.344	-0.24	-0.224
	97.5%	1.443	1.146	1.306	0.936	0.946	0.857	0.169	0.175	0.225	0.213	0.182	0.182	0.249	0.248
P	$\widehat{\mu}$	0.002	-0.002	0.006	-0.004	0.009	-0.008	-0.204	-0.127	-0.393	-0.27	-0.219	-0.219	< .001	< .001
	$\widehat{M}$	0.003	-0.002	0.007	-0.004	0.01	-0.109	-0.209	-0.131	-0.447	-0.303	-0.219	-0.219	< .001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.064	0.047	0.249	0.211	0.116	0.116	0.001	0.001
	2.5%	-0.01	-0.015	-0.025	-0.035	-0.053	-0.083	-0.678	-0.529	-1.234	-1.061	-0.86	-0.86	-0.06	-0.056
	97.5%	0.014	0.012	0.037	0.032	0.063	0.08	0.339	0.35	0.749	0.708	0.456	0.456	0.062	0.062
Skew		-0.084	0.407	0.031	0.127	-1.173	0.468	0.154	0.213	0.7	0.533	0.151	0.151	0.05	0.183
Kurtosis		3.134	2.263	2.415	3.263	7.523	6.561	0.717	0.518	1.156	1.046	0.509	0.509	0.082	0.245

Condition 1; Scenario 1; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.247	-0.157	0.229	-0.116	0.137	-0.084	-0.076	-0.093	-0.07	-0.002
	$\widehat{M}$	0.257	-0.155	0.259	-0.127	0.151	-0.11	-0.082	-0.106	-0.076	-0.007
	$\widehat{\sigma}^2$	0.416	0.386	0.308	0.265	0.224	0.201	0.01	0.016	0.013	0.007
	2.5%	-1.004	-1.404	-0.868	-1.026	-0.804	-0.88	-0.252	-0.3	-0.272	-0.157
	97.5%	1.456	1.138	1.321	0.939	0.951	0.856	0.171	0.209	0.16	0.177
P	$\widehat{\mu}$	0.002	-0.002	0.007	-0.004	0.009	-0.008	-0.153	-0.308	-0.174	< .001
	$\widehat{M}$	0.003	-0.002	0.007	-0.004	0.01	-0.01	-0.165	-0.354	-0.19	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.041	0.18	0.081	< .001
	2.5%	-0.01	-0.015	-0.025	-0.035	-0.054	-0.082	-0.503	-0.999	-0.68	-0.039
	97.5%	0.015	0.012	0.038	0.032	0.063	0.08	0.343	0.696	0.399	0.044
Skew		-0.061	0.414	0.042	0.13	-1.156	0.47	0.581	0.959	0.399	0.193
Kurtosis		3.125	2.266	2.434	3.258	7.416	6.566	1.079	2.041	0.714	0.147

Condition1; Scenario1; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.225	-0.2	0.195	-0.13	0.121	-0.101	-0.1	-0.067	-0.117	-0.086	-0.084	-0.064	0.003	0.001
	$\widehat{M}$	0.245	-0.185	0.235	-0.142	0.136	-0.125	-0.103	-0.068	-0.134	-0.096	-0.084	-0.063	0.004	-0.004
	$\widehat{\sigma}^2$	0.352	0.328	0.277	0.225	0.18	0.168	0.016	0.011	0.021	0.018	0.018	0.015	0.015	0.015
	2.5%	-0.968	-1.395	-0.844	-0.989	-0.769	-0.893	-0.336	-0.265	-0.369	-0.328	-0.341	-0.303	-0.239	-0.239
	97.5%	1.333	0.965	1.234	0.832	0.898	0.753	0.167	0.151	0.217	0.191	0.183	0.176	0.248	0.248
P	$\widehat{\mu}$	0.002	-0.002	0.006	-0.004	0.008	-0.009	-0.201	-0.134	-0.39	-0.288	-0.21	-0.161	0.001	< .001
	$\widehat{M}$	0.002	-0.002	0.007	-0.005	0.009	-0.125	-0.206	-0.136	-0.445	-0.318	-0.209	-0.158	0.001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.001	0.064	0.046	0.237	0.202	0.111	0.096	0.001	0.001
	2.5%	-0.01	-0.015	-0.024	-0.034	-0.051	-0.083	-0.673	-0.529	-1.229	-1.093	-0.852	-0.757	-0.06	-0.06
	97.5%	0.013	0.01	0.035	0.028	0.06	0.07	0.334	0.301	0.724	0.637	0.457	0.44	0.062	0.062
Skew		-0.014	0.218	-0.285	0.424	-0.56	0.479	0.164	0.285	0.602	0.463	-0.006	0.037	0.1	0.144
Kurtosis		1.864	2.132	1.523	1.599	2.453	1.844	0.806	0.829	0.879	0.935	0.109	0.065	0.189	0.227

Condition 1; Scenario 1; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.235	-0.197	0.202	-0.132	0.129	-0.103	-0.079	-0.096	-0.07	-0.002
	$\widehat{M}$	0.251	-0.181	0.233	-0.145	0.144	-0.126	-0.084	-0.109	-0.076	-0.007
	$\widehat{\sigma}^2$	0.377	0.337	0.288	0.24	0.189	0.178	0.01	0.015	0.012	0.007
	2.5%	-0.988	-1.404	-0.868	-1.026	-0.77	-0.88	-0.252	-0.3	-0.267	-0.157
	97.5%	1.38	1.007	1.232	0.847	0.931	0.755	0.153	0.188	0.156	0.177
P	$\widehat{\mu}$	0.002	-0.002	0.006	-0.004	0.009	-0.01	-0.158	-0.32	-0.176	< .001
	$\widehat{M}$	0.003	-0.002	0.007	-0.005	0.01	-0.012	-0.169	-0.362	-0.19	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.039	0.168	0.076	< .001
	2.5%	-0.01	-0.015	-0.025	-0.035	-0.051	-0.082	-0.503	-1	-0.668	-0.039
	97.5%	0.014	0.011	0.035	0.029	0.062	0.07	0.306	0.627	0.39	0.044
Skew		-0.153	0.157	-0.278	0.032	-0.524	0.002	0.543	0.843	0.226	0.193
Kurtosis		2.641	1.616	1.554	3.477	2.567	5.088	1.029	1.775	0.144	0.146

Condition 1; Scenario 1; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.072	0.118	0.059	0.088	0.024	0.085	0.021	0.033	0.014	–
	$\widehat{M}$	0.104	0.098	0.088	0.076	0.042	0.058	0.021	0.028	0.011	–
	$\widehat{\sigma}^2$	0.339	0.337	0.254	0.23	0.186	0.176	0.006	0.01	0.009	–
	2.5%	-1.097	-1.003	-0.985	-0.764	-0.865	-0.633	-0.128	-0.153	-0.157	–
	97.5%	1.105	1.38	0.993	1.088	0.779	1.001	0.188	0.25	0.198	–
P	$\widehat{\mu}$	0.001	0.001	0.002	0.003	0.002	0.008	0.043	0.111	0.035	–
	$\widehat{M}$	0.001	0.001	0.003	0.003	0.003	0.005	0.042	0.092	0.026	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.025	0.113	0.053	–
	2.5%	-0.011	-0.011	-0.028	-0.026	-0.058	-0.059	-0.256	-0.51	-0.393	–
	97.5%	0.011	0.015	0.028	0.037	0.052	0.093	0.375	0.833	0.494	–
Skew		-0.475	0.588	-0.37	0.453	-1.184	0.794	0.123	0.478	0.155	–
Kurtosis		2.587	1.93	1.279	1.537	6.077	4.472	0.393	1.107	0.507	–

Note: The column heading "D" represents the discrepancy (i.e., the estimate minus the population value) and the column heading "Stat." represents the statistics used to summarize the discrepancies. B represents the bias whereas P represents the relative bias. Summaries of B and P are given by $\widehat{\mu}$, $\widehat{M}$, $\widehat{\sigma}^2$, 2.5%, and 97.5%, which represent the mean, median, variance, 2.5th quantile and the 97.5th quantile, respectively. Skew and kurtosis for each of the observed parameter estimates is equal to the skew and kurtosis, respectively, of B and P. Thus, only a single value for skew and kurtosis is given for each of the parameters.

As can be seen in Table 4, $|\hat{B}|$ for α_1 of Methods 1, 2, 3.1, and 3.2 are all .225 or greater. Notice that the value of $\hat{B}$ for α_1 of Method 4 is .072. Thus, for α_1 in C1S1, the $\hat{B}$ is more than three times larger for Methods 1, 2, 3.1, and 3.2 than it is for Method 4. Similar results favoring Method 4 emerge for β_1 and γ_1 in C1S1. The results for α_2, β_2, and γ_2 are not so clearly in favor of Method 4, even though $|\hat{B}|$ is smaller for Method 4 than it is for ten of the 12 fixed effects from the other methods. In particular, $|\hat{B}|$ for γ_2 in Method 1 is .002 units smaller, and γ_2 in Method 2 is .001 units smaller than the $|\hat{B}|$ for γ_2 in Method 4. The largest $|\hat{P}|$ for any of the fixed effects was .01, which was for γ_2 in Method 3.2. This translates into the absolute largest $\hat{B}$ being only 1% of the size of the population value. Similar values for $\tilde{B}$ and $\tilde{P}$ are obtained as they were for $\hat{B}$ and $\hat{P}$, respectively. The similarity between the mean and the median discrepancy measures is because the skewness was not large, which lead to distributions of discrepancies that were nearly symmetric, implying that the mean and the median were of similar magnitude. In fact, the mean of the skewness for the 30 fixed effects (6 parameters by 5 methods) within C1S1 was only -.036 (with standard deviation .515), with the largest absolute skewness in C1S1 was -1.184 for γ_1 in Method 4.

The worst case in terms of the relative bias for any of the methods within C1S1, which was for γ_2 from Method 3.2, illustrates the effectiveness of all of the methods at recovering the population values of the fixed effects in C1S1. Recall that in the worst case within C1S1, the estimated parameter value was only .103 units too small (10.610 when the true value was 10.713). Such a trivial difference between the true and empirically obtained values illustrates that all of the methods were effective at recovering the population values of the fixed effects. Although each of the recovered parameters was shown to be accurate for C1S1, 71 other conditions exist. Treating C1S1 as if it were representative of all of the other conditions would be inappropriate without explicitly examining the other situations. However, it would be difficult report the results for each scenario within each condition as thoroughly as it was done for C1S1. Nevertheless, inspection of Tables C-2 through C-72, which are contained in Appendix C, reveals that discrepancies are generally not large in an absolute sense, and that the discrepancies are not large relative to the value of the parameter. In fact, the mean relative bias across all methods for all of the fixed effects are 0.0094, 0.0168, and -0.0044, respectively for Series 1, 2, and 3 (with standard deviations 0.0376, 0.0235, 0.0096, respectively). The median relative bias across all methods for all of the fixed effects are 0.003, 0.011, and -0.002, respectively for Series 1, 2, and 3. Thus, it is clear from examination of Table 4 and Tables C-2 through C-72 that the mean and median relative biases for the fixed effects were trivially small and that the methods were all effective at recovering the population values of the fixed effects accurately.

Further evidence that the methods are effective at recovering the population fixed effect values across all 72 situations is that, with the exception of two fixed effects (α_2 & γ_2 for Method 4 in C16S3), the 2.5% and 97.5% quantiles from the distribution of the discrepancies bracket zero. This implies that each of the methods is close to providing what is functionally an unbiased estimate of the fixed effects, in the sense that zero does not tend to be excluded from the bounds bracketing the population bias of the fixed effects. This is not to say that the simulation study showed unequivocally that all five methods were literally unbiased, but it has shown that in the situations examined the bias was not so large that zero (i.e., the case of an unbiased estimate) was generally excluded from the 95% interval bracketing the population bias. Of course, as Equation 58 shows, an estimate can be unbiased but very imprecise, creating a situation where inaccurate measures are obtained. Because of this fact, it is important to examine the precision and the bias simultaneously so that the accuracy of the estimate can be determined and ultimately compared across estimation procedures. This comparative examination is important in order to help determine if any of the methods generally outperforms the other methods so that a general recommendation regarding the best method can be made.

Given that the methods are generally accurate at recovering the fixed effect parameters, determining which of the methods is optimal in the sense of being most accurate is important. Indeed, a major reason for conducting the simulation study was to see which of the five proposed estimation methods recovers the fixed effects most accurately and under what circumstances. In determining the most accurate method, three measures of generalized accuracy are used so that as much information as is reasonably possible can be obtained. The information generated from the measures of accuracy are plotted so that comparisons can be readily made across the different estimation procedures. In particular, a figure for each of the 24 conditions is provided so that accuracy comparisons can be made visually rather than by the tedious examination of tabled values. The series of accuracy figures illustrates the individual accuracies for each fixed effect as well as for the three generalized accuracy measures, all as a function of scenario and estimation method. Recall that the three generalized accuracy measures were outlined in the previous section. In each case, the generalized accuracy measures are standardized within scenario so that relative comparisons of the methods within each scenario can be made. Because the relation among the accuracy measures is what is important, not the raw value of the generalized accuracy measure, standardization does not introduce any artifacts. Rather, standardization avoids the problem of relating different measurement scales (for each generalized accuracy measure), and of plotting raw generalized accuracy measures where some differences are very large across the different methods and some are very small.

The first three plots in Figure 12 show the raw measures of accuracy for each of the fixed effects as a function of estimation method for each of the three scenarios in Condition 1 (recall that smaller values are better). The bottom row of plots show the three different ways generalized accuracy has been conceptualized. The first measure of generalized accuracy, illustrated in the left plot of the bottom row, is described beginning on page 120. Recall that the first measure of generalized accuracy is based on the determinant of the covariance matrix of the discrepancies. The second measure of generalized accuracy, illustrated in the middle plot of the bottom row, is described beginning on page 121. Recall that the second measure of generalized accuracy is a modification of the first generalized accuracy measures that is based only on the determinant of the principal diagonal of the covariance matrix of the discrepancies. The third measure of generalized accuracy, illustrated in the right plot in the bottom row, is described beginning on page 120. Recall that the third measure of generalized accuracy attempts to provide an estimate of the RMS directly by averaging the squared deviations between the true and predicted change curve. Although each of the three generalized accuracy measures is fundamentally different, they each attempt to measure the overall accuracy of the methods. Ideally the pattern of results would be the same across each of the three plots of generalized accuracy. An inconsistent pattern of results across the accuracy measures would be analogous to an interaction in a factorial analysis of variance context. In situations where inconsistencies across the accuracy methods exist, averaging the generalized accuracy measures, analogous to interpreting the main effects in a factorial analysis of variance context, would be clouded by the inconsistency in the relative ordering of the pattern of generalized accuracies.

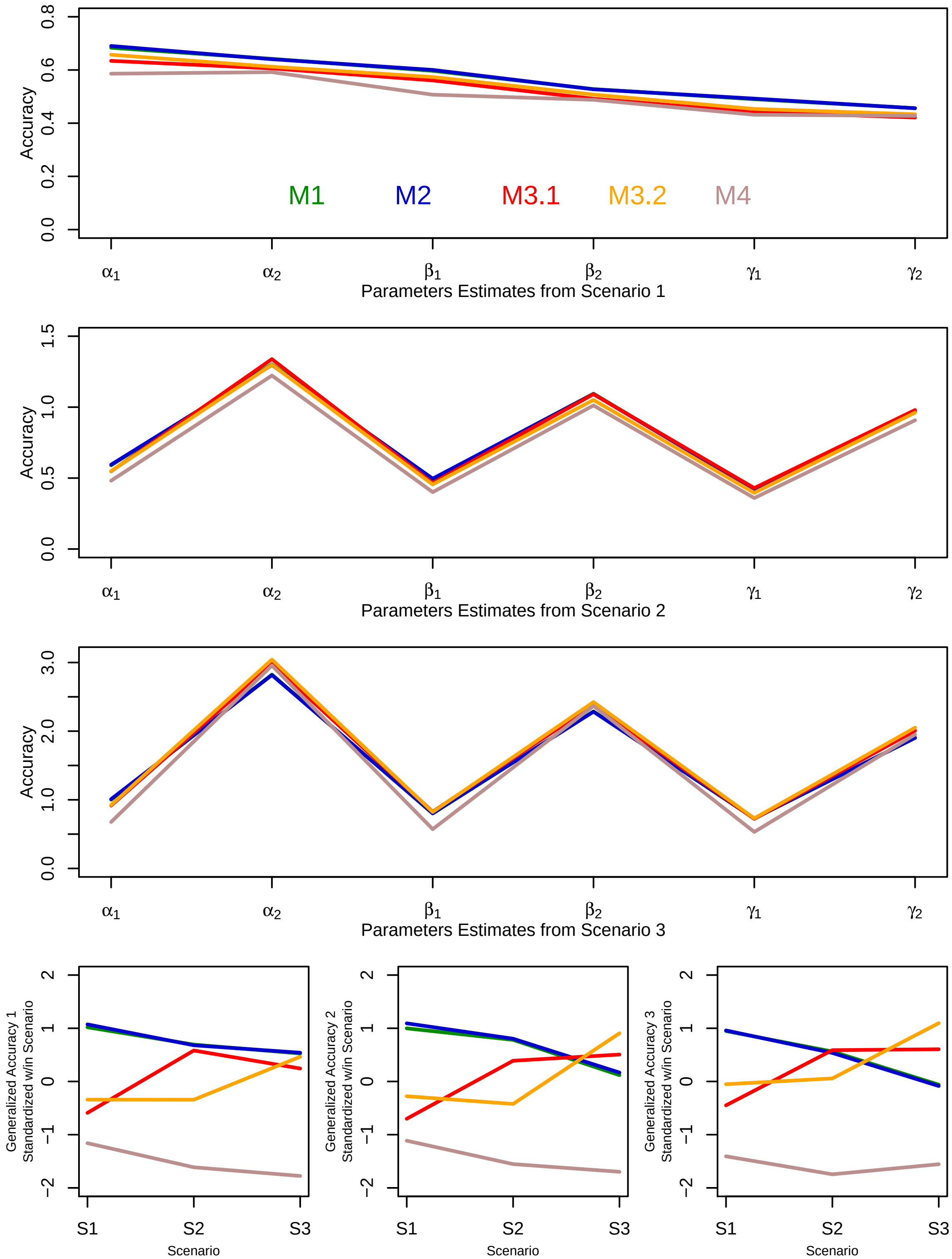

Figure 12: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 1.

The usefulness of Figure 12 is that it allows the accuracy of the estimation procedures to be compared to one another. As can be seen in each of the three scenarios of Figure 12, there is not much of a difference in any of the methods in terms of the individual accuracies. Method 4, however, is generally the most accurate method for the individual accuracies of the fixed effects. When all of the fixed effects are simultaneously taken into consideration, Method 4 is clearly the most accurate method, as judged by the three measures of generalized accuracy in each of the three scenarios of Condition 1. Given the results contained in Table 4 and Figure 12, for the parameters contained in C1S1, C1S2, and C1S3, Method 4 is clearly the best method in terms of accuracy.[35] However, it is also clearly the case that all of the methods were effective at recovering the fixed effect parameters.

As can be seen by examining the bottom row of Figure 12, the results in this particular condition are consistent across the three measures of generalized accuracy. These plots clearly show that Method 4 is the most accurate method overall in terms of recovering the fixed effect parameters of the model. Although Method 4 is clearly the best method for Condition 1, generalizing across the remaining 23 conditions without examining the analogous figures would be inappropriate. Nevertheless, examination of Figures 13 through 27 reveals similar results as those discussed regarding Figure 12 with regards to Method 4 being the most accurate and thus best method. Although there is some fluctuation in terms of the method that yields the most accurate individual fixed effect estimate in certain cases, Method 4 is uniformly the most accurate method for C1S1 through C16S3. In fact, for each of the scenarios across all three generalized accuracy measures in the 16 conditions, Method 4 was shown to be the best method.

---

[35]The reason for the relative similarity in for the individual accuracies for each of the three pairs of fixed effects in Scenario 1 is because $\pi_1 = .5$. Thus, there was approximately the same amount of information (i.e., sample size) in each of the two classes. However, the jagged pattern that emerges for the three pairs of fixed effects for Scenario 2 and Scenario 3 is because $\pi_1 > \pi_2$, and thus less information is known for Class 2 in these scenarios (since Class 1 tends to have a larger sample size).

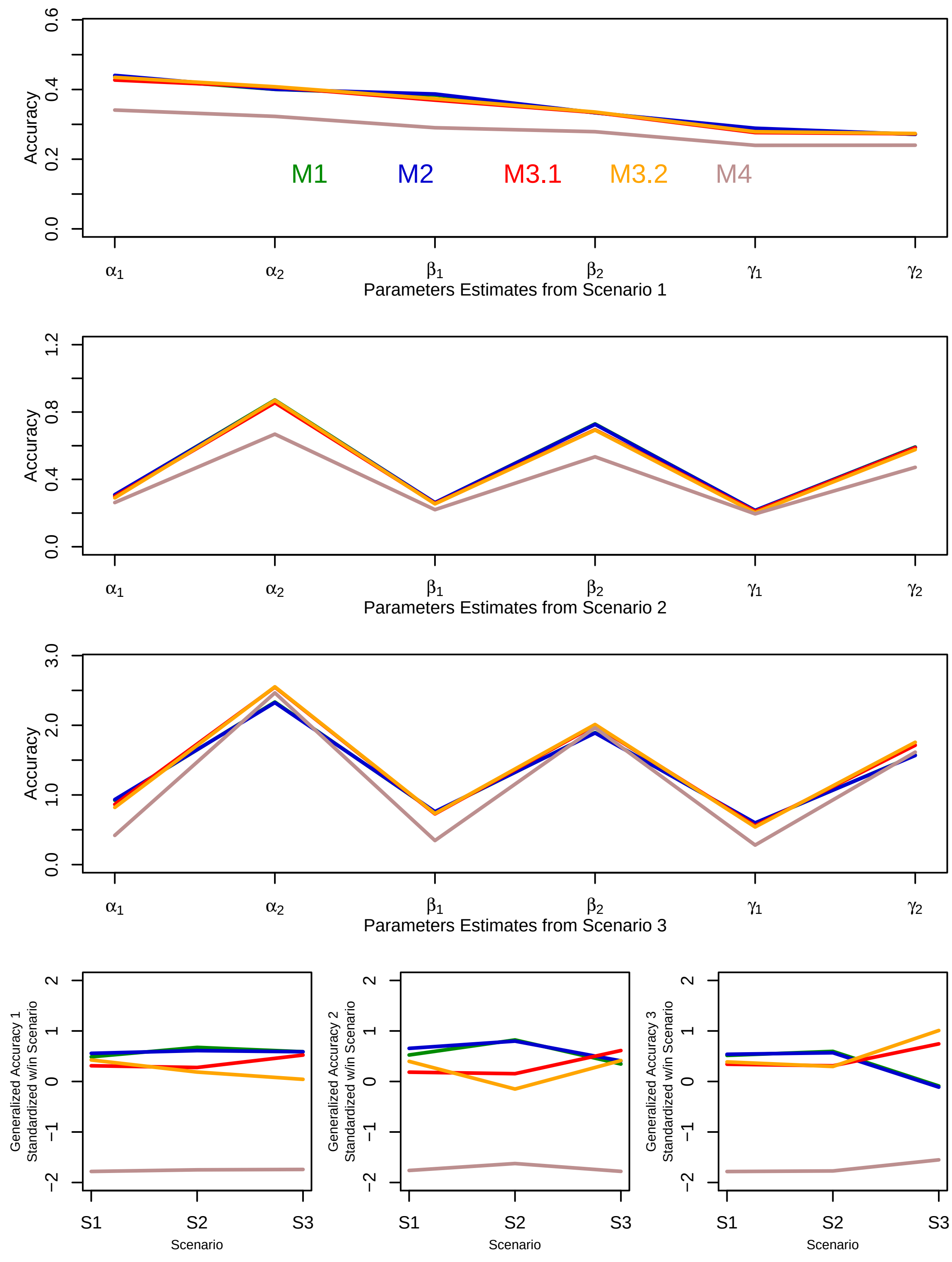

Figure 13: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 2.

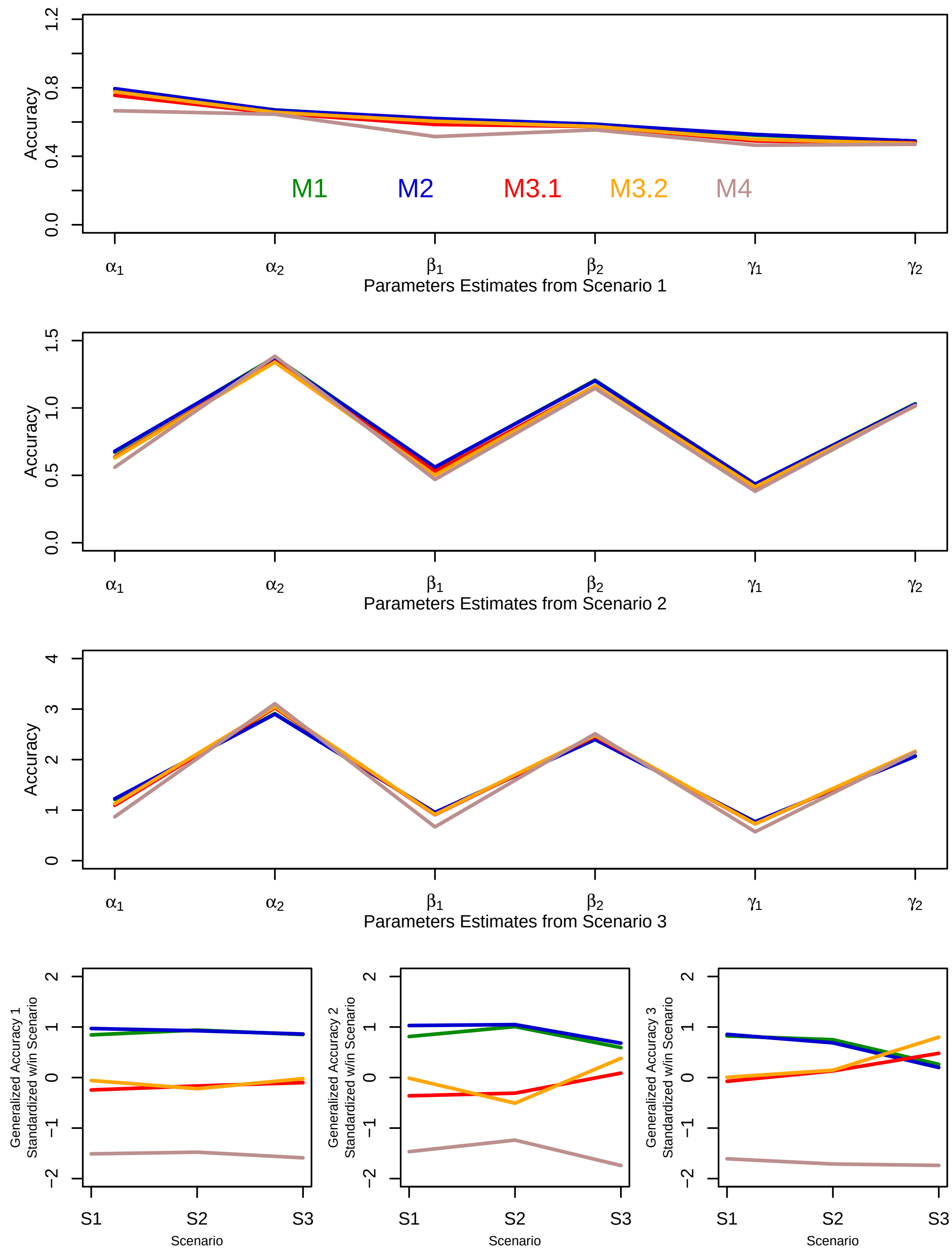

Figure 14: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 3.

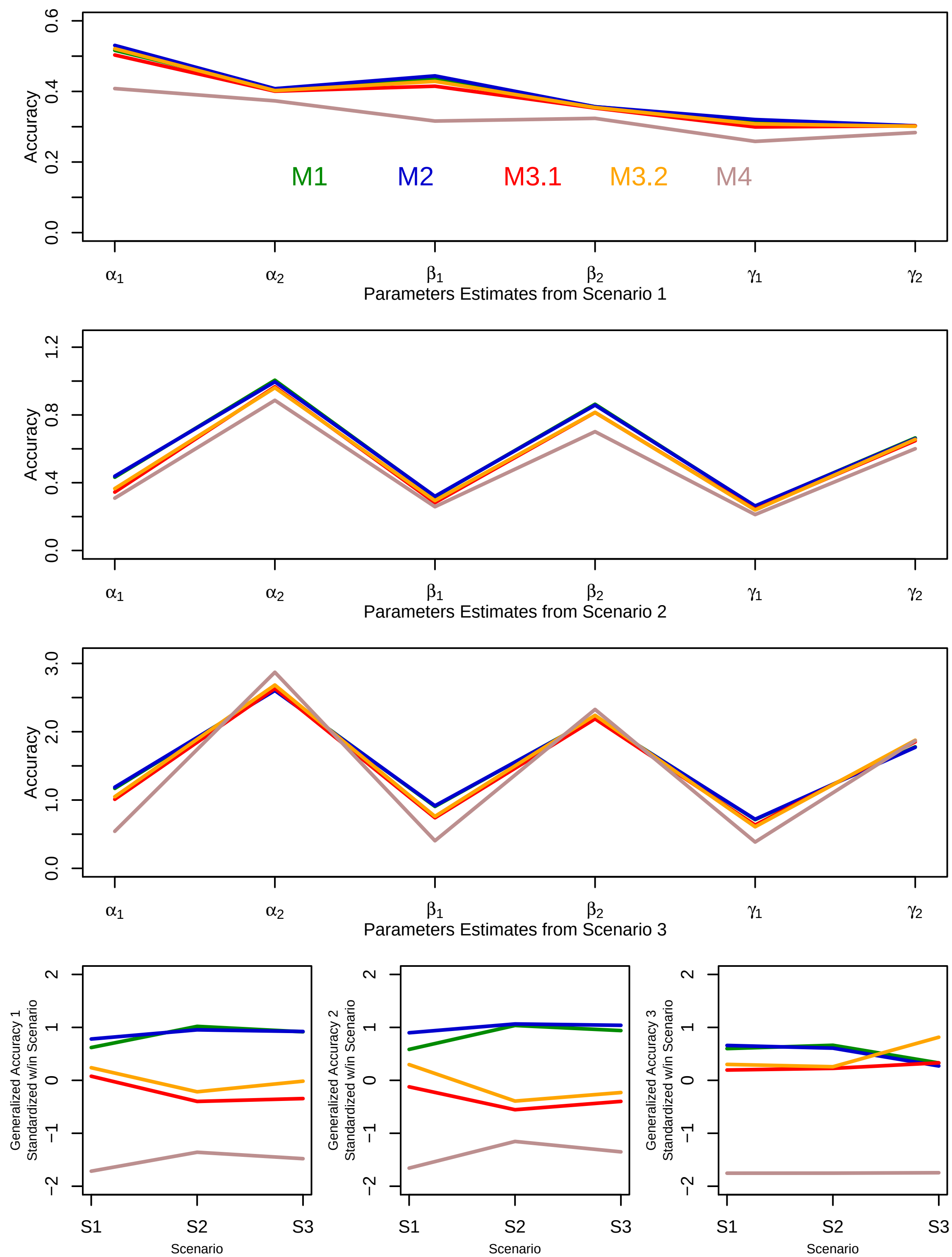

Figure 15: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 4.

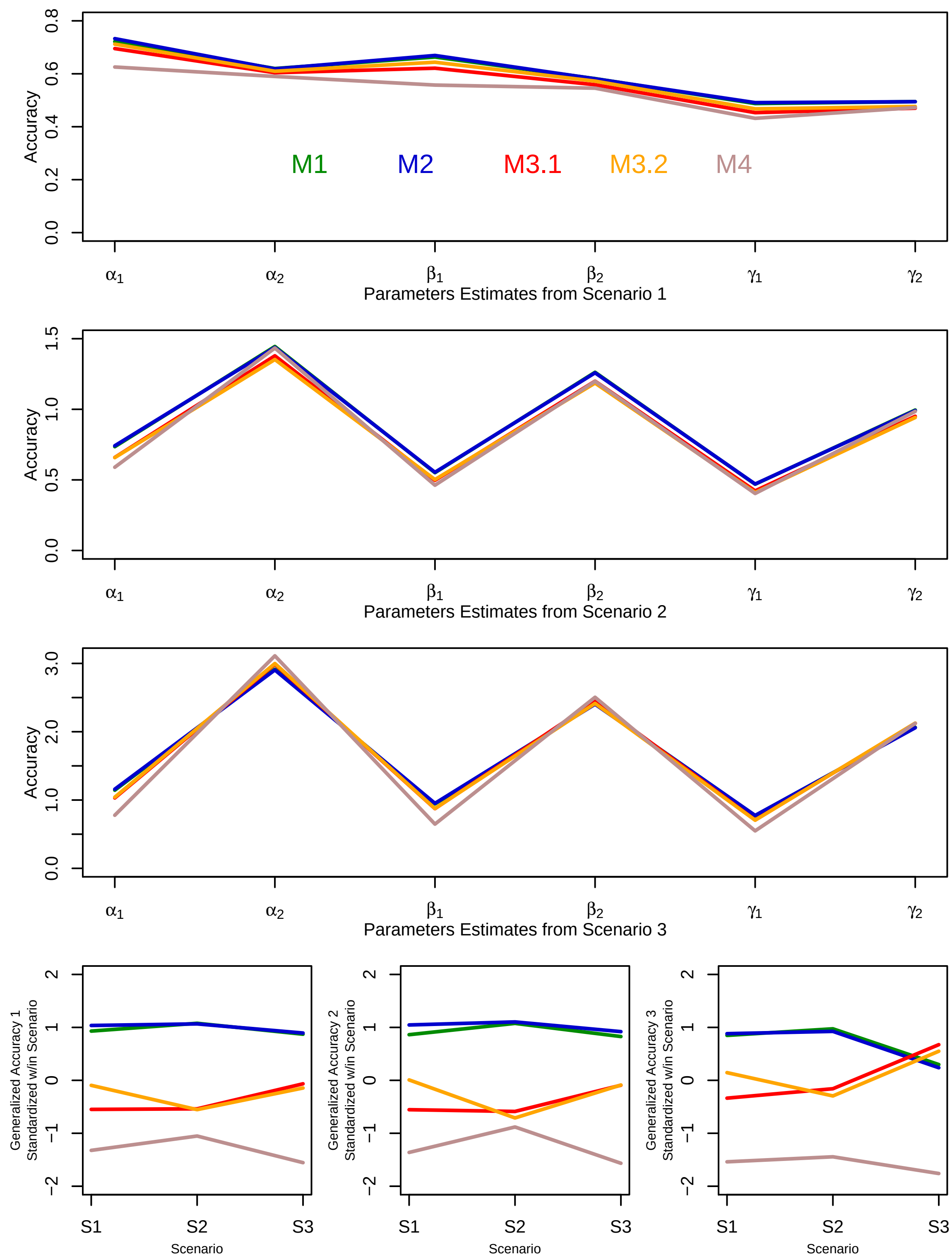

Figure 16: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 5.

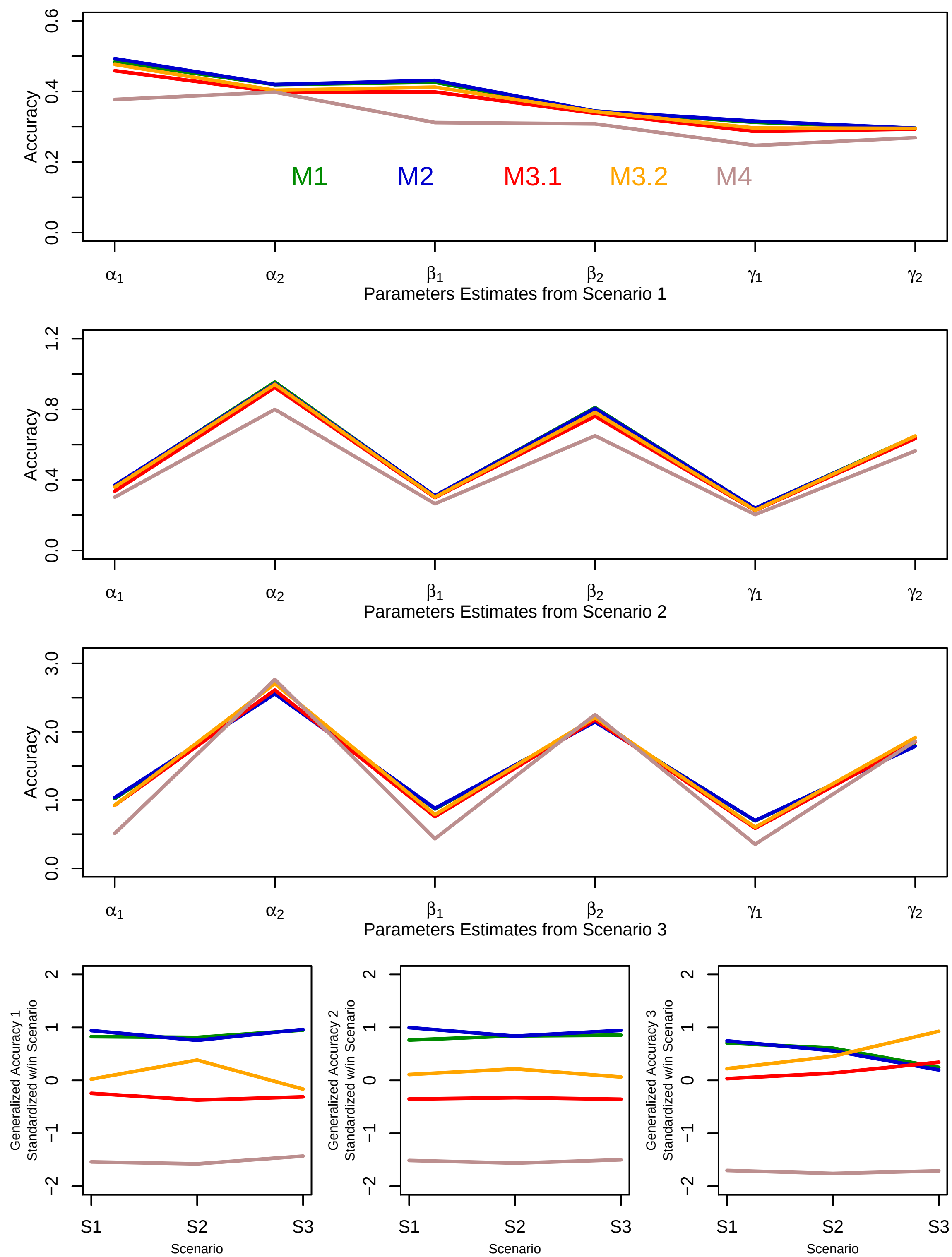

Figure 17: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 6.

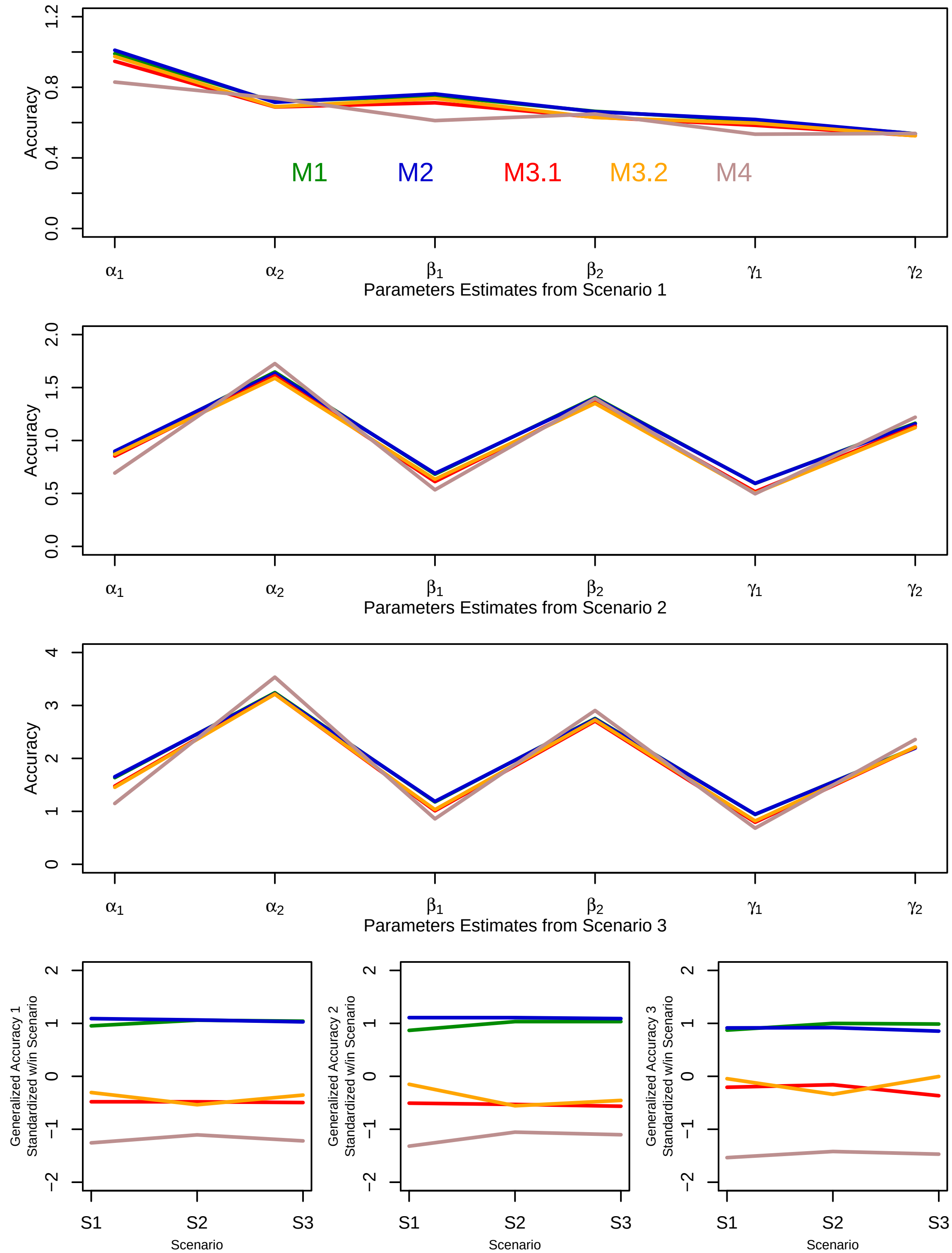

Figure 18: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 7.

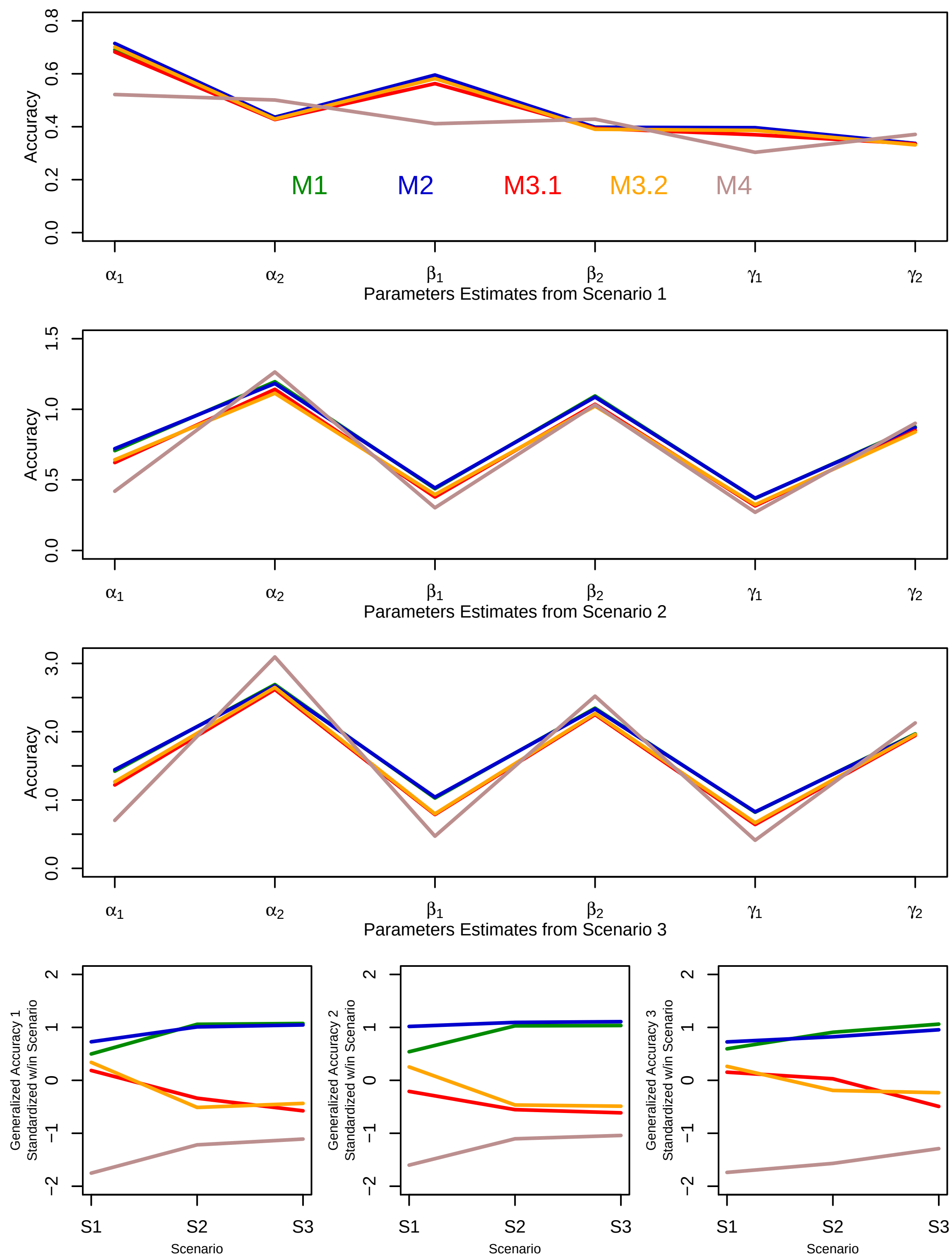

Figure 19: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 8.

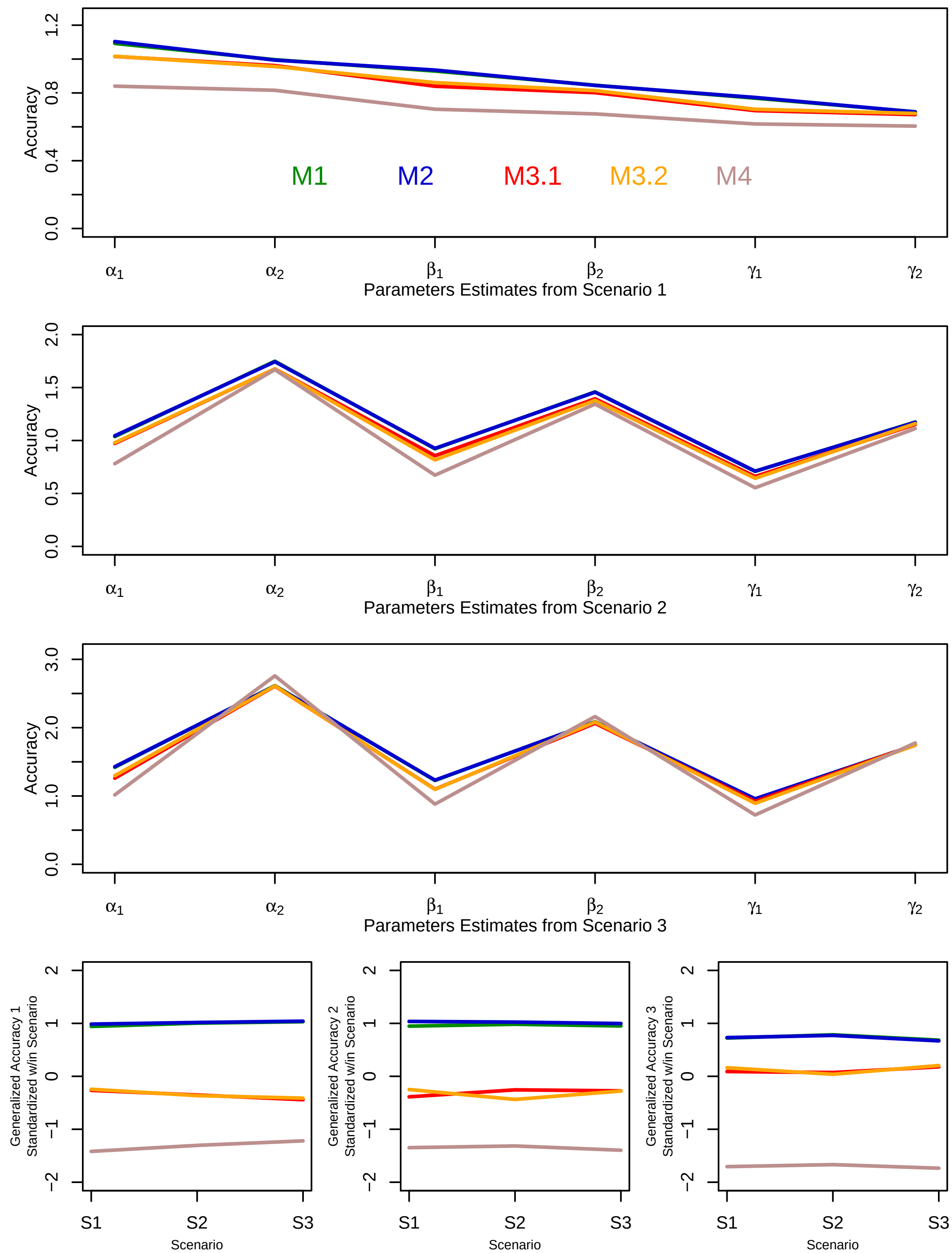

Figure 20: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 9.

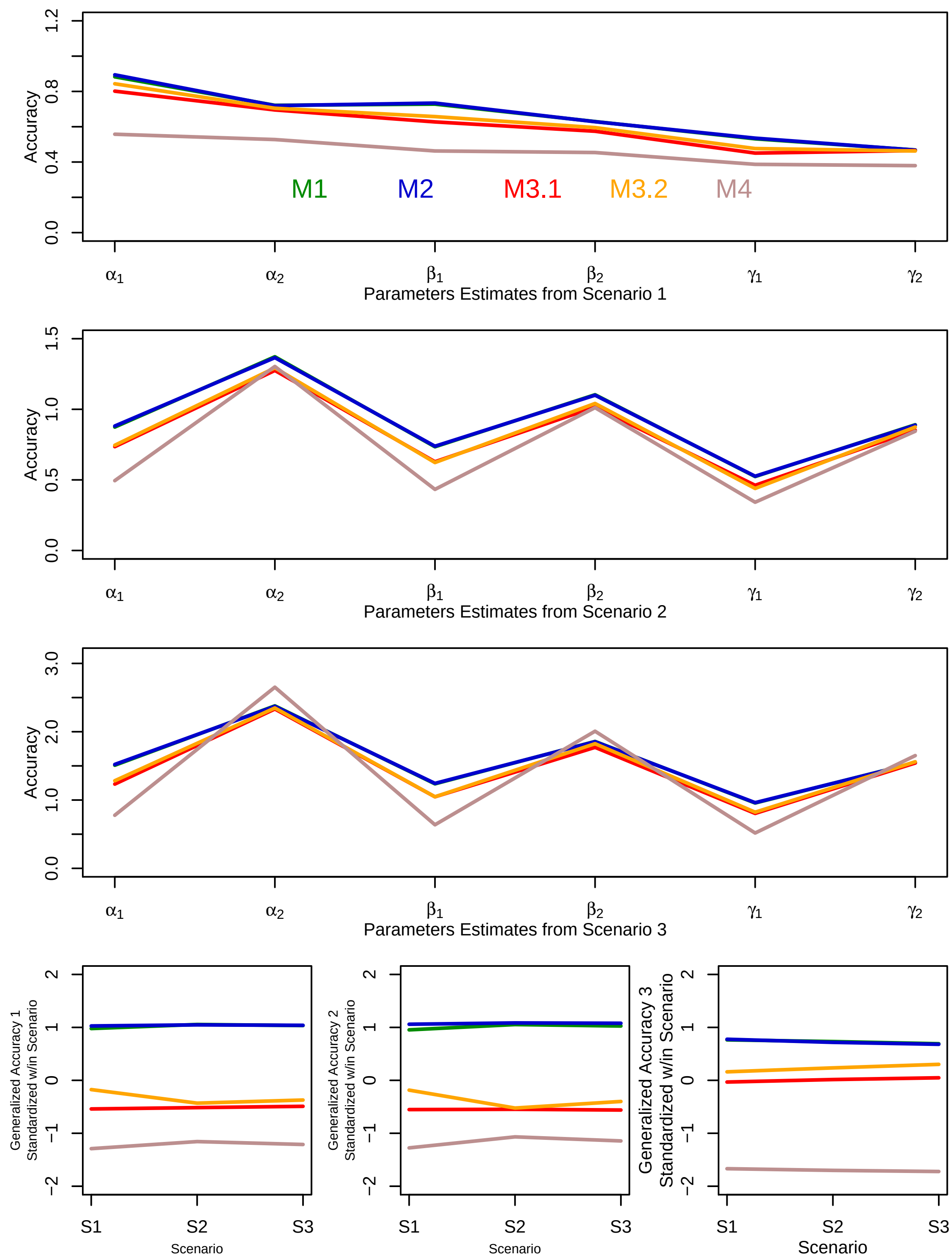

Figure 21: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 10.

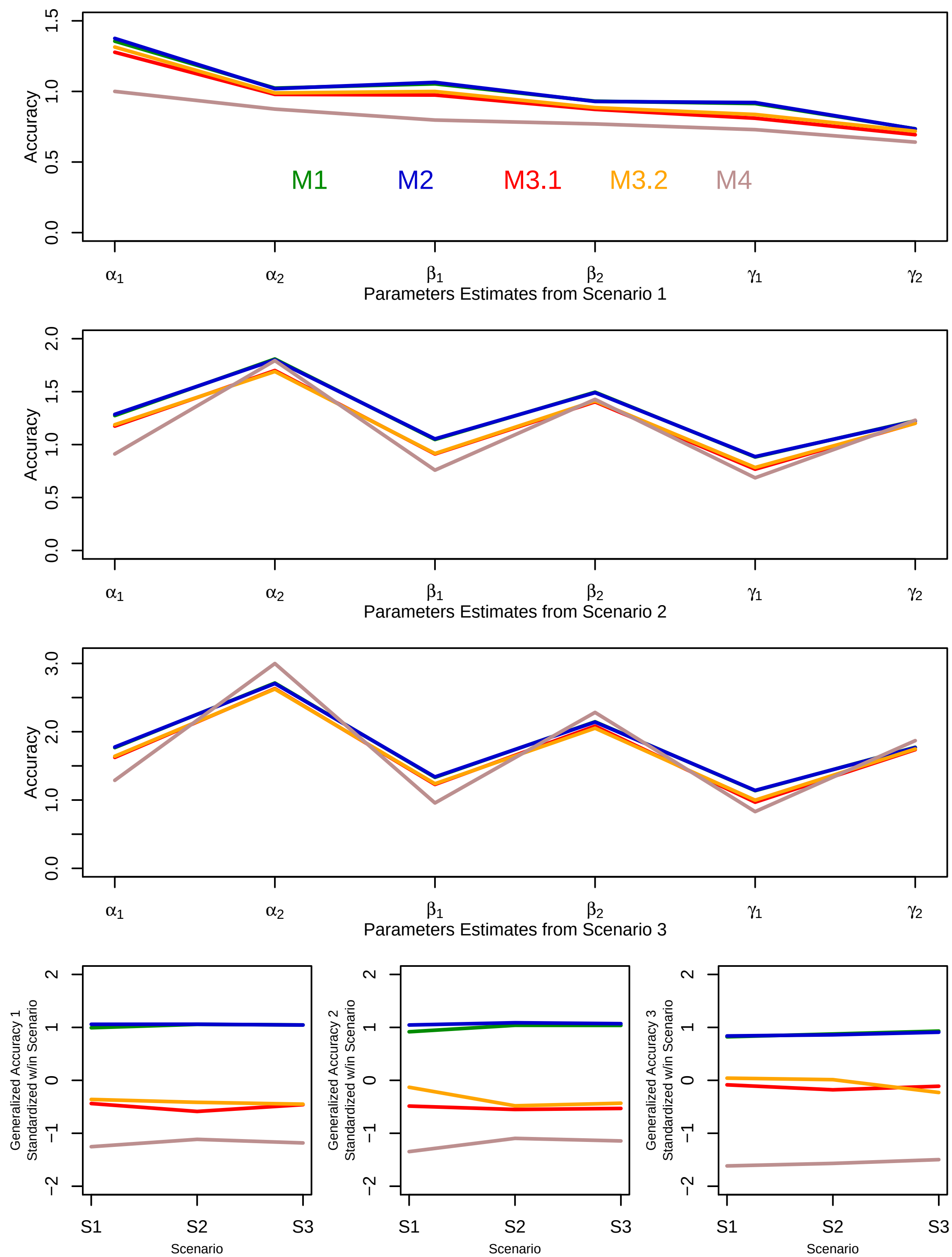

Figure 22: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 11.

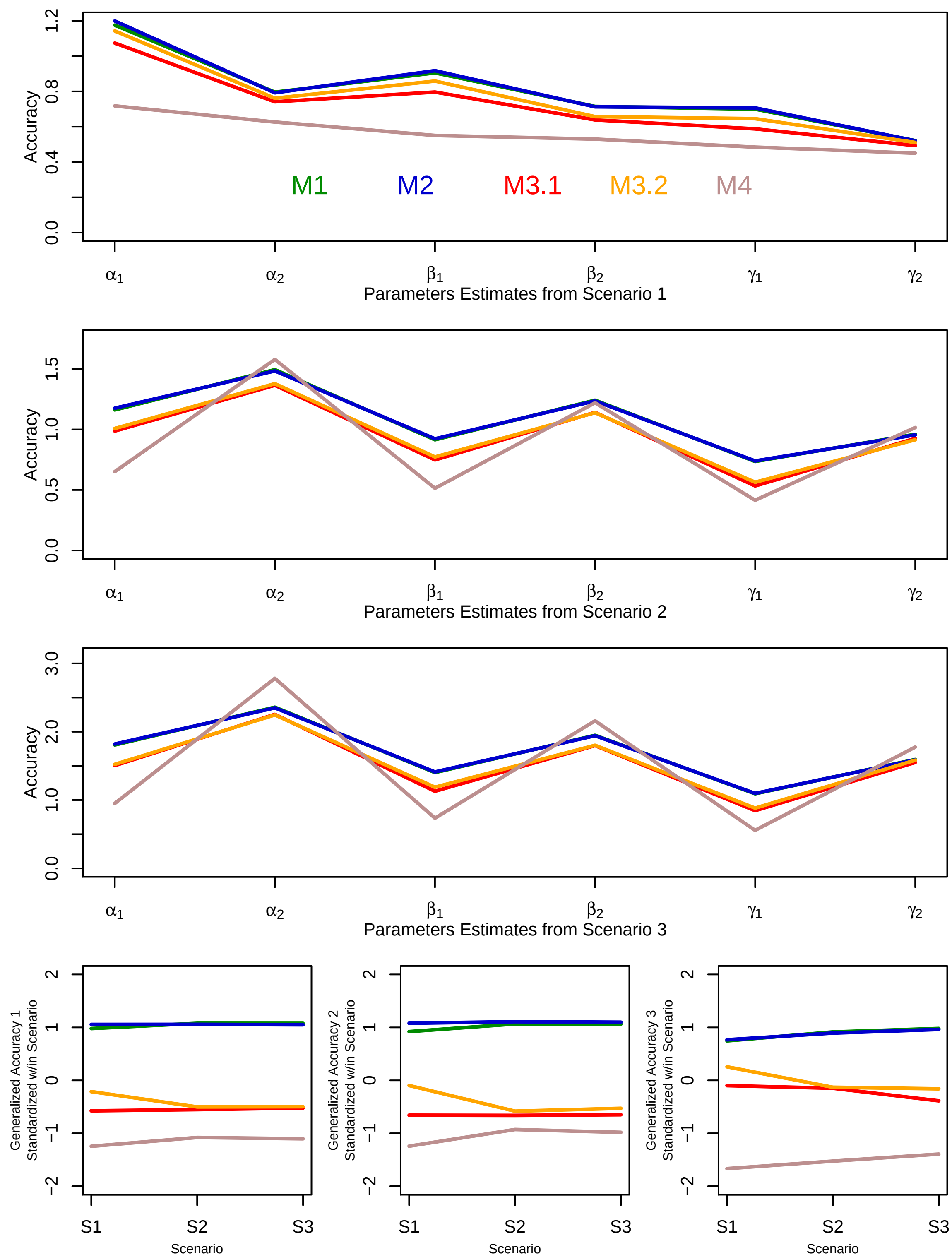

Figure 23: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 12.

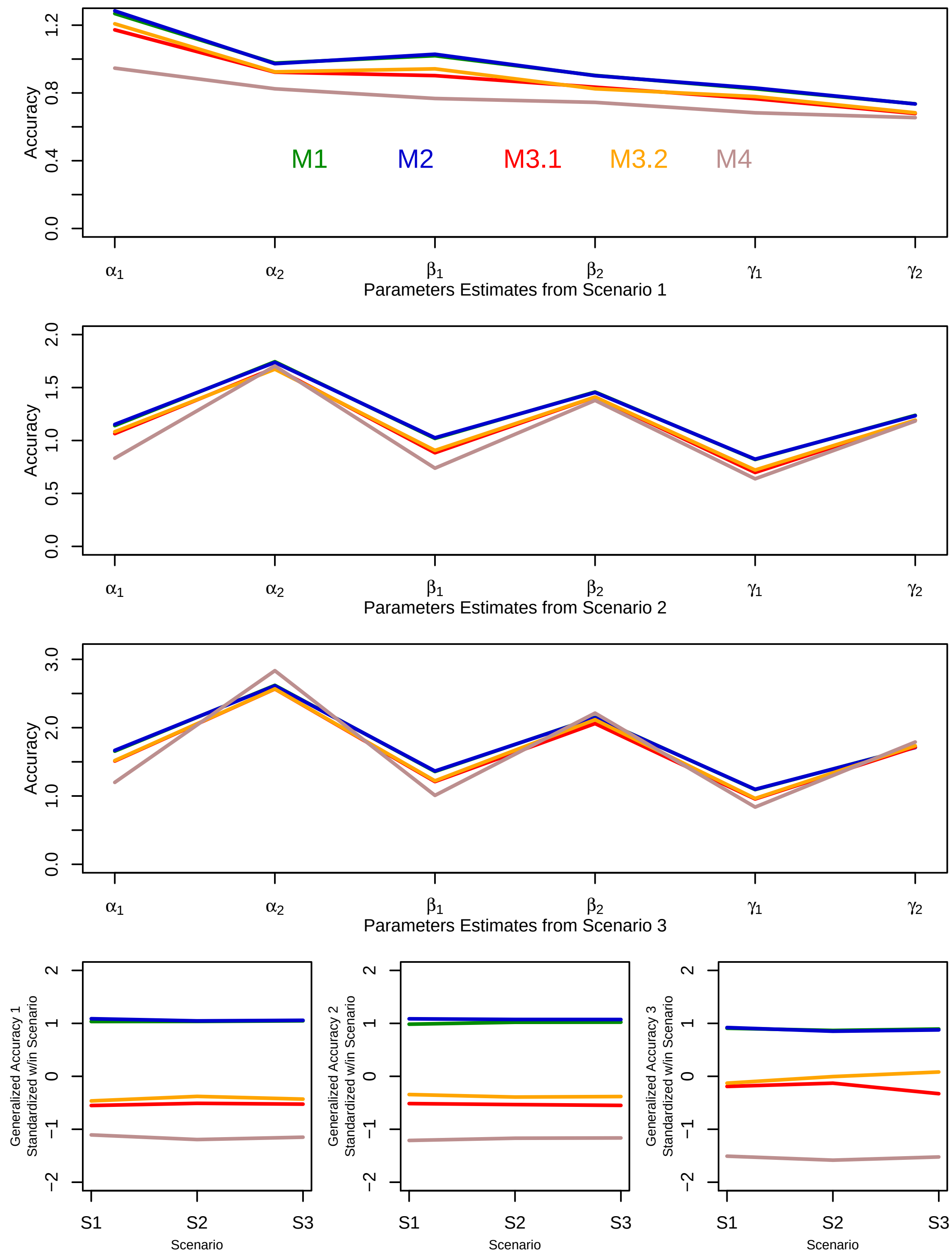

Figure 24: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 13.

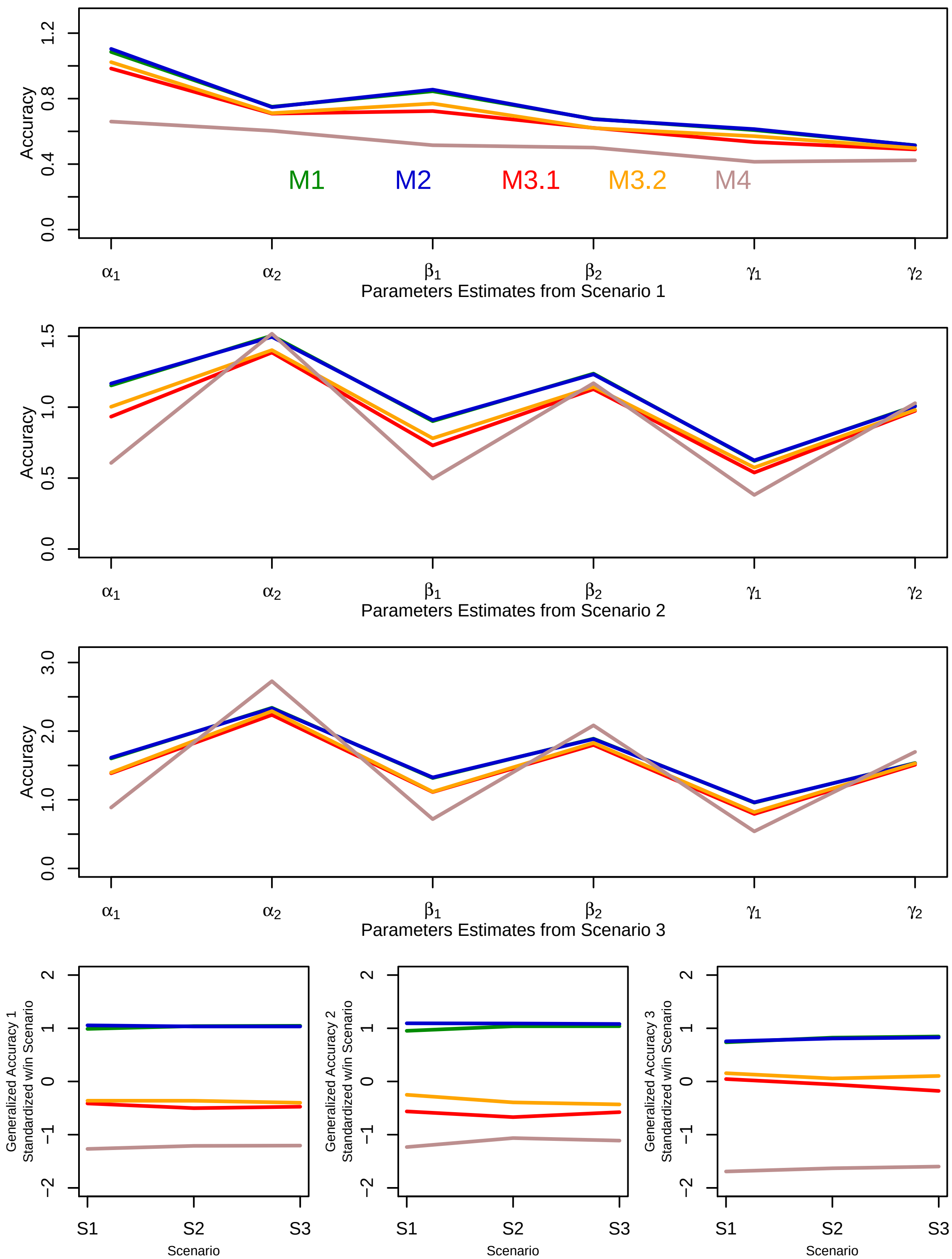

Figure 25: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 14.

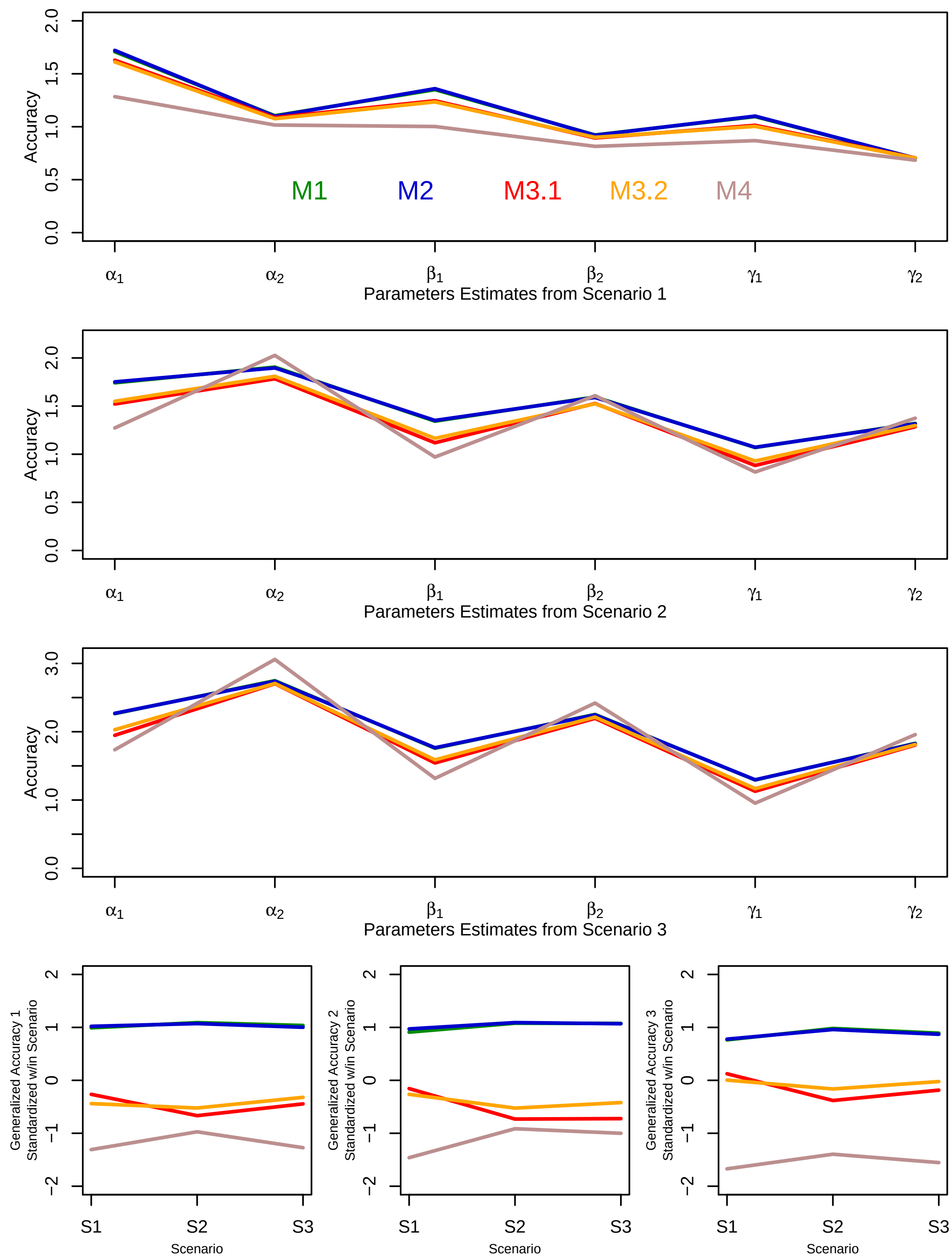

Figure 26: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 15.

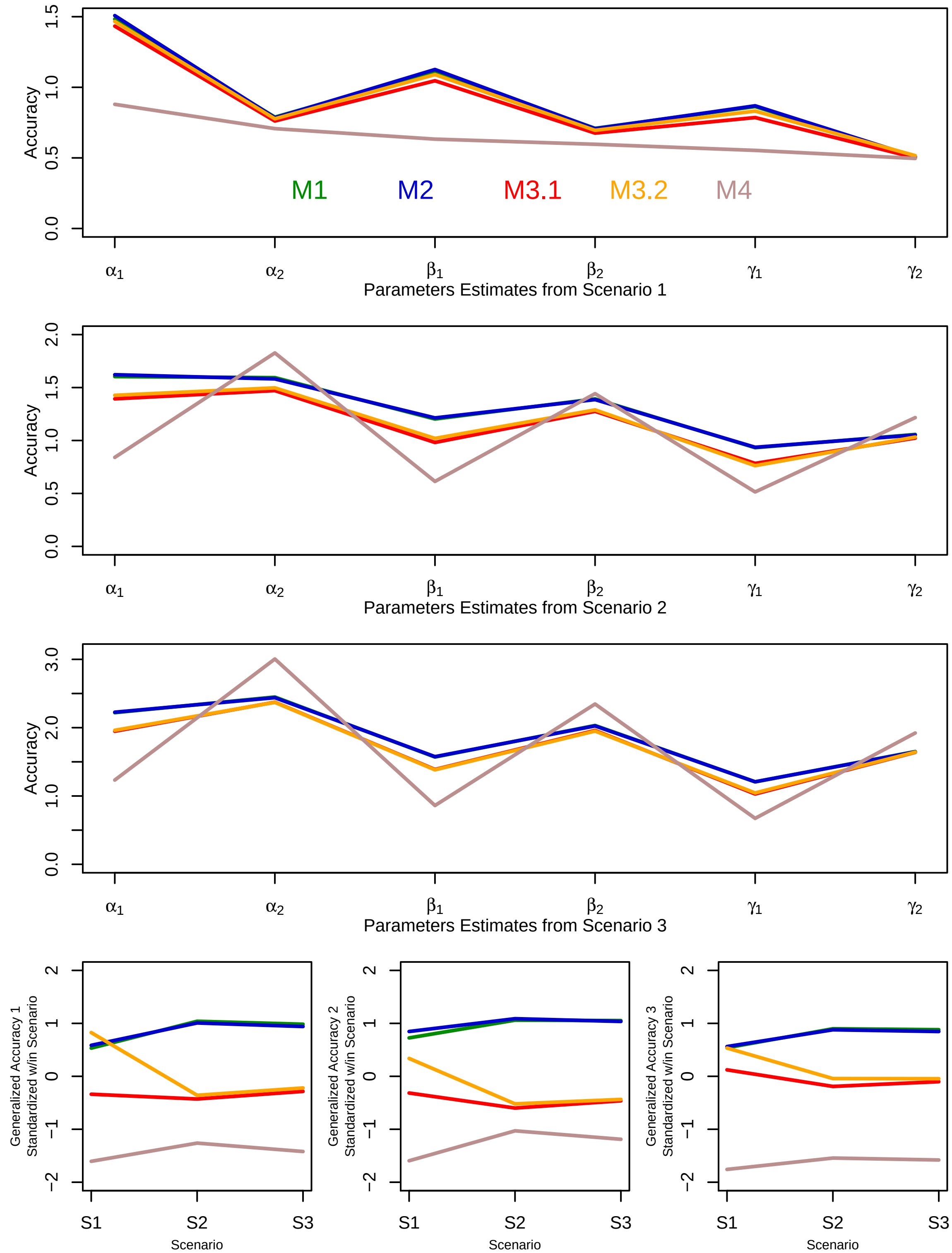

Figure 27: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 16.

Even though Method 4 was shown to be superior for all situations within Conditions 1 through 16, examining Figures 28 through 35 reveals that Method 3.2 is generally the most accurate method, as judged by the three generalized accuracy measures. Because of the cross-class constraints in Conditions 17 through 24 (specifically $\alpha_1 = \alpha_2 = \alpha$ and $\gamma_1 = \gamma_2 = \gamma$), Method 1, 3.1, and 4 might not have been optimally specified (because independent estimates for α_g, β_g, and γ_g were obtained). These three methods treat each class in isolation, whereas Method 2 and 3.2 estimate the parameters of the latent classes simultaneously. Holding parameters constant across class for Methods 1, 3.1, and 4 was done using a post hoc procedure where weighted means of the fixed effects of the two classes were used to estimate the constrained parameters. Specifically, the fixed effects for the two classes were combined using π_1 and π_2 as the weighting factors. Although reasonable, this method of imposing cross-class constraints might not have been the best approach.[36] Had a more appropriate approach been used to impose the cross-class constraints, Method 4 (as well as Methods 1 and 3.1) might have fared better. Nevertheless, at the present time the most accurate method in situations where cross-class constraints are imposed is Method 3.2 (recall that cross-class constraints were imposed only when it was known that the cross-class parameters were equal). At the present time, no other viable alternative seems to exists for cross-class constraints other than to estimate the parameters for each class separately and then combine the separate estimates to obtain a single estimate by using the weighted mean (based on the πs). Although the ad hoc cross-class constraints based on this approach was shown to be effective at recovering the fixed effects for Method 4, Figures 28–35 show that Method 3.2 generally provides the most overall accurate method based on the ranking of the generalized accuracy measures.

[36]Had there been more than two classes, the same principal of using weighted means to estimate any cross-class constraints, whether for all classes or only a subset, could be carried out using the same weighted means procedure as used here. As mentioned in the text, at the present time it is unclear if this method of post hoc weighting is optimal from a perspective of accuracy. Further research is needed to determine if such a procedure is optimal, and to determine if alternative methods might outperform the method of post hoc weighting that is used in the present work.

It seems that the reason Method 3.2 was so successful is that cross-class constraints are capable of being specified directly in the model. The ability to impose cross-class constraints a priori is an advantage realized only by Method 2 and Method 3.2. However, all of the methods were generally effective at recovering the fixed effect parameters even though Method 3.2 was shown to be more accurate in the majority of the situations within Series 3.

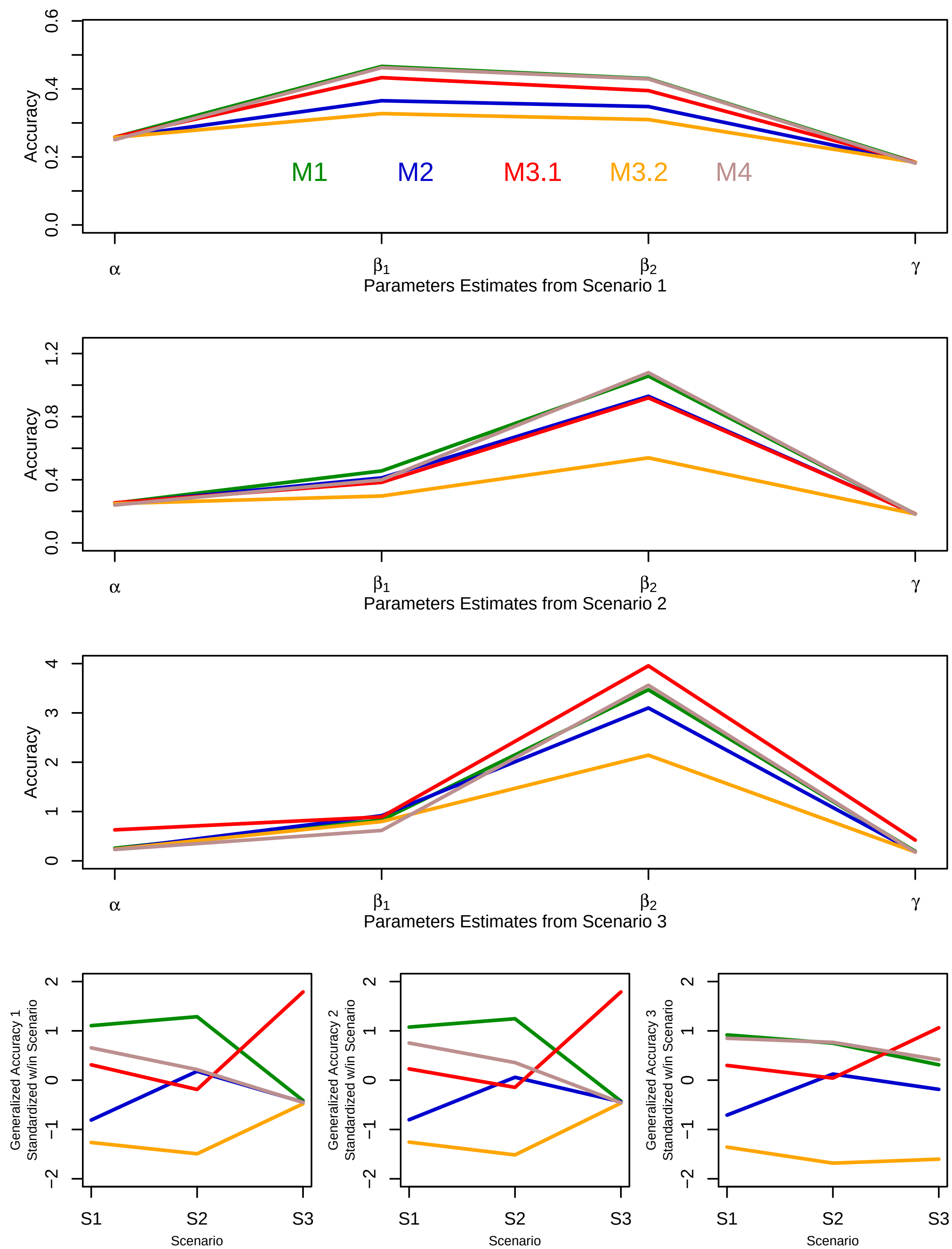

Figure 28: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 17.

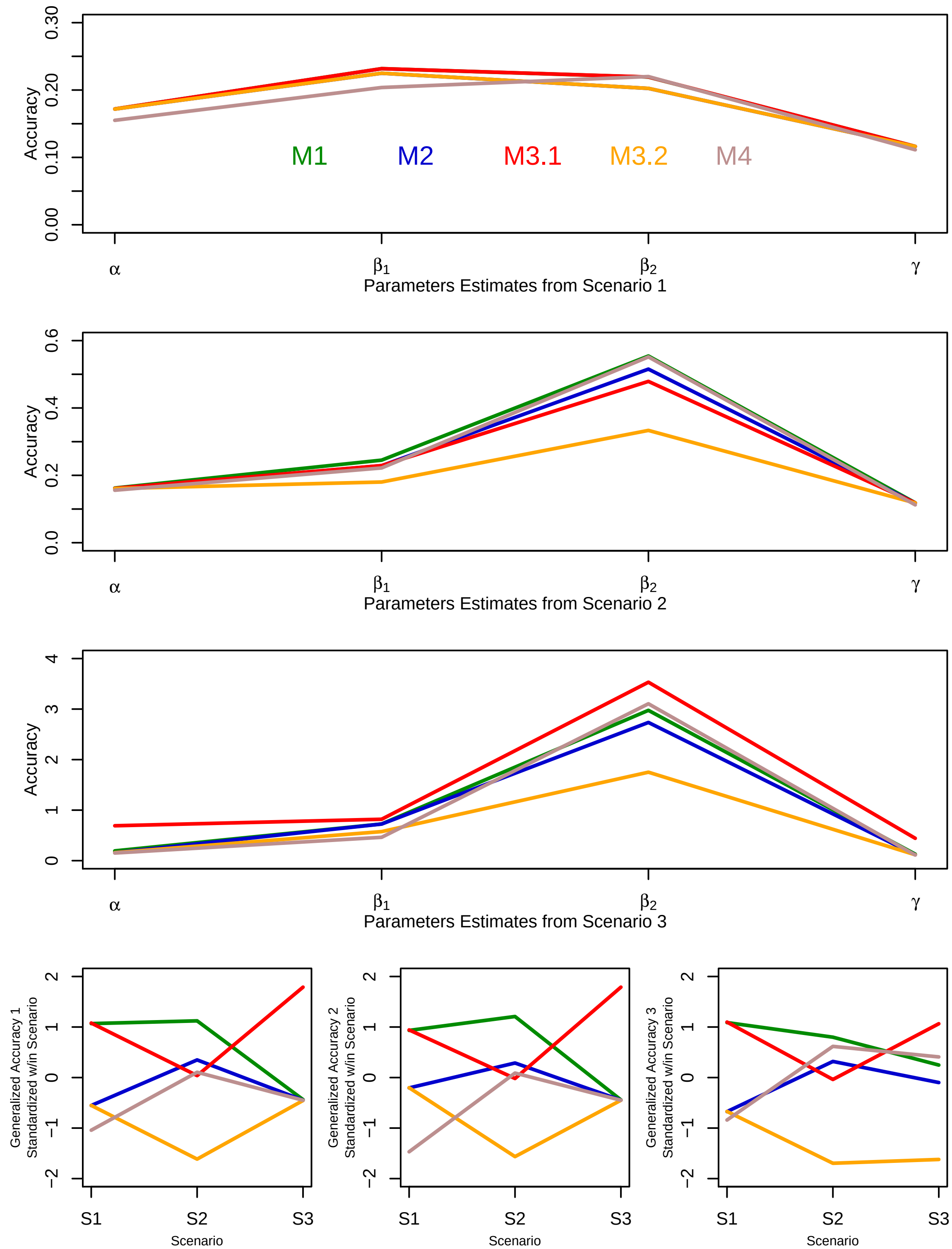

Figure 29: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 18.

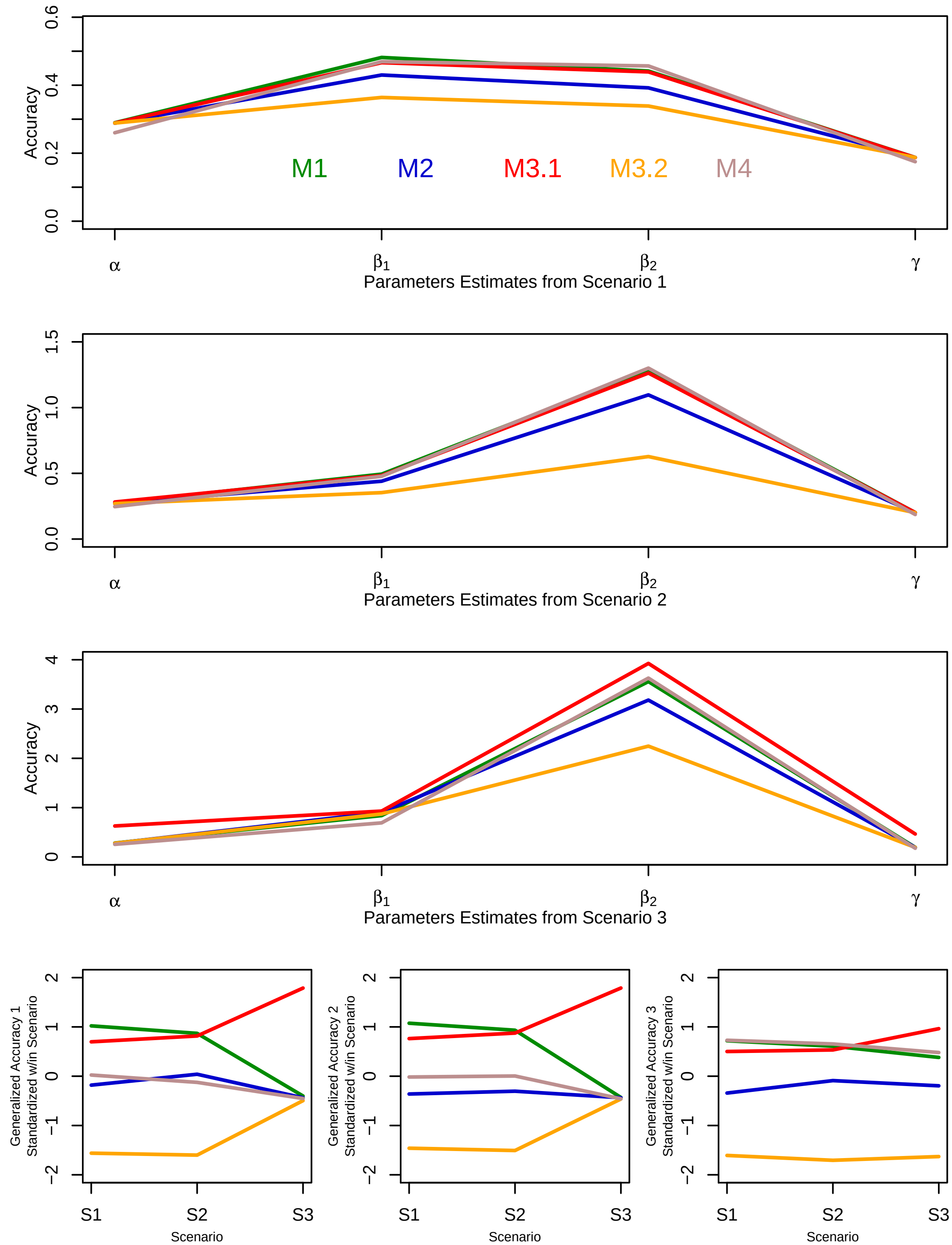

Figure 30: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 19.

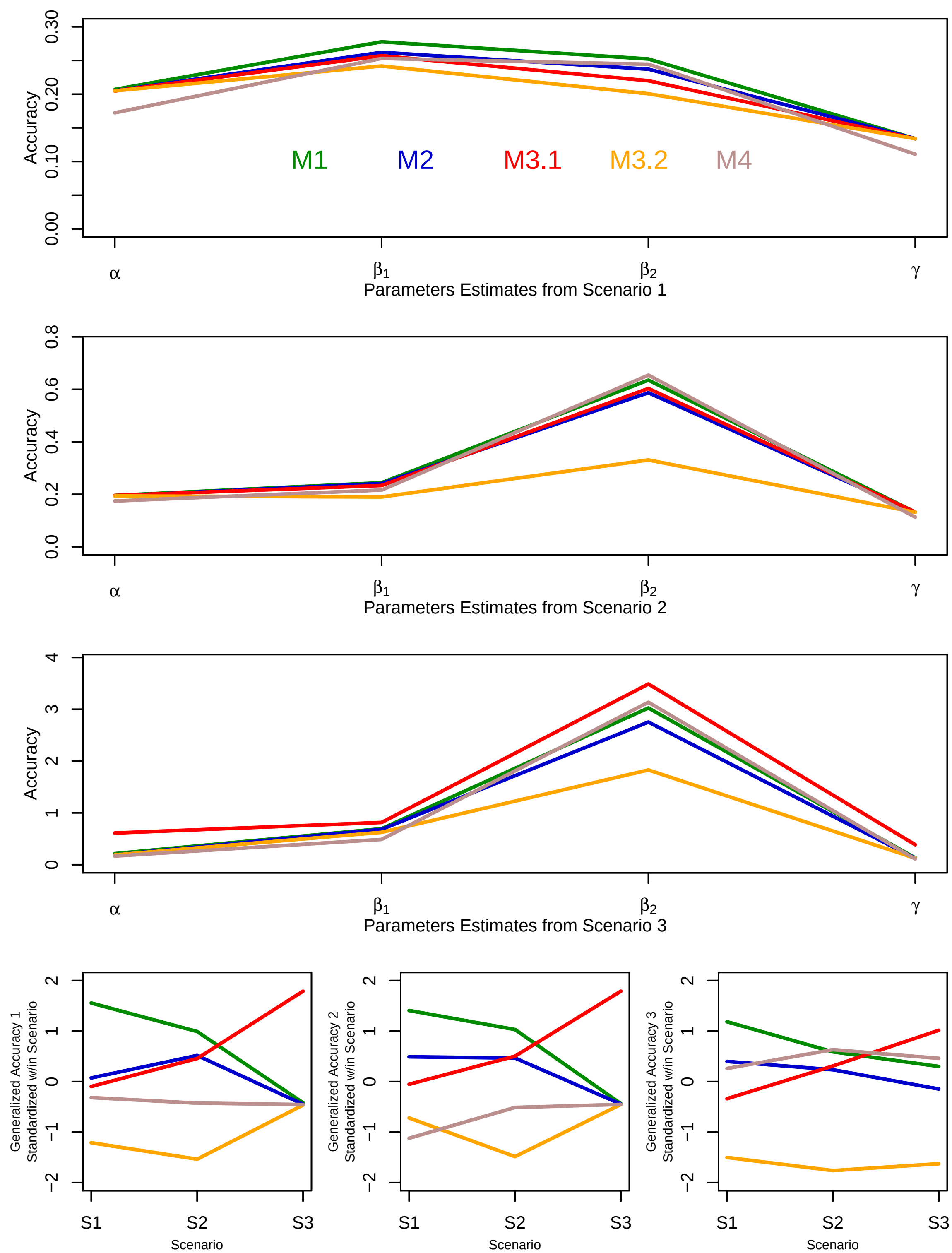

Figure 31: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 20.

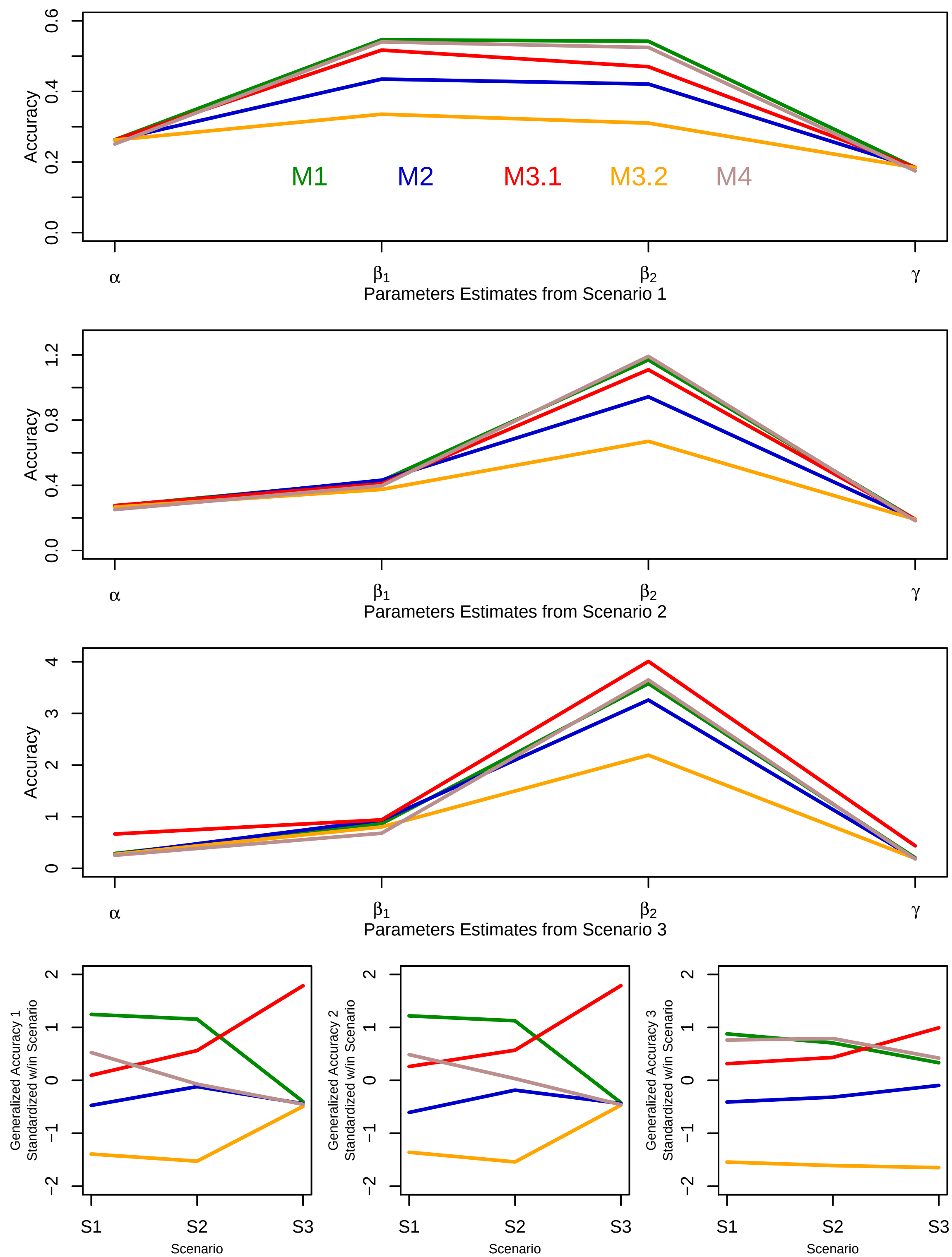

Figure 32: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 21.

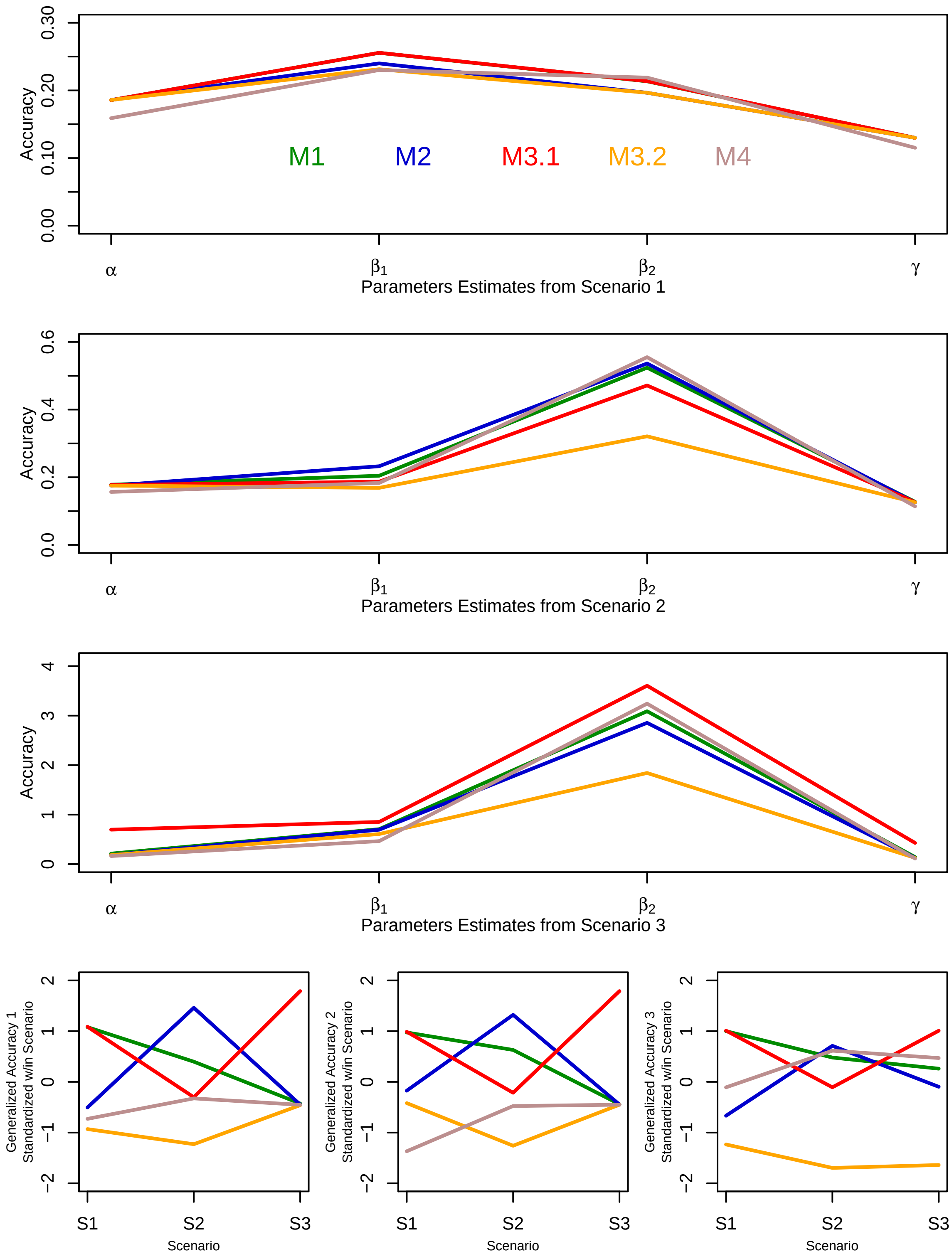

Figure 33: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 22.

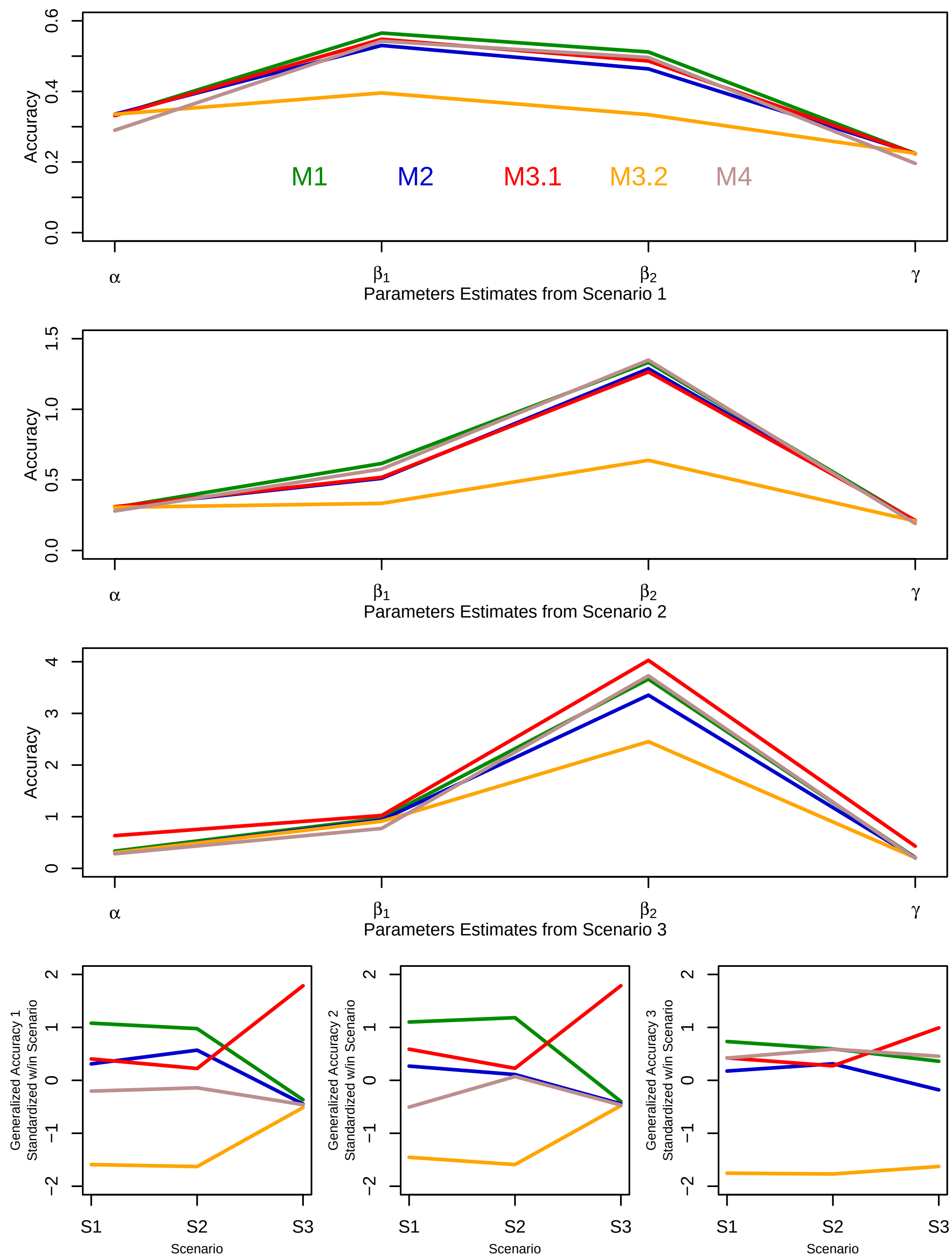

Figure 34: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 23.

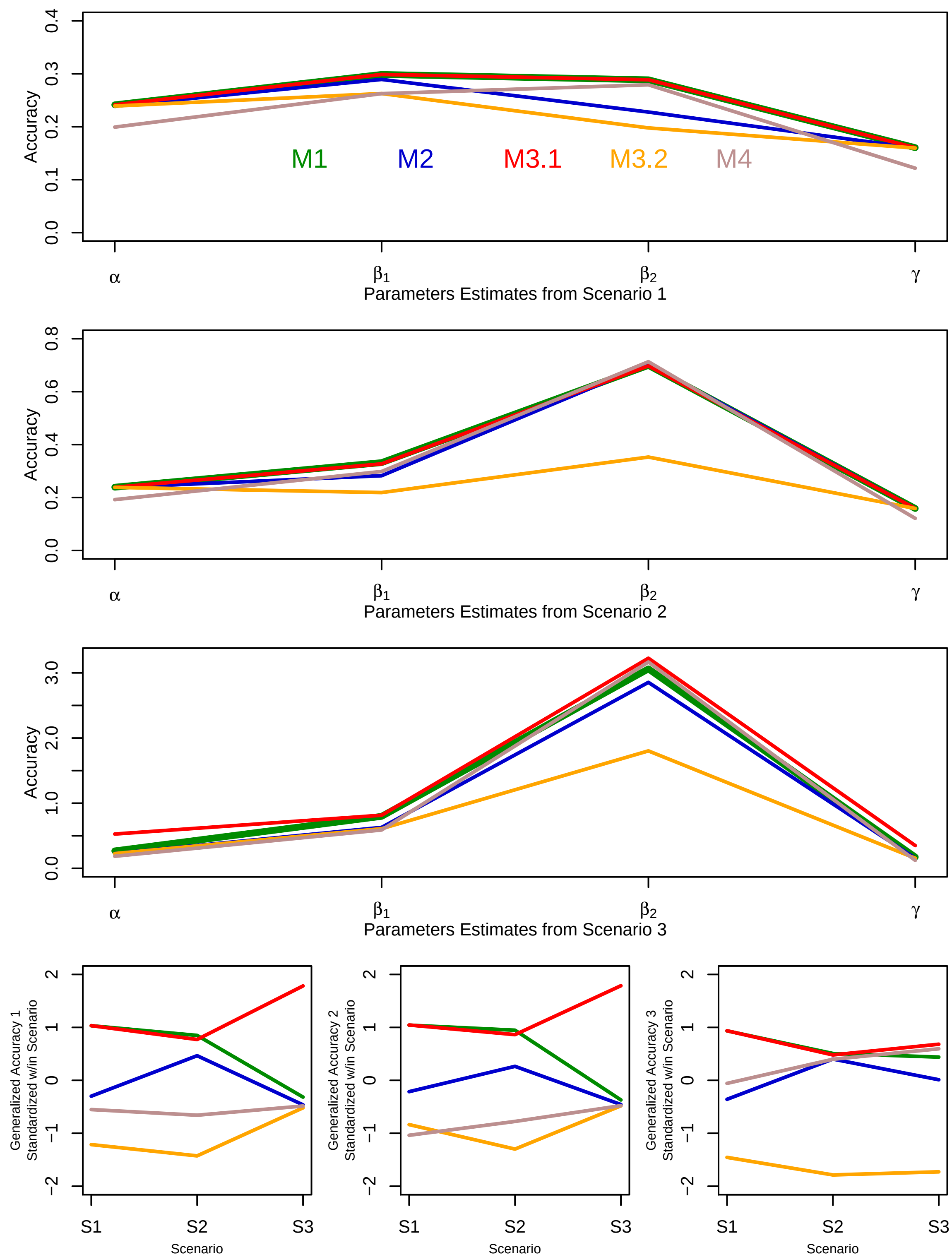

Figure 35: Specific and overall measures of accuracy for each of the five methods and each of the three scenarios within Condition 24.

Given the results manifested in Figures 12 through 35, it becomes evident that Method 4 and Method 3.2 are generally more accurate than the other methods. Nevertheless, each of the methods was generally effective at recovering the population values of the fixed effects. One problem with the figures illustrating the accuracy of the individual fixed effects and the three measures of generalized accuracy is that they require the reader to examine 24 figures, where each figure contains multiple plots. Because no method was shown to have a generalized accuracy measure better than Method 4 or method 3.2, having so many plots may not be overly burdensome since the pattern of results was so repetitive. Nevertheless, an omnibus summary can be provided that aggregates across each of the three series for each of the fixed effects. Figures 36, 37, and 38 provide such a summary.

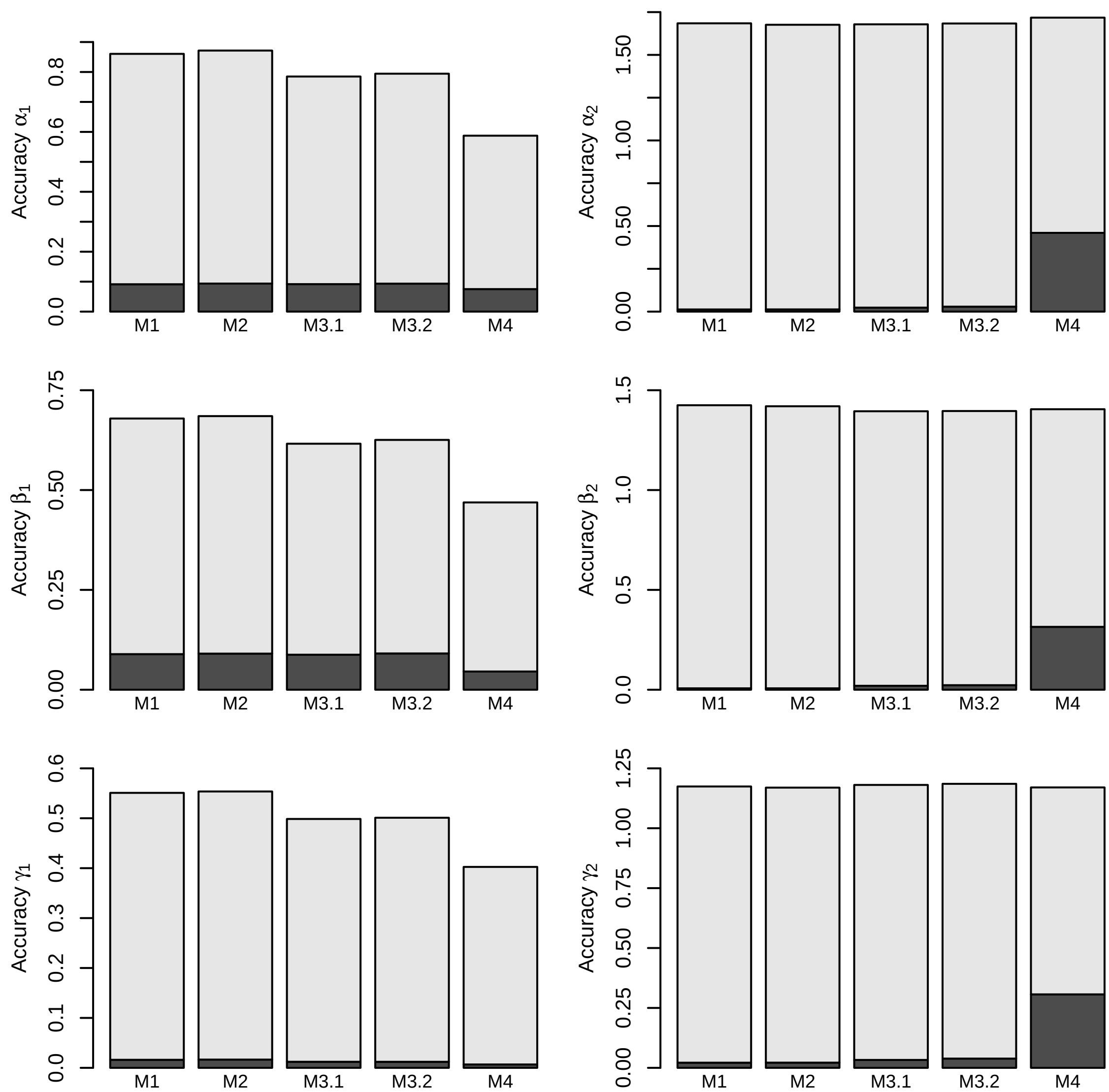

Figure 36: Mean accuracy as a function of bias (dark gray) and precision (light gray) across each scenario in Condition 1 through Condition 8 (i.e., Series 1) for each of the fixed effects.

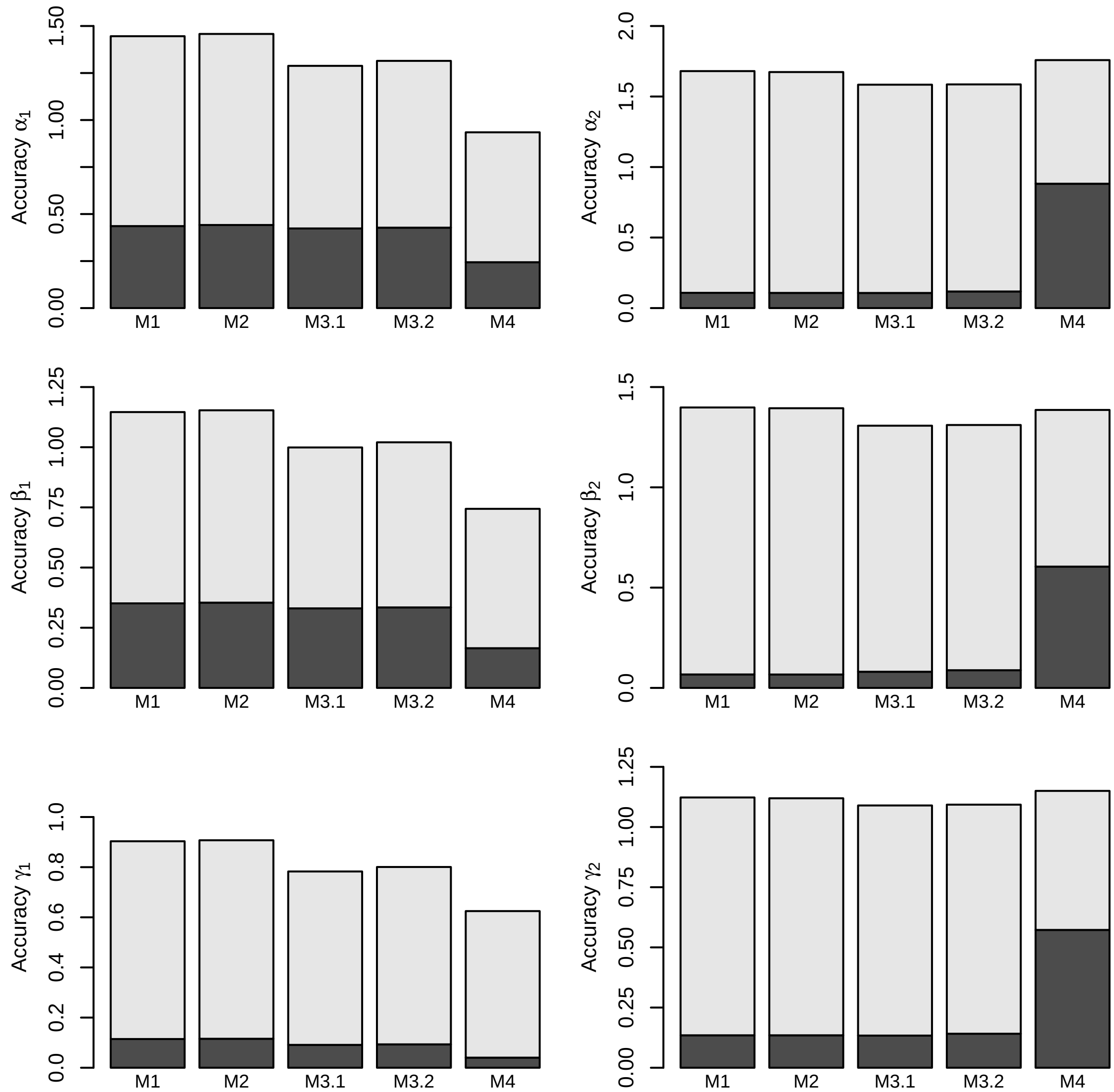

Figure 37: Mean accuracy as a function of bias (dark gray) and precision (light gray) across each scenario in Condition 9 through Condition 16 (i.e., Series 2) for each of the fixed effects.

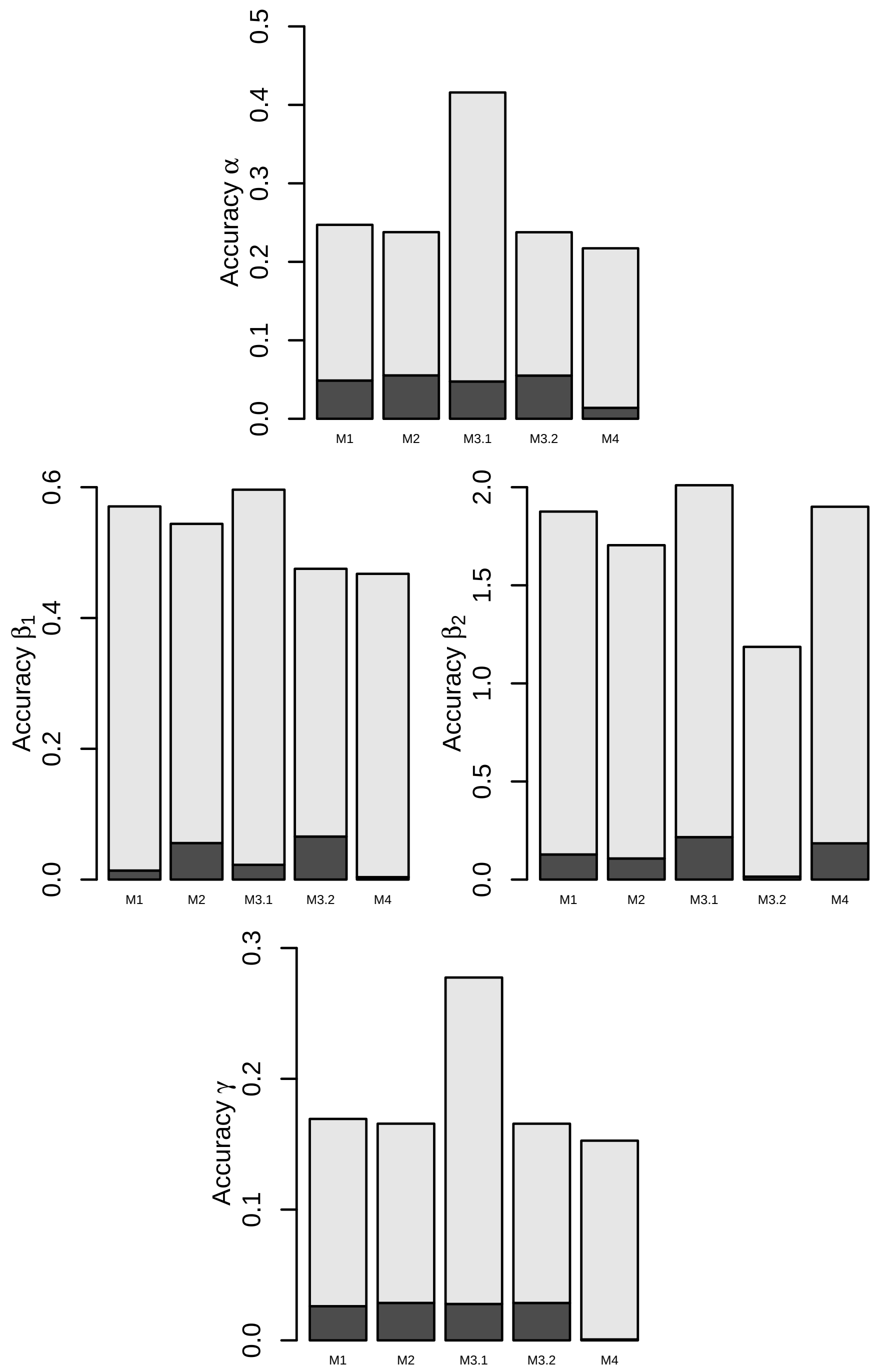

Figure 38: Mean accuracy as a function of bias (dark gray) and precision (light gray) across each scenario in Condition 17 through Condition 24 (i.e., Series 3) for each of the fixed effects.

The height of the bars in the three figures represent accuracy for each of the fixed effects averaged across the entire series of eight conditions (and thus 24 scenarios) within each series (recall each series consists of the eight conditions having the same set of fixed effects parameter values). The dark gray part of the bars represents the proportion of bias contributing to the overall measure of accuracy and the light gray represents the proportion of precision that contributes to the overall measure of accuracy.[37] As can be seen in Figure 36, Method 4 is more accurate than the other methods for each of the parameters in Class 1. However, all of the methods yielded approximately the same accuracy for the parameters of Class 2. As can be seen, Method 4 was more biased than the other methods for the fixed effects of Class 2, but more precise. The end result is essentially the same amount of accuracy in Class 2. However, Figure 36 illustrates a characteristic that manifests itself in Figures 37 and 38. That is, in situations where sample size is small, Method 4 is more biased but more precise than the other methods. The precision of Method 4 usually outweighs the bias and thus leads to estimates that are overall still approximately as accurate as the other methods.

An analogous pattern emerges in Figure 37 as it did in Figure 36. In particular, Figure 37 illustrates that Method 4 is more accurate than the other methods for the parameters of Class 1, and is approximately equivalent to each of the other methods for the parameters of Class 2. Notice the pattern of greater bias but also greater precision for the fixed effect parameters for Class 2 from Method 4.

As previously mentioned, Series 3 provides interesting results. This is because there are cross-class constraints leading to fixed effects being based on weighted means for some methods (specifically Methods 1, 3.1, and 4). While at this time it is unclear if

[37]The proportion of accuracy that the bias and precision contributes was determined for the squared measures of bias, precision, and accuracy. Accuracy is calculated as the square root of the sum of the bias squared and the precision squared. The proportion of each of these measures that contributes to the accuracy squared was used to represent the proportion of the measures in the transformed (i.e., after taking the square root) accuracy metric.

such weighted means for constrained parameter estimates is optimal, Figure 38 shows that Method 4 leads to more accurate estimates for all but one of the fixed effects. In particular, it is shown that β_2 is more accurately estimated with Method 3.2 than it was with Method 4. However, for each of the other three fixed effects Method 4 is more accurate. Recall that examination of Figures 28 through 35 showed that Method 3.2 was generally the most accurate method. Aggregation across these conditions, however, reveals that Method 3.2 is better than Method 4 only for β_2. Recall from Figures 36 and 37 that the accuracy of Method 4 is sensitive to small sample size. In the estimation of β_2, which tends to be based on smaller sample sizes, Method 3.2 was clearly the winner. However, this fact seems to be based in large part on Method 3.2 being less sensitive to small sample size than Method 4.

Across the 16 fixed effects for the 72 situations, Method 4 was inferior for only one fixed effect. In particular, Method 4 was better than or approximately equal to the other methods consistently and across a wide variety of situations. The closest competitor to Method 4 was Method 3.2. Method 3.2 was shown to be more accurate for most of the situations within Series 3, but when aggregating across Series 3 Method 4 was most accurate for three out of the four parameters. While Method 4 was shown to be clearly the most accurate of the methods examined, perhaps more important is the fact that each of the methods recovered the fixed effect parameters well. This is evidenced in Table 4 and Tables C-2 through C-72 more clearly than it is in the plots of accuracy. While there has been much attention devoted to separating the methods by determining which of the methods is most accurate, the effectiveness of each method at recovering the parameters is actually similar in most cases.

The effectiveness of each method should not be overlooked because of the overall accuracy and success of Method 4. What should be emphasized is the advantage each of the methods offers. The way in which each of the methods attempts to recover the parameters of the LCDC model offers interesting possibilities in terms of which method might be the most appropriate for a specific question of interest as it relates to the theoretical framework of interest. The following subsection discusses the problems and benefits of each of the methods. It also addresses the fact that the particular method chosen should be based on the question that is being addressed.

Although the successfulness of the FMM is not directly of interest in the present study, tables illustrating the effectiveness of classification and recovery of the π_g parameters is given in Appendix D. The tables in Appendix D are included mainly for completeness and archival purposes. At some future time the results contained in these tables could be used to help explain some of the differences that exist between the methods. However, given that all of the methods were so successful at recovering the fixed effect parameters, coupled with the fact that Method 4 clearly outperformed the other methods in terms of accuracy, such an analysis is unnecessary at the present time. Had the best method depended on π_1 or how accurately π_1 was recovered, conditioning on the results given in Appendix D might be helpful. Since the results apply almost uniformly across the situations, the substantial amount of additional analyses would not provide much beyond what has already been learned.

Making Use of the Methods

Method 1 and Method 3.1 treat the G classes in isolation. Although this is not optimal in the sense of obtaining the best estimate of the root mean square error and thus estimation of the standard errors and confidence intervals when the model is identical across class, treating the latent classes in isolation potentially has advantages. In particular, once crisp classifications have been carried out, the functional form of change for the classes need not be constant. For example, a four parameter logistic growth curve can be fit to one class whereas a three parameter logistic growth curve could be fit to another. More fundamental differences between the functional form of change can also be carried out. For example, in a two class situation one latent class could be based on an asymptotic regression growth curve whereas the other can be based on a Gompertz growth curve. The flexibility of treating the latent classes in isolation is also realized when modeling the error structure. For example, one latent class can have an auto-regressive error structure whereas the other class can have a standard error structure of the form $\sigma^2_\varepsilon \mathbf{I}$. The point is that if some differences in the model occur across the classes, those differences can be evaluated using Method 1 or Method 3.1.

Method 2 and Method 3.2 model each of the classes in a single *G*-group MLM. Having a single model that conditions on latent class yet takes into account information from all classes is beneficial when the model is the same across class. When the functional form of change and the covariance structure of the errors is the same for all of the *G* classes, Method 2 and Method 3.2 are more powerful and precise than Method 1 and Method 3.1. The power and precision of Method 2 and Method 3.2 is a function of all individuals being used in the estimation of the root mean square error and thus the standard errors of the fixed effects. Because of the overall greater amount of information used, the design is more powerful and more precise. Perhaps the biggest benefit of using Method 2 or Method 3.2 is that hypothesis tests and confidence intervals can be carried out directly on differences in fixed effect parameters in a multiple group MLM approach. Along these same lines, Method 2 and Method 3.2 can constrain parameters a priori to be set equal to one another. Recall that α and γ in Series 3 of the simulation had parameters that were equal to one another. Method 2 and method 3.2 allowed estimated parameters to be based on the data from all individuals. However, Method 1, Method 3.1, and Method 4 had applied the cross-class constraint to those parameters in a post hoc, and therefore less elegant, fashion. The ability to test hypotheses about differences in parameters (e.g., "Is the asymptote of Class 1 larger than the asymptote of Class 2?" or "Does Class 1 have a steeper curvature parameter than Class 2?", etc.) and to directly constrain them makes Method 2 and Method 3.2 very flexible.

The main difference between Method 4 and the other methods is that Method 4 is based on fuzzy sets where the contribution an individual makes to the parameter estimates within a class is a function of the likelihood the individual belongs to that class. Rather than assigning an individual entity to one and only one class, Method 4 allows what is functionally partial inclusion. This translates into the class specific parameter estimates being based to some degree on data from all of the individuals. In particular, the amount in which an individual entity contributes to the fixed effects of a particular class is based on the probability of that individual belonging to the particular class. The benefits of fuzzy classification are most prevalent when there is not a large difference in the multivariate distributions of individual change coefficients. When the difference in the distributions are large, the probabilities of class membership approach zero and one, which means that Method 4 becomes more like the other methods (which are based on crisp classification). The real advantage of fuzzy classification manifests itself when there is more class ambiguity in the classification of individuals into the most likely class (e.g., posteriori probabilities for Class 1 and Class 2 of .55 and .45, respectively). The clear advantage of Method 4 is in terms of the accuracy in which the fixed effect parameter estimates can be recovered. For standard applications of the LCDC model, Method 4 is the recommended method.

Given the effectiveness of each of the estimation procedures, coupled with their pros and cons, the particular question of interest and/or the theory that is being evaluated should dictate which of the five procedures should be used. Even though Method 4 was shown to be the overall most accurate method, a blanket statement recommending Method 4 cannot be given because the other methods have advantages in certain situations. Because of the positive effect iterating back and forth between Stage 2 and 3, Method 3.1

and Method 3.2 are universally recommended over Method 1 and Method 2, respectively. Although accuracy is important, use of the best method to answer the particular question of interest is also necessary. Because of the success each method showed in the simulation study and the fact that they each have their own advantages, the method of choice depends on the question of interest. To summarize, when the same model is fit to each of the classes and all fixed effects are thought to be unique to the class, Method 4 is the recommended method. When a different model is supposed across the latent classes (e.g., a different functional form of change or a different structure for the errors) Method 3.1 should be used. When specific constraints and cross-class hypotheses are to be evaluated, Method 3.2 should be used.

FUTURE DIRECTIONS

Perhaps in some ways surprising, Method 4 was shown to be the overall most accurate method for recovering the fixed effects of the LCDC model in the conditions that were examined in the present study. It is surprising that Method 4, a two-stage method, outperformed the three-stage and four-stage methods because the latter stages of the other methods used the theory of MLMs. The use of MLMs is commonly regarded as *the* way to model longitudinal data (recall the discussion in the *Standard Approaches for the Analysis of Change* section which began on page 9). The major difference between the Method 4 and the other four methods is that Method 4 allows fuzzy classification. In the fuzzy classification approach, the latent class parameter estimates are calculated as a weighted mean of the individual change coefficients, where the weighting factor is based on the posterior probability of class membership. This is contrasted with the other approaches where crisp classifications of individuals contribute only to the fixed effects within the class in which the individuals were most likely to belong. Such a crisp classification method works best when the posterior probabilities are large for the estimated class and thus little ambiguity in terms of class membership exists. However, when the posterior probabilities are close to .50 (in the two group situation), classification successes is about the same as it would be with the flip of a fair coin. In cases of such ambiguous classification, methods based on crisp classifications do not provide the best parameter estimates, because such methods assume that class membership is known without error.

Another method that better capitalizes on the MLM framework and fuzzy classification is one where the class specific parameter estimates are weighted based on the posterior probability of class membership. Such an estimation method would combine the benefits of the FMM approach with the advantages of the MLM framework. However, using weighted fixed effects in a MLM framework is quite difficult and has not received any real attention in the literature. While at the present time it does not seem as if a maximum likelihood method exists for using weights in a nonlinear MLM, an alternative method that might work well until such a maximum likelihood method is developed is based on the resampling literature. In particular, individuals can be assigned to class based on the posterior probability of them belonging to the particular class, the fixed effects calculated, and then stored in a results matrix. This same procedure can be replicated a large number of times (e.g., 50,000) with assignment to class membership being based on the original posteriori probability of membership. After all of the replications have been carried out, the mean and variance could be obtained in an analogous manner as is done in standard applications of the bootstrap method (e.g., Efron & Tibshirani, 1993). The major difference between the proposed method and the bootstrap methodology is that standard applications of the bootstrap methodology sample individual data vectors with an equal probability of selection. The proposed sampling procedure given here is a generalization of the bootstrap methodology because sampling depends on the original posterior probability of membership as estimated from the FMM.

As has been alluded to numerous times throughout the work, the theoretically best estimation method is one based on a full information maximum likelihood approach when the model is correctly specified. Such an approach would be similar to Method 3.2, only all three stages would be estimated simultaneously and the individuals' contribution to the parameter estimates would be based on the likelihood that the individual was a member of the particular class. It is hard to imagine a more elegant statistical solution to estimating the LCDC model than a full information maximum likelihood approach. It is also hard to say if and when such a solution might ultimately exist. Hopefully as the theory of estimation advances, algorithms that implement maximum likelihood estimation (such as the EM algorithm, Dempster et al., 1977) become more general, and computer software better integrates nonlinear estimation as a core component, such an estimation procedure and methods to implement it might come to light.

Regardless of if and when a fully maximum likelihood framework might be developed for estimating the parameters of the LCDC model, as it currently stands no computer software exists for directly carrying out the LCDC model with the methods proposed in the current work. Implementation of the LCDC model can be done in any statistical programming language that has the capability of performing nonlinear least squares, FMMs, and nonlinear MLMs in tandem. However, the model can only be realized with customized programming. Because many applied researchers whose expertise is not in statistical programming might be interested in using the LCDC model, to make the model accessible to such researchers computer procedures and/or functions are necessary. While functions were written to implement the methods for the simulation study reported in the present work, those functions were embedded in a larger program. Thus, the functions written in R for purposes of the Monte Carlo simulation study are not directly usable in stand-alone applications of the LCDC model. However, modification of the functions already written can be done so that researchers can make use of the proposed model and methods.

When parametric statistical procedures are proposed, they are almost always conditional on several assumptions (this is the major difference between parametric and nonparametric procedures). Like many proposed methods, the estimation methods presented here for carrying out the LCDC model are based on assumptions. One of the assumptions is that the model is correctly specified. If the model is misspecified in any way, it is not clear how well a particular estimation methods might work. Areas of concern on the topic of misidentification are the functional form of change, the number of latent classes, the covariance structure of the errors, and the covariance structure of the unique effects. When specification errors are introduced into any model, the validity of the parameter estimates must be called into question. Given that five estimation procedures were introduced and presented as reasonable ways to estimate the parameters of the LCDC model, evaluation of the robustness of the estimation procedures should also be considered. Perhaps one of the estimation methods is overly sensitive to specification error. Knowing information relating to robustness is helpful in determining which estimation method should be used in what type of context. Because specification error can manifest itself in numerous ways, nonparametric methods might also be considered as a viable estimation and hypothesis testing procedure.

The rationale of conducting the simulation study was to evaluate the effectiveness of the proposed methods in recovering the fixed effect parameters. Of course, the fixed effects alone are not the only parameters of interest from the LCDC model of change. In particular, the covariance structure of the unique effects is especially interesting when evaluating interindividual differences in change. For example, it might be important to examine the variability of the asymptote, the point of inflection, or how the two parameters covary. Further research should examine the effectiveness of the methods at recovering the covariance structure of the unique effects. Of course, recovery of the covariance structure of the errors and the error variance itself is also important and should be evaluated in future work.

The standard errors of the fixed effects are also important and should be better addressed in future research. The integrity of the hypothesis tests and confidence intervals are in part a function of the standard errors of the fixed effects. When the standard errors are not appropriately estimated, the empirical Type I error rate will generally differ from the nominal value. Provided that fixed effects can be accurately recovered, which was shown to be the case with the proposed methods, the bootstrap methodology can be useful in obtaining accurate standard errors, hypothesis tests, and confidence intervals. Thus, in some ways the recovery of the model based standard errors is less important than the model based recovery of the fixed effects and covariance structures. While not trying to say that the recovery of the standard errors is unimportant, any problems with estimation of standard errors is theoretically overcome with applications of the bootstrap methodology (e.g., Efron & Tibshirani, 1993). Nevertheless, future work should attempt to evaluate the methods' effectiveness at recovering the standard errors analytically.

DISCUSSION

An overarching theme in science is the study of change. When the behavior of individuals is studied in isolation without regard for the population from which they are a member, a science of specific develops and its application to the population is limited. When only the population trend is considered without regard to the individuals from whom the trend is supposed to represent, interindividual differences are ignored and the uniqueness of the individual is lost. In attempting to realistically model behavior, it seems reasonable to combine an idiographic approach with a nomothetic approach (e.g., Dunn, 1994; Muthén & Muthén, 2000).

The way in which the idiographic and nomothetic approach to studying behavior relates to the analysis of change is in how the fixed effects and unique effects are defined and evaluated. Although allowing unique effects to exist around a set of fixed effects is inline with the idea of combining idiographic and nomothetic approaches, it does so in only a limited way. In particular, a standard application of a MLM (or LGC) assumes that heterogeneity is known or that the population of individuals is a representative sample from a single homogeneous population. Generally speaking, however, "it is often unlikely that all individuals in our sample have the same set of parameter values" (p. 558 Muthén, 1989). Applying this idea to the analysis of change, it can mean that unknown homogeneous classes exist in a heterogeneous population. This implies that multiple "mean trajectories" exist, where variability around the parameters defining the mean trajectories may or may not also exist. Thus, the unique effects are a function of the class from which the individual belongs.

It was illustrated that models of change that are nonlinear in their parameters oftentimes provide a more realistic description of change than their counterparts that are linear in their parameters. It seems that if models nonlinear in their parameters are used by behavioral researchers, a better understanding of behavior might be realized. Even if a nonlinear MLM is used, like standard methods used to analyze change, it is assumed that the population is homogeneous or that heterogeneity is known and explicitly incorporated into the model.

When heterogeneity in an analysis of change context is unknown, the GMM provides a realistic way to model change by conditioning parameter estimates on latent classes. Although the GMM is conceptually very appealing, it is necessarily restricted to models of change where the parameters enter linearly. Thus, if a researcher believes latent classes exist and that the functional form of change is governed by a model nonlinear in its parameters, the GMM does not apply. Because of the limitations of the GMM, another model of change is required that explicitly allows unknown heterogeneity to exist in a population where change follows a functional form nonlinear in its parameters.

The LCDC model of change was developed, introduced, and evaluated in the present work so that the necessary limitations of the GMM could be overcome. The LCDC model of change explicitly acknowledges heterogeneity when change follows a functional form nonlinear in its parameters. The LCDC model of change is a conceptual extension of the GMM in that it extends the ideas of the GMM into the context of models of change where the parameters enter nonlinearly. It should be clear, however, that the GMM cannot simply be "tweaked" so that it can be made equal to the LCDC model. This fact is necessarily so because the GMM is built on a system of equations linear in its parameters and is thus fundamentality a different model than the LCDC model. The flexibility of the LCDC model of change becomes apparent when it is realized that special cases of the LCDC model can be made to equal many of the commonly used models of change. Although the estimation methods are different, when the functional form of change is linear in its parameters, the LCDC model of change reduces to the GMM. When heterogeneity exists and the within class variability is restricted to zero, the LCDC model of change reduces to the latent class growth model. When class membership is known, the LCDC model of change is equivalent to a multiple group nonlinear MLM. When the population is homogeneous, the LCDC model of change is equivalent to a standard nonlinear MLM (or LGC when the functional form of change is linear in its parameters). Thus, the LCDC models of change can subsume the most commonly used models of change as special cases. Because of this, the LCDC model of change fits nicely into the analysis of change literature and provides a needed extension. As can be seen, the LCDC model of change is an overarching model that makes connections to many commonly used models of change.

Although the LCDC model of change might be considered conceptually reasonable, due to the nature of combining a nonlinear MLM with a FMM, closed form solutions are not available at the present time. Because of this, five estimation procedures were proposed and evaluated via a comprehensive Monte Carlo simulation study. The results showed that each of the methods were effective at recovering the parameters of the LCDC model. The fact that each of the methods were generally accurate at recovering the parameters of the model is beneficial, because the methods can address fundamentally different types of questions researchers might have. In a standard application of the LCDC model where no cross-class constraints are imposed and the functional form of change is constant across the G classes, Method 4 was shown to be the most accurate and thus recommended method. When there is an interest in using different functional forms of change for different latent classes, or if there is an interest in specifying different error structures or a different covariance structure for unique effects, Method 3.1 can be used. When an interest exists in imposing cross-class constraints or testing the difference between parameters of change for different classes and the error structure is constant across class, Method 3.2 can be used. Thus, the estimation method that should be used corresponds to the particular question of interest. Note that Method 1 and Method 2, although reasonable in their own right, are superseded by Methods 3.1 and 3.2, respectively.

When heterogeneity exists in an analysis of change context, ignoring the heterogeneity introduces bias into the obtained results. In such a situation the parameter estimates that are supposed to represent a single homogeneous population instead represent a mixture of homogeneous subpopulations. This potentially translates into a model of change that is meant to represent everyone with a single trajectory but in actuality fits no one. Fitting a homogeneous model in a heterogeneous population leads to biased and thus misleading results. When theory or data suggest the existence of latent classes in an analysis of change context, combining a MLM or LGC framework with the FMM is necessary to ensure the parameter estimates are conditional on the latent classes. In cases where the functional form governing change is linear in its parameters, the GMM provides a powerful and elegant model to help understand change. However, in cases where the functional form of change is governed by a nonlinear functional form, which is probably more often than not true in the behavioral sciences, use of the LCDC model of change is appropriate. The LCDC model of change and its estimation methods can address a diverse set of questions arising from data having a wide range of complicated structures. The hope is that by explicitly modeling unknown heterogeneity in populations where the functional form of change has parameters that enter nonlinearly will help better address the questions asked by applied researchers who are interested in understanding, and not simply analyzing, change.

APPENDIX A

The joint probability of a set of observations, which is conditional on a set of parameters, is the likelihood function. The likelihood function of a set of N independent observations, given a vector of parameters for the model of interest, is given by

$$\mathcal{L}(\mathbf{W}|\mathbf{\Omega}) \equiv \prod_{i=1}^{N} \mathrm{d}(\mathbf{w}_i|\mathbf{\Omega}), \tag{60}$$

where $\mathbf{w}_i = [w_1, \ldots, w_P]'$ is a P dimensional vector for the ith individual, $\mathbf{W} = [\mathbf{w}_1, \ldots, \mathbf{w}_N]'$ is an N by P matrix of observed scores, and $\mathcal{L}(\mathbf{W}|\mathbf{\Omega})$ is the likelihood function for the observed set of data given the parameters contained in $\mathbf{\Omega}$ that identify the model of interest (Pawitan, 2001; Stuart et al., 1999, chapter 18). Generally of most interest is the $\mathbf{\Omega}$ that yields the largest likelihood function of the observed data and that also consists of a set of permissible parameter estimates. The set of parameter estimates that maximize the likelihood function is known as the maximum likelihood estimate of $\mathbf{\Omega}$.

Given that the likelihood function is twice differentiable, the maximum likelihood parameters contained in $\mathbf{\Omega}$ can be obtained with the methods proposed by Fisher (1922). Given the likelihood function, the score function and Fisher's information matrix are needed to derive parameter estimates and their variance. The score function is defined as the partial derivative of the log of the likelihood function:

$$\mathcal{S}(\mathbf{W}|\mathbf{\Omega}) \equiv \frac{\partial \log \mathcal{L}(\mathbf{\Omega})}{\partial \mathbf{\Omega}}. \tag{61}$$

The maximum likelihood estimate of $\boldsymbol{\Omega}$ is then the solution to the score function:

$$\mathcal{S}(\mathbf{W}|\boldsymbol{\Omega}) = \mathbf{0}. \tag{62}$$

At the maximum likelihood estimate, the second derivative of the likelihood (or log-likelihood) function is negative. In general, the more gradual the slope from minimal values of the likelihood function to the maximum value of the likelihood function, the less confidence can be placed in the estimated values as being an accurate representation of the population values. A gradual rise to the maximum value thus illustrates increased variability of the estimates, as less information is known about the parameters. The degree of variability of the parameter estimates is quantified by the Fisher information matrix, which is defined as minus the second partial derivative of the log-likelihood function and yields a covariance matrix of the maximum likelihood parameter estimates obtained by setting the score function to zero and evaluating:

$$\mathcal{I}(\mathbf{W}|\boldsymbol{\Omega}) \equiv -\frac{\partial^2 \log L(\boldsymbol{\Omega})}{\partial\boldsymbol{\Omega}\partial\boldsymbol{\Omega}'}. \tag{63}$$

Equation 63 represents the observed Fisher information matrix, which is generally better to use than the expected Fisher information matrix (Efron & Hinkley, 1978; Pawitan, 2001).

The parameters of the FMM, as previously mentioned, are generally estimated with maximum likelihood methods. The parameter vector $\boldsymbol{\Omega}$ from Equations 60 through 63, applied to the FMM is essentially equal to $\boldsymbol{\Psi}$ from Equation 40, the difference being that $\boldsymbol{\Psi}$ has a redundant π_g parameter that is thus not necessary to estimate (because $\pi_g = 1 - \sum_{g=1}^{G-1} \pi_g$). Although the elements of $\boldsymbol{\Omega}$ will depend on the model of interest, the general form when multivariate normality of errors is assumed can be written as

$$\boldsymbol{\Omega}_{\text{FMM}} = [\pi_1, \ldots, \pi_{G-1}, \boldsymbol{\mu}_1', \ldots, \boldsymbol{\mu}_G', \text{vech}(\boldsymbol{\Sigma}_1)', \ldots, \text{vech}(\boldsymbol{\Sigma}_G)']', \tag{64}$$

where $\boldsymbol{\Omega}_{\text{FMM}}$ is the parameter vector defining the particular FMM, $\boldsymbol{\mu}_g$ is the P length vector of means for the gth class, and $\boldsymbol{\Sigma}_g$ is the P by P covariance matrix for the gth class (McLachlan & Peel, 2001).

The parameters of the nonlinear MLM can generally be estimated only with approximate procedures. This is the case because the integral of the likelihood function of a MLM containing nonlinear unique effects cannot generally be written (Pinheiro & Bates, 2000, section 7.2.1; Davidian & Giltinan, 1995; Davidian & Giltinan, 2003). Although theoretically, advances in the theory of integral calculus, specifically nonlinear integration, may lead to methods of writing the exact likelihood function for nonlinear MLMs, at the present time it seems that the estimation of the parameters from the nonlinear MLM is necessarily limited. Although the parameter vector $\boldsymbol{\Omega}$ from Equations 60 through 63 applied to the MLM will depend on the model of interest, the general form of the model conditional on any predictor variables can (theoretically) be written as

$$\boldsymbol{\Omega}_{\text{MLM}} = [\boldsymbol{\beta}', \text{vech}(\boldsymbol{\Sigma}_{\varepsilon})', \text{vech}(\boldsymbol{\Sigma}_{\upsilon})']', \tag{65}$$

where $\boldsymbol{\Omega}_{\text{MLM}}$ is the parameter vector defining the particular MLM, $\boldsymbol{\beta}$ is the P length vector of fixed effects, $\boldsymbol{\Sigma}_{\varepsilon}$ is the covariance matrix of errors, and $\boldsymbol{\Sigma}_{\upsilon}$ is the covariance matrix of the unique effects. Unfortunately, given the current state of knowledge, when attempting to find the integral of the log of Equation 65, an exact analytic solution cannot always be written, in particular when nonlinear unique effects enter the likelihood equation. The complications and methods to overcome them are further explained in Appendix B.

APPENDIX B

A vast and varied literature on the estimation of the parameters from nonlinear models exists. As is generally the case in the estimation of parameters, unless there are obvious reasons not to do so, maximum likelihood methods provide a "natural starting point" for the parameters of nonlinear models (Davidian & Giltinan, 2003, p. 403). As far as the estimation theory of nonlinear models goes, the literature can be (essentially) separated into two parts. These two parts are (a) when interest is in models containing only fixed effects and (b) when models contain both fixed and unique effects. Models containing only fixed effects are those where multiple individuals contribute only a single timepoint in a cross-sectional design or when only the change coefficients of a single individual are of interest (see, for example, Bates & Watts, 1988; Gallant, 1987; Ratkowsky, 1983; Seber & Wild, 1989, for a comprehensive review of nonlinear models containing only fixed effects). Models containing fixed and unique effects are those where multiple individuals are measured multiple times (i.e., panel data), and interest is in the value of fixed effects as well as functions (e.g., variances, correlations) of the unique effects (see, for example, Davidian & Giltinan, 1995; Pinheiro & Bates, 2000; Vonesh & Chinchilli, 1997, for a comprehensive review of nonlinear MLMs). The present work is concerned foremost with both fixed effects and unique effects for the general nonlinear MLM when multiple individuals are measured multiple times. Understanding estimation procedures for the nonlinear MLM, however, necessarily requires understanding estimation procedures for the fixed effects model.

Parameter estimation from the fixed effects model, that is a model where the υs from Equation 4 are all zero, is typically carried out based on maximum likelihood methods. Seber and Wild (1989; their equation 2.45, notation changed to reflect present system) show that when errors are independent and normally distributed with expectation zero and constant variance σ^2_ε for each of the T measurement occasions, the likelihood function for the fixed effects model can be written as

$$\mathcal{L}(\mathbf{W}|\boldsymbol{\beta}, \sigma^2_\varepsilon) = (2\pi\sigma^2_\varepsilon)^{T/2} \exp\left(-\frac{1}{2}\sum_{t=1}^{T} \frac{[\mathbf{y}_t - f(\mathbf{A}\boldsymbol{\beta}, \mathbf{x})]^2}{\sigma^2_\varepsilon}\right). \qquad (66)$$

Given the likelihood contained in Equation 66 and $f(\mathbf{A}\boldsymbol{\beta}, \mathbf{x}) + \boldsymbol{\varepsilon}$ (i.e., Equation 4 for the fixed effects model), the question that remains is how to estimate the parameters.

The estimation of the parameters contained in Equation 66 are typically obtained by nonlinear least squares methods. Nonlinear least squares are the analog to ordinary least squares obtained by solving the "normal equations" in the general linear model (Graybill, 1976). Unlike ordinary least squares, however, no closed form solution generally exists and estimation of nonlinear least squares almost always requires iterative methods. As with ordinary least squares, the goal of nonlinear least squares is to minimize the sum of squared deviations between the observed scores and the predicted scores.

The Gauss-Newton method is generally the method of choice for minimizing the sum of squared errors and thus estimating the parameters of the likelihood function for the nonlinear fixed effects model.[38] The Gauss-Newton algorithm is Ratowsky's "favored" method, which is used throughout his book (1983, p. 15), is the default method in the NLS package for the R (R, 2004) and S-Plus (S-Plus, 2002) computer programs, and is the default method for the NLIN procedure in SAS (SAS, 2004). For estimation of

[38] The Gauss-Newton method was improved upon by Hartley (1961), who called his improved algorithm the modified Gauss-Newton algorithm. Generally the modified Gauss-Newton algorithm is simply called the Gauss-Newton algorithm. It if for this reason that the present work uses the term Gauss-Newton to refer to what is technically the modified Gauss-Newton algorithm. Note that the modified Gauss-Newton algorithm is the method popular software programs (e.g., R, S-Plus, SAS) use as the default algorithm for estimation of the parameters of nonlinear regression models.

the parameters of the nonlinear fixed effects model, the present work makes use of the Gauss-Newton algorithm.

Although there is general consensus in the literature that the Gauss-Newton algorithm is a reasonable estimation procedure for the fixed effects nonlinear model, estimation of the nonlinear MLM is extremely difficult. The first difficulty in the estimation of the parameters of the nonlinear MLM arises because the likelihood function cannot be solved analytically. The lack of intractability of a closed form expression for the likelihood function is because the function $f(\cdot)$ in Equation 4 is generally nonlinear in unique effects (see section 7.2.1 of Pinheiro & Bates, 2000, and section 4.1 of Lindstrom & Bates, 1990). Because Equation 4 generally has nonlinear unique effects, the likelihood function itself must be approximated. In order to obtain maximum likelihood estimates, a likelihood function must be specified and then maximized. If a likelihood function cannot be specified, an approximate likelihood function can be derived and then an attempt can be made at maximizing the approximated likelihood function. Alternatively, methods other than maximum likelihood can be used to estimate the parameters contained in the (unwritable) likelihood function (e.g., two-stage methods, generalized least squares, etc.). The specification of the approximate likelihood has been subjected to much debate and many proposals have been given. Because the likelihood function is based on an approximate likelihood function, the parameter estimates are necessarily approximate. What is important, however, is to use methods that are, although approximate, still reasonable with desirable properties.

One popular method to approximate the parameters in the likelihood function is the iterative two-stage algorithm of Lindstrom and Bates (1990), which is the method used in the NLME package for the S-language (Pinheiro et al., 2004). Davidian and Giltinan (2003, section 3.2; see also chapter 5 of Davidian and Giltinan, 1995) discuss another two-stage method, where estimation of the parameters of the likelihood function are based on the means and variances of the individual parameter estimates. Another recommended and promising method is the adaptive Gaussian quadrature approximation discussed in Pinheiro and Bates (2000, beginning on p. 319). As the number of quadrature points on the abscissa is increased, estimation of the integral in the likelihood function by way of the Gaussian quadrature approximation improves. However, the Gaussian quadrature method "quickly becomes prohibitively computationally intensive" (Pinheiro & Bates, 2000, p. 321). Although the adaptive Gaussian quadrature approximation is theoretically better than any of the two-stage or generalized least squares algorithms, it is not always practical, especially in large scale simulation studies (it is also not implemented in the NLME package). The present work will make use of the methods proposed by Lindstrom and Bates (1990) and the NLME package for obtaining estimates of the parameters from nonlinear MLMs.

In summary, for fixed effects nonlinear models the likelihood function can generally be written analytically, yet no closed form solution generally exists. However, the MLM has an integral in the likelihood function that depends in part on the unique effects. When the unique effects are nonlinear, the likelihood function (and thus the log-likelihood function) cannot be written analytically. Thus, estimation of the parameters of a nonlinear MLM generally requires that an approximation to the likelihood function be calculated. Conditional on the approximated likelihood function, iterative methods are then generally required in order to estimate the parameters of the (approximated) likelihood function. Estimation of the parameters of the nonlinear MLM is not trivial, cannot generally be done analytically, and is oftentimes computationally intensive.

APPENDIX C

Table C-1 identifies the specific situations where failures occurred. As can be seen, only 128 failures of the 72,128 total replications occurred. Notice that more failures occurred in the third scenario of the conditions, where the sample size for Class 2 was the smallest of the three scenarios of the conditions. As can be seen, failures were more prevalent for larger σ^2_εs and larger π_1s (implying smaller π_2s)

TABLE C-1

TABLE IDENTIFYING THE NUMBER OF FAILURES THAT OCCURRED IN THE SIMULATION STUDY FOR EACH OF THE SITUATIONS (CONDITION BY SCENARIO)

Condition	Scenario 1	Scenario 2	Scenario 3
1	0	0	2
2	2	1	2
3	2	1	2
4	2	0	2
5	0	0	0
6	0	0	0
7	0	0	2
8	2	1	3
9	0	1	2
10	2	1	0
11	0	1	3
12	1	2	4
13	1	0	3
14	0	0	2
15	8	4	6
16	1	4	7
17	0	0	0
18	0	0	0
19	0	1	6
20	1	1	0
21	0	0	3
22	0	0	1
23	1	4	16
24	2	3	12

The following 71 tables are analogous to Table 4 from the *Results* section. The tables contained in this appendix are provided so that the reader may examine specific results from the simulation study that were not included or aggregated over in the figures used to summarize the results. A detailed discussion of Tables C-2 through C-72, as was done for Table 4, is not practical or necessary. Furthermore, because the greatest benefit comes from generalizations across situations and not for any specific situation, a thorough examination of each specific condition is not very helpful in determining which method is optimal and thus which method is to be ultimately recommended.

TABLE C-2

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 1 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 1; Scenario 2; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.092	-0.261	0.101	-0.225	0.027	-0.122	-0.045	-0.118	-0.043	-0.118	-0.034	-0.034	0.001	-0.006
	$\widehat{M}$	0.069	-0.452	0.084	-0.398	0.026	-0.242	-0.049	-0.127	-0.053	-0.146	-0.04	-0.04	-0.003	-0.002
	$\widehat{\sigma}^2$	0.34	1.629	0.234	1.149	0.174	0.928	0.011	0.031	0.02	0.044	0.014	0.014	0.011	0.029
	2.5%	-0.882	-2.176	-0.689	-1.778	-0.709	-1.505	-0.248	-0.437	-0.286	-0.462	-0.263	-0.263	-0.2	-0.363
	97.5%	1.303	3.285	1.075	2.997	0.775	2.774	0.18	0.256	0.304	0.399	0.246	0.246	0.216	0.302
P	$\widehat{\mu}$	0.001	-0.003	0.003	-0.008	0.002	-0.011	-0.09	-0.236	-0.144	-0.393	-0.084	-0.084	< .001	-0.001
	$\widehat{M}$	0.001	-0.005	0.002	-0.014	0.002	-0.242	-0.098	-0.253	-0.176	-0.485	-0.099	-0.099	-0.001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.008	0.044	0.122	0.218	0.489	0.087	0.087	0.001	0.002
	2.5%	-0.009	-0.023	-0.02	-0.061	-0.047	-0.14	-0.496	-0.874	-0.953	-1.541	-0.657	-0.657	-0.05	-0.091
	97.5%	0.013	0.035	0.031	0.102	0.052	0.259	0.359	0.512	1.015	1.33	0.615	0.615	0.054	0.075
Skew		1.526	1.35	1.535	1.49	-0.017	1.376	0.493	0.221	0.752	0.654	0.62	0.62	0.064	-0.239
Kurtosis		9.586	3.475	18.043	3.819	8.818	3.219	1.46	0.076	1.977	0.598	2.026	2.026	0.333	0.366

Condition 1; Scenario 2; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.089	-0.257	0.099	-0.223	0.026	-0.12	-0.057	-0.055	-0.045	< .001
	$\widehat{M}$	0.064	-0.449	0.082	-0.395	0.025	-0.241	-0.067	-0.075	-0.056	-0.001
	$\widehat{\sigma}^2$	0.346	1.619	0.236	1.145	0.175	0.924	0.012	0.022	0.015	0.008
	2.5%	-0.883	-2.168	-0.692	-1.776	-0.708	-1.497	-0.24	-0.289	-0.256	-0.183
	97.5%	1.338	3.282	1.078	2.987	0.775	2.771	0.216	0.358	0.281	0.179
P	$\widehat{\mu}$	0.001	-0.003	0.003	-0.008	0.002	-0.011	-0.113	-0.184	-0.112	< .001
	$\widehat{M}$	0.001	-0.005	0.002	-0.013	0.002	-0.022	-0.135	-0.249	-0.14	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.008	0.049	0.239	0.095	0.001
	2.5%	-0.009	-0.023	-0.02	-0.061	-0.047	-0.14	-0.481	-0.964	-0.641	-0.046
	97.5%	0.013	0.035	0.031	0.102	0.052	0.259	0.432	1.194	0.703	0.045
Skew		1.58	1.35	1.565	1.49	0.03	1.375	0.8	1.183	0.87	0.064
Kurtosis		9.82	3.487	18.074	3.828	8.915	3.223	1.476	2.333	1.66	-0.088

Condition1; Scenario2; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.089	-0.215	0.098	-0.168	0.029	-0.092	-0.044	-0.113	-0.043	-0.116	-0.032	-0.098	-0.006	-0.003
	$\widehat{M}$	0.081	-0.426	0.086	-0.364	0.043	-0.236	-0.049	-0.126	-0.056	-0.148	-0.039	-0.112	-0.004	-0.001
	$\widehat{\sigma}^2$	0.292	1.748	0.208	1.164	0.184	0.953	0.011	0.032	0.021	0.047	0.014	0.037	0.024	0.029
	2.5%	-0.88	-2.149	-0.729	-1.735	-0.778	-1.486	-0.248	-0.437	-0.287	-0.462	-0.258	-0.451	-0.333	-0.347
	97.5%	1.253	3.378	1.075	3.109	0.775	2.877	0.199	0.29	0.32	0.434	0.251	0.34	0.283	0.304
P	$\widehat{\mu}$	0.001	-0.002	0.003	-0.006	0.002	-0.009	-0.088	-0.226	-0.144	-0.387	-0.08	-0.245	-0.002	-0.001
	$\widehat{M}$	0.001	-0.005	0.002	-0.012	0.003	-0.236	-0.097	-0.252	-0.184	-0.493	-0.097	-0.28	-0.001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.008	0.045	0.126	0.229	0.523	0.088	0.229	0.002	0.002
	2.5%	-0.009	-0.023	-0.021	-0.059	-0.052	-0.139	-0.496	-0.873	-0.956	-1.541	-0.646	-1.129	-0.083	-0.087
	97.5%	0.013	0.036	0.031	0.106	0.052	0.268	0.399	0.579	1.068	1.446	0.629	0.85	0.071	0.076
Skew		0.246	1.437	0.176	1.498	-0.584	1.449	0.548	0.284	0.946	0.76	0.626	0.408	-0.269	-0.213
Kurtosis		4.009	3.702	5.201	3.638	8.046	3.39	1.488	0.096	2.198	0.75	1.652	0.233	0.826	0.418

Condition 1; Scenario 2; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.079	-0.239	0.092	-0.197	0.019	-0.106	-0.057	-0.058	-0.045	< .001
	$\widehat{M}$	0.07	-0.424	0.084	-0.367	0.037	-0.234	-0.068	-0.08	-0.056	-0.001
	$\widehat{\sigma}^2$	0.296	1.632	0.199	1.066	0.157	0.914	0.012	0.023	0.016	0.008
	2.5%	-0.855	-2.11	-0.731	-1.733	-0.726	-1.483	-0.238	-0.29	-0.259	-0.183
	97.5%	1.255	3.233	1.03	2.873	0.713	2.759	0.226	0.382	0.293	0.179
P	$\widehat{\mu}$	0.001	-0.003	0.003	-0.007	0.001	-0.01	-0.115	-0.192	-0.111	< .001
	$\widehat{M}$	0.001	-0.005	0.002	-0.013	0.003	-0.022	-0.136	-0.266	-0.141	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.008	0.049	0.253	0.101	0.001
	2.5%	-0.009	-0.023	-0.021	-0.059	-0.048	-0.138	-0.477	-0.966	-0.648	-0.046
	97.5%	0.013	0.035	0.029	0.098	0.048	0.258	0.452	1.275	0.734	0.045
Skew		0.157	1.459	0.047	1.456	-1.029	0.008	0.856	1.317	0.983	0.063
Kurtosis		4.484	4.357	5.321	3.848	7.097	3.547	1.59	2.709	1.985	-0.089

Condition 1; Scenario 2; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.098	0.374	0.075	0.268	0.026	0.265	0.027	0.052	0.025	–
	$\widehat{M}$	0.113	0.176	0.084	0.115	0.039	0.146	0.024	0.042	0.019	–
	$\widehat{\sigma}^2$	0.223	1.357	0.155	0.954	0.129	0.754	0.007	0.012	0.009	–
	2.5%	-0.798	-1.435	-0.671	-1.126	-0.723	-0.968	-0.119	-0.13	-0.149	–
	97.5%	0.981	3.662	0.884	3.222	0.642	2.89	0.22	0.314	0.262	–
P	$\widehat{\mu}$	0.001	0.004	0.002	0.009	0.002	0.025	0.055	0.173	0.062	–
	$\widehat{M}$	0.001	0.002	0.002	0.004	0.003	0.013	0.048	0.14	0.046	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.007	0.027	0.129	0.057	–
	2.5%	-0.008	-0.015	-0.019	-0.038	-0.048	-0.09	-0.238	-0.434	-0.371	–
	97.5%	0.01	0.039	0.025	0.11	0.043	0.27	0.44	1.049	0.656	–
Skew		0.091	1.37	-0.341	1.504	-1.015	1.394	0.475	0.756	0.588	–
Kurtosis		2.384	3.149	3.399	3.535	6.003	2.996	0.871	1.312	1.054	–

Note: The column heading "D" represents the discrepancy (i.e., the estimate minus the population value), "Stat." represents the statistics used to summarize the discrepancies. B represents the bias whereas P represents the relative bias. Summaries of B and P are given by $\widehat{\mu}$, $\widehat{M}$, $\widehat{\sigma}^2$, 2.5%, and 97.5%, which represent the mean, median, variance, 2.5th quantile and the 97.5th quantile, respectively. Skew and kurtosis for each of the observed parameter estimates is equal to the skew and kurtosis, respectively, of B and P. Thus, only a single value for skew and kurtosis is given for each of the parameters. This note applies to each of the following 70 discrepancy summary tables.

TABLE C-3

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 1 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 1; Scenario 3; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.341	0.969	0.302	0.721	0.146	0.696	-0.016	-0.078	0.002	-0.035	-0.005	-0.005	0.004	0.011
	$\widehat{M}$	0.139	0.284	0.143	0.186	0.041	0.333	-0.026	-0.057	-0.017	-0.024	-0.014	-0.014	-0.001	0.002
	$\widehat{\sigma}^2$	0.888	7.045	0.548	4.716	0.508	3.149	0.019	0.07	0.034	0.093	0.025	0.025	0.012	0.051
	2.5%	-0.984	-2.894	-0.8	-2.808	-1.124	-2.061	-0.258	-0.618	-0.31	-0.612	-0.304	-0.304	-0.199	-0.455
	97.5%	2.929	6.137	2.181	4.902	2.014	4.215	0.302	0.338	0.421	0.484	0.327	0.327	0.212	0.48
P	$\widehat{\mu}$	0.003	0.01	0.009	0.025	0.01	0.065	-0.032	-0.155	0.006	-0.116	-0.013	-0.013	0.001	0.003
	$\widehat{M}$	0.001	0.003	0.004	0.006	0.003	0.333	-0.052	-0.115	-0.055	-0.079	-0.035	-0.035	< .001	< .001
	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.005	0.002	0.027	0.077	0.281	0.378	1.032	0.157	0.157	0.001	0.003
	2.5%	-0.01	-0.031	-0.023	-0.096	-0.075	-0.192	-0.517	-1.236	-1.036	-2.04	-0.76	-0.76	-0.05	-0.114
	97.5%	0.029	0.066	0.062	0.167	0.134	0.393	0.603	0.675	1.404	1.612	0.816	0.816	0.053	0.12
Skew		1.704	0.487	1.332	0.403	1.509	0.422	0.02	-0.453	0.008	-0.104	-0.399	-0.399	0.285	0.512
Kurtosis		5.056	-0.643	4.534	-0.655	7.002	-0.575	3.345	-0.19	3.715	-0.811	3.096	3.096	3.467	3.072

Condition 1; Scenario 3; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.345	0.97	0.305	0.722	0.147	0.696	-0.003	0.02	0.015	0.004
	$\widehat{M}$	0.139	0.292	0.144	0.192	0.041	0.341	-0.024	-0.011	-0.005	0.001
	$\widehat{\sigma}^2$	0.901	7.004	0.555	4.698	0.511	3.133	0.02	0.034	0.023	0.007
	2.5%	-0.978	-2.882	-0.794	-2.794	-1.118	-2.057	-0.239	-0.276	-0.244	-0.158
	97.5%	2.947	6.13	2.205	4.899	2.032	4.202	0.305	0.427	0.357	0.181
P	$\widehat{\mu}$	0.003	0.01	0.009	0.025	0.01	0.065	-0.007	0.068	0.036	0.001
	$\widehat{M}$	0.001	0.003	0.004	0.007	0.003	0.032	-0.048	-0.036	-0.013	< .001
	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.005	0.002	0.027	0.079	0.381	0.143	< .001
	2.5%	-0.01	-0.031	-0.023	-0.095	-0.074	-0.192	-0.479	-0.921	-0.61	-0.039
	97.5%	0.029	0.066	0.063	0.167	0.135	0.392	0.61	1.425	0.892	0.045
Skew		1.716	0.489	1.348	0.403	1.538	0.421	0.471	0.59	0.54	0.137
Kurtosis		5	-0.635	4.528	-0.651	7.102	-0.57	-0.195	-0.06	0.123	-0.115

Condition 1; Scenario 3; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.347	1.294	0.33	1	0.151	0.905	-0.009	-0.052	0.011	-0.014	0.004	-0.016	0.004	0.009
	$\widehat{M}$	0.191	0.563	0.179	0.523	0.065	0.55	-0.025	-0.026	-0.015	0.003	-0.013	0.009	0.002	0.005
	$\widehat{\sigma}^2$	0.72	7.203	0.576	4.674	0.5	3.212	0.02	0.07	0.034	0.098	0.027	0.075	0.031	0.045
	2.5%	-0.98	-2.74	-1.151	-2.547	-1.21	-1.787	-0.247	-0.6	-0.303	-0.616	-0.297	-0.538	-0.351	-0.432
	97.5%	2.432	6.229	2.193	5.06	2.004	4.317	0.317	0.346	0.44	0.511	0.374	0.428	0.362	0.437
P	$\widehat{\mu}$	0.003	0.014	0.009	0.034	0.01	0.084	-0.018	-0.105	0.035	-0.048	0.009	-0.04	0.001	0.002
	$\widehat{M}$	0.002	0.006	0.005	0.018	0.004	0.55	-0.05	-0.051	-0.05	0.01	-0.034	0.022	< .001	0.001
	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.005	0.002	0.028	0.079	0.281	0.379	1.092	0.166	0.469	0.002	0.003
	2.5%	-0.01	-0.029	-0.033	-0.087	-0.081	-0.167	-0.494	-1.201	-1.011	-2.052	-0.742	-1.345	-0.088	-0.108
	97.5%	0.024	0.067	0.063	0.172	0.134	0.403	0.633	0.691	1.465	1.704	0.936	1.071	0.091	0.109
Skew		1.146	0.332	0.738	0.304	0.772	0.347	0.472	-0.464	0.457	-0.166	0.076	-0.367	0.12	0.168
Kurtosis		3.514	-0.921	2.213	-0.823	2.832	-0.712	0.378	-0.348	0.638	-0.871	1.28	-0.303	2.288	1.64

Condition 1; Scenario 3; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.347	1.406	0.344	1.097	0.143	0.985	0.013	0.042	0.03	0.003
	$\widehat{M}$	0.184	0.712	0.187	0.679	0.062	0.643	-0.011	0.003	0.007	0.001
	$\widehat{\sigma}^2$	0.734	7.289	0.558	4.676	0.513	3.239	0.023	0.041	0.027	0.007
	2.5%	-1.18	-2.731	-0.978	-2.52	-1.244	-1.75	-0.238	-0.283	-0.245	-0.158
	97.5%	2.571	6.351	2.136	5.058	1.99	4.357	0.32	0.47	0.389	0.182
P	$\widehat{\mu}$	0.003	0.015	0.01	0.037	0.01	0.092	0.025	0.14	0.075	0.001
	$\widehat{M}$	0.002	0.008	0.005	0.023	0.004	0.06	-0.022	0.009	0.018	< .001
	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.005	0.002	0.028	0.092	0.458	0.172	< .001
	2.5%	-0.012	-0.029	-0.028	-0.086	-0.083	-0.163	-0.477	-0.945	-0.613	-0.039
	97.5%	0.026	0.068	0.061	0.172	0.133	0.407	0.641	1.568	0.973	0.045
Skew		0.91	0.288	0.748	0.289	0.906	0.028	0.405	0.45	0.455	0.139
Kurtosis		1.94	-0.962	1.925	-0.806	4.041	-0.738	-0.499	-0.504	-0.28	-0.116

Condition 1; Scenario 3; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.275	1.772	0.228	1.358	0.114	1.169	0.048	0.085	0.052	–
	$\widehat{M}$	0.211	1.187	0.169	0.948	0.073	0.919	0.035	0.069	0.039	–
	$\widehat{\sigma}^2$	0.382	5.607	0.276	3.764	0.27	2.438	0.01	0.017	0.012	–
	2.5%	-0.859	-1.859	-0.747	-1.896	-0.939	-1.265	-0.124	-0.12	-0.139	–
	97.5%	1.719	6.229	1.509	4.907	1.359	4.162	0.251	0.363	0.289	–
P	$\widehat{\mu}$	0.003	0.019	0.006	0.046	0.008	0.109	0.097	0.282	0.13	–
	$\widehat{M}$	0.002	0.013	0.005	0.032	0.005	0.086	0.071	0.23	0.097	–
	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.004	0.001	0.021	0.039	0.185	0.077	–
	2.5%	-0.009	-0.02	-0.021	-0.065	-0.063	-0.118	-0.247	-0.4	-0.348	–
	97.5%	0.017	0.067	0.043	0.167	0.091	0.388	0.503	1.211	0.721	–
Skew		0.436	0.368	0.368	0.293	0.614	0.335	0.334	0.464	0.41	–
Kurtosis		2.326	-0.752	2.108	-0.75	4.423	-0.666	-0.389	-0.137	-0.015	–

TABLE C-4

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 2 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 2; Scenario 1; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.256	-0.23	0.24	-0.174	0.139	-0.123	-0.129	-0.089	-0.145	-0.102	-0.108	-0.108	0.005	0.003
	$\widehat{M}$	0.25	-0.222	0.24	-0.183	0.135	-0.118	-0.134	-0.086	-0.146	-0.104	-0.107	-0.107	0.003	0.004
	$\widehat{\sigma}^2$	0.124	0.108	0.09	0.081	0.063	0.059	0.005	0.004	0.007	0.006	0.006	0.006	0.006	0.006
	2.5%	-0.424	-0.865	-0.33	-0.712	-0.362	-0.568	-0.263	-0.22	-0.308	-0.242	-0.258	-0.258	-0.143	-0.16
	97.5%	0.942	0.437	0.815	0.383	0.642	0.363	0.014	0.044	0.021	0.048	0.051	0.051	0.163	0.157
P	$\widehat{\mu}$	0.003	-0.002	0.007	-0.006	0.009	-0.011	-0.259	-0.178	-0.483	-0.338	-0.27	-0.27	0.001	0.001
	$\widehat{M}$	0.002	-0.002	0.007	-0.006	0.009	-0.118	-0.267	-0.174	-0.487	-0.348	-0.267	-0.267	0.001	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.019	0.017	0.083	0.065	0.04	0.04	< .001	< .001
	2.5%	-0.004	-0.009	-0.009	-0.024	-0.024	-0.053	-0.526	-0.44	-1.025	-0.805	-0.645	-0.645	-0.036	-0.04
	97.5%	0.009	0.005	0.023	0.013	0.043	0.034	0.028	0.088	0.072	0.16	0.127	0.127	0.041	0.039
Skew		0.187	0.062	0.082	0.082	-0.039	0.199	0.172	-0.006	0.262	0.364	0.121	0.121	0.106	-0.073
Kurtosis		0.416	0.75	0.342	0.48	0.314	0.778	0.315	0.31	1.051	1.104	0.178	0.178	-0.039	0.022

Condition 2; Scenario 1; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.261	-0.231	0.243	-0.175	0.141	-0.123	-0.105	-0.119	-0.092	0.003
	$\widehat{M}$	0.255	-0.222	0.242	-0.184	0.136	-0.119	-0.108	-0.124	-0.095	0.001
	$\widehat{\sigma}^2$	0.126	0.107	0.091	0.081	0.064	0.058	0.003	0.005	0.004	0.003
	2.5%	-0.419	-0.863	-0.328	-0.712	-0.364	-0.568	-0.209	-0.237	-0.216	-0.1
	97.5%	0.955	0.433	0.823	0.38	0.644	0.361	0.009	0.014	0.04	0.114
P	$\widehat{\mu}$	0.003	-0.002	0.007	-0.006	0.009	-0.012	-0.211	-0.398	-0.229	0.001
	$\widehat{M}$	0.003	-0.002	0.007	-0.006	0.009	-0.011	-0.216	-0.413	-0.236	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.012	0.052	0.027	< .001
	2.5%	-0.004	-0.009	-0.009	-0.024	-0.024	-0.053	-0.418	-0.791	-0.539	-0.025
	97.5%	0.01	0.005	0.024	0.013	0.043	0.034	0.017	0.046	0.1	0.029
Skew		0.194	0.062	0.083	0.082	-0.038	0.198	0.311	0.646	0.28	0.063
Kurtosis		0.421	0.753	0.344	0.479	0.314	0.778	0.599	2.228	0.493	-0.181

Condition 2; Scenario 1; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.248	-0.254	0.221	-0.181	0.125	-0.129	-0.128	-0.09	-0.143	-0.103	-0.102	-0.08	0.004	0.003
	$\widehat{M}$	0.241	-0.239	0.222	-0.188	0.12	-0.128	-0.132	-0.089	-0.143	-0.105	-0.102	-0.082	0.005	0.004
	$\widehat{\sigma}^2$	0.121	0.101	0.088	0.079	0.061	0.058	0.005	0.004	0.008	0.006	0.007	0.006	0.006	0.006
	2.5%	-0.412	-0.882	-0.35	-0.715	-0.366	-0.581	-0.263	-0.22	-0.304	-0.242	-0.257	-0.222	-0.149	-0.161
	97.5%	0.941	0.391	0.803	0.363	0.62	0.36	0.015	0.047	0.021	0.048	0.057	0.07	0.157	0.157
P	$\widehat{\mu}$	0.002	-0.003	0.006	-0.006	0.008	-0.012	-0.256	-0.18	-0.477	-0.344	-0.256	-0.201	0.001	0.001
	$\widehat{M}$	0.002	-0.003	0.006	-0.006	0.008	-0.128	-0.264	-0.177	-0.477	-0.35	-0.253	-0.204	0.001	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.02	0.018	0.084	0.065	0.041	0.035	< .001	< .001
	2.5%	-0.004	-0.009	-0.01	-0.024	-0.024	-0.054	-0.525	-0.441	-1.014	-0.808	-0.644	-0.554	-0.037	-0.04
	97.5%	0.009	0.004	0.023	0.012	0.041	0.034	0.03	0.094	0.07	0.16	0.142	0.176	0.039	0.039
Skew		0.177	-0.01	0.025	0.062	-0.095	0.204	0.203	0.003	0.294	0.421	0.127	0.168	-0.033	-0.078
Kurtosis		0.341	0.631	0.216	0.498	0.273	0.874	0.39	0.378	1.206	1.463	0.214	0.142	0.118	0.024

Condition 2; Scenario 1; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.255	-0.254	0.224	-0.181	0.126	-0.128	-0.106	-0.12	-0.089	0.003
	$\widehat{M}$	0.249	-0.244	0.225	-0.188	0.12	-0.128	-0.108	-0.124	-0.092	0.002
	$\widehat{\sigma}^2$	0.124	0.102	0.089	0.08	0.062	0.059	0.003	0.005	0.004	0.003
	2.5%	-0.414	-0.883	-0.346	-0.715	-0.369	-0.583	-0.209	-0.238	-0.214	-0.1
	97.5%	0.955	0.368	0.803	0.371	0.63	0.361	0.006	0.012	0.044	0.114
P	$\widehat{\mu}$	0.003	-0.003	0.006	-0.006	0.008	-0.012	-0.211	-0.399	-0.222	0.001
	$\widehat{M}$	0.003	-0.003	0.006	-0.006	0.008	-0.012	-0.216	-0.414	-0.231	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.012	0.053	0.027	< .001
	2.5%	-0.004	-0.01	-0.01	-0.024	-0.025	-0.054	-0.418	-0.794	-0.536	-0.025
	97.5%	0.01	0.004	0.023	0.013	0.042	0.034	0.012	0.039	0.11	0.029
Skew		0.188	0.009	0.065	0.08	-0.059	0.001	0.336	0.704	0.276	0.062
Kurtosis		0.38	0.702	0.301	0.499	0.225	0.864	0.725	2.512	0.504	-0.181

Condition 2; Scenario 1; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.072	0.072	0.06	0.053	0.018	0.063	0.012	0.027	0.008	–
	$\widehat{M}$	0.065	0.073	0.061	0.051	0.02	0.066	0.015	0.027	0.007	–
	$\widehat{\sigma}^2$	0.111	0.099	0.081	0.075	0.057	0.054	0.002	0.003	0.003	–
	2.5%	-0.572	-0.528	-0.506	-0.46	-0.47	-0.372	-0.078	-0.08	-0.094	–
	97.5%	0.713	0.688	0.626	0.605	0.486	0.528	0.101	0.14	0.113	–
P	$\widehat{\mu}$	0.001	0.001	0.002	0.002	0.001	0.006	0.025	0.091	0.019	–
	$\widehat{M}$	0.001	0.001	0.002	0.002	0.001	0.006	0.029	0.088	0.018	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	< .001	0.008	0.036	0.017	–
	2.5%	-0.006	-0.006	-0.014	-0.016	-0.031	-0.035	-0.155	-0.265	-0.235	–
	97.5%	0.007	0.007	0.018	0.021	0.032	0.049	0.202	0.466	0.282	–
Skew		0.037	0.098	-0.021	0.112	-0.137	0.194	-0.061	0.215	0.092	–
Kurtosis		0.186	0.613	0.23	0.543	0.181	0.61	0.008	0.593	0.188	–

TABLE C-5

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 2 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 2; Scenario 2; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	0.015	-0.552	0.044	-0.47	-0.002	-0.331	-0.059	-0.169	-0.064	-0.18	-0.053	-0.053	0.003	0.001
	$\widehat{M}$	-0.006	-0.621	0.034	-0.504	0.004	-0.374	-0.058	-0.171	-0.067	-0.19	-0.055	-0.055	0.004	-0.001
B	$\widehat{\sigma}^2$	0.094	0.455	0.066	0.312	0.047	0.241	0.003	0.01	0.006	0.015	0.005	0.005	0.004	0.012
	2.5%	-0.522	-1.634	-0.387	-1.346	-0.359	-1.091	-0.178	-0.35	-0.207	-0.391	-0.182	-0.182	-0.11	-0.22
	97.5%	0.551	0.743	0.491	0.506	0.347	0.573	0.044	0.042	0.079	0.056	0.078	0.078	0.12	0.221
	$\widehat{\mu}$	< .001	-0.006	0.001	-0.016	< .001	-0.031	-0.119	-0.337	-0.213	-0.601	-0.132	-0.132	0.001	< .001
	$\widehat{M}$	< .001	-0.007	0.001	-0.017	< .001	-0.374	-0.117	-0.342	-0.222	-0.635	-0.138	-0.138	0.001	< .001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.002	0.013	0.041	0.066	0.163	0.028	0.028	< .001	0.001
	2.5%	-0.005	-0.018	-0.011	-0.046	-0.024	-0.102	-0.357	-0.699	-0.689	-1.304	-0.454	-0.454	-0.028	-0.055
	97.5%	0.006	0.008	0.014	0.017	0.023	0.053	0.089	0.084	0.262	0.187	0.194	0.194	0.03	0.055
Skew		1.42	2.295	1.888	2.152	-1.666	2.352	0.265	0.581	1.256	1.079	0.667	0.667	-0.058	-0.004
Kurtosis		9.705	12.111	13.719	12.073	24.888	14.244	1.686	1.969	8.458	4.169	4.527	4.527	-0.1	0.071

Condition 2; Scenario 2; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.01	-0.546	0.041	-0.467	-0.004	-0.329	-0.083	-0.09	-0.073	0.003
	$\widehat{M}$	-0.013	-0.614	0.029	-0.502	0.004	-0.371	-0.085	-0.095	-0.075	0.005
B	$\widehat{\sigma}^2$	0.096	0.452	0.067	0.311	0.047	0.24	0.004	0.006	0.005	0.003
	2.5%	-0.524	-1.627	-0.389	-1.342	-0.363	-1.089	-0.186	-0.222	-0.19	-0.097
	97.5%	0.548	0.752	0.491	0.505	0.343	0.571	0.023	0.06	0.051	0.106
	$\widehat{\mu}$	< .001	-0.006	0.001	-0.016	< .001	-0.031	-0.166	-0.299	-0.182	0.001
	$\widehat{M}$	< .001	-0.007	0.001	-0.017	< .001	-0.035	-0.171	-0.316	-0.189	0.001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.002	0.015	0.068	0.029	< .001
	2.5%	-0.005	-0.018	-0.011	-0.046	-0.024	-0.102	-0.373	-0.741	-0.476	-0.024
	97.5%	0.005	0.008	0.014	0.017	0.023	0.053	0.047	0.2	0.129	0.027
Skew		1.491	2.287	1.942	2.146	-1.618	2.348	1.013	1.962	1.142	0.02
Kurtosis		10.249	12.054	14.23	12.031	24.766	14.236	4.937	11.911	5.774	-0.128

Condition 2; Scenario 2; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	0.027	-0.545	0.047	-0.44	0.003	-0.325	-0.06	-0.166	-0.068	-0.182	-0.053	-0.144	< .001	< .001
	$\widehat{M}$	0.005	-0.614	0.037	-0.488	0.011	-0.369	-0.059	-0.169	-0.07	-0.194	-0.055	-0.143	-0.002	-0.001
B	$\widehat{\sigma}^2$	0.088	0.435	0.064	0.289	0.044	0.24	0.003	0.01	0.006	0.015	0.004	0.013	0.009	0.012
	2.5%	-0.496	-1.583	-0.365	-1.251	-0.334	-1.081	-0.177	-0.35	-0.208	-0.396	-0.181	-0.365	-0.18	-0.22
	97.5%	0.575	0.703	0.496	0.509	0.348	0.573	0.045	0.041	0.077	0.046	0.077	0.081	0.205	0.221
	$\widehat{\mu}$	< .001	-0.006	0.001	-0.015	< .001	-0.03	-0.12	-0.332	-0.225	-0.606	-0.132	-0.361	< .001	< .001
	$\widehat{M}$	< .001	-0.007	0.001	-0.017	0.001	-0.369	-0.117	-0.337	-0.234	-0.646	-0.137	-0.358	-0.001	< .001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.002	0.013	0.041	0.065	0.163	0.027	0.081	0.001	0.001
	2.5%	-0.005	-0.017	-0.01	-0.043	-0.022	-0.101	-0.354	-0.7	-0.692	-1.319	-0.452	-0.912	-0.045	-0.055
	97.5%	0.006	0.008	0.014	0.017	0.023	0.053	0.089	0.082	0.255	0.156	0.193	0.201	0.051	0.055
Skew		1.368	2.229	1.79	2.225	-2.378	2.467	0.275	0.577	1.334	1.096	0.587	0.443	0.083	0.019
Kurtosis		9.217	11.647	13.038	12.582	27.199	14.654	1.757	1.875	9.129	4.252	3.985	1.593	0.483	0.047

Condition 2; Scenario 2; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.01	-0.536	0.042	-0.445	0.002	-0.328	-0.082	-0.093	-0.073	0.003
	$\widehat{M}$	-0.01	-0.606	0.029	-0.487	0.011	-0.371	-0.085	-0.098	-0.075	0.005
B	$\widehat{\sigma}^2$	0.084	0.469	0.063	0.282	0.041	0.226	0.004	0.006	0.004	0.003
	2.5%	-0.508	-1.598	-0.369	-1.266	-0.336	-1.084	-0.186	-0.224	-0.187	-0.097
	97.5%	0.548	0.745	0.485	0.518	0.345	0.556	0.023	0.049	0.051	0.106
	$\widehat{\mu}$	< .001	-0.006	0.001	-0.015	< .001	-0.031	-0.165	-0.309	-0.184	0.001
	$\widehat{M}$	< .001	-0.007	0.001	-0.017	0.001	-0.035	-0.17	-0.325	-0.189	0.001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.002	0.016	0.067	0.028	< .001
	2.5%	-0.005	-0.017	-0.011	-0.043	-0.022	-0.101	-0.373	-0.748	-0.467	-0.024
	97.5%	0.005	0.008	0.014	0.018	0.023	0.052	0.046	0.165	0.129	0.027
Skew		0.528	2.65	1.738	1.998	-2.364	0.002	1.427	1.92	0.922	0.021
Kurtosis		4.981	15.542	12.71	11.023	27.583	13.217	7.418	11.25	4.447	-0.13

Condition 2; Scenario 2; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.047	0.192	0.039	0.112	0.009	0.124	0.017	0.037	0.011	–
	$\widehat{M}$	0.038	0.124	0.036	0.066	0.013	0.083	0.016	0.035	0.009	–
B	$\widehat{\sigma}^2$	0.067	0.411	0.047	0.273	0.038	0.207	0.002	0.004	0.003	–
	2.5%	-0.477	-0.778	-0.374	-0.667	-0.341	-0.596	-0.066	-0.069	-0.086	–
	97.5%	0.552	1.301	0.461	1.043	0.344	0.987	0.101	0.158	0.116	–
	$\widehat{\mu}$	< .001	0.002	0.001	0.004	0.001	0.012	0.034	0.123	0.028	–
	$\widehat{M}$	< .001	0.001	0.001	0.002	0.001	0.008	0.031	0.117	0.023	–
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.002	0.008	0.04	0.018	–
	2.5%	-0.005	-0.008	-0.011	-0.023	-0.023	-0.056	-0.133	-0.229	-0.215	–
	97.5%	0.006	0.014	0.013	0.036	0.023	0.092	0.202	0.528	0.291	–
Skew		-0.089	2.41	0.141	2.321	-2.047	2.108	0.629	1.026	0.678	–
Kurtosis		0.361	12.605	0.658	12.14	19.08	11.542	2.271	4.707	2.506	–

TABLE C-6

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 2 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 2; Scenario 3; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.258	0.217	0.238	0.137	0.081	0.228	-0.028	-0.158	-0.013	-0.151	-0.019	-0.019	< .001	-0.008
	$\widehat{M}$	0.01	-0.554	0.039	-0.491	-0.01	-0.286	-0.031	-0.189	-0.026	-0.217	-0.021	-0.021	-0.001	-0.006
	$\widehat{\sigma}^2$	0.788	5.404	0.514	3.573	0.348	2.418	0.013	0.046	0.02	0.069	0.016	0.016	0.004	0.022
	2.5%	-0.566	-2.46	-0.403	-2.165	-0.801	-1.652	-0.242	-0.508	-0.239	-0.546	-0.217	-0.217	-0.133	-0.329
	97.5%	2.815	5.849	2.453	4.701	1.863	3.81	0.253	0.296	0.335	0.418	0.246	0.246	0.126	0.276
P	$\widehat{\mu}$	0.003	0.002	0.007	0.005	0.005	0.021	-0.056	-0.316	-0.042	-0.504	-0.049	-0.049	< .001	-0.002
	$\widehat{M}$	< .001	-0.006	0.001	-0.017	-0.001	-0.286	-0.062	-0.378	-0.085	-0.722	-0.053	-0.053	< .001	-0.002
	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.004	0.002	0.021	0.054	0.185	0.223	0.771	0.103	0.103	< .001	0.001
	2.5%	-0.006	-0.026	-0.012	-0.074	-0.053	-0.154	-0.484	-1.016	-0.796	-1.821	-0.542	-0.542	-0.033	-0.082
	97.5%	0.028	0.063	0.07	0.16	0.124	0.356	0.506	0.592	1.115	1.393	0.616	0.616	0.031	0.069
Skew		2.926	1.252	2.77	1.199	3.283	1.131	-0.936	0.464	-0.37	0.66	-2.564	-2.564	-0.238	-0.13
Kurtosis		12.134	0.458	9.772	0.393	18.954	0.202	11.146	-0.396	9.317	-0.333	24.712	24.712	2.963	0.7

Condition 2; Scenario 3; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.26	0.223	0.239	0.14	0.082	0.231	-0.02	-0.004	-0.01	-0.001
	$\widehat{M}$	0.007	-0.546	0.036	-0.485	-0.012	-0.281	-0.041	-0.038	-0.033	-0.002
	$\widehat{\sigma}^2$	0.805	5.364	0.521	3.554	0.352	2.403	0.011	0.019	0.012	0.003
	2.5%	-0.565	-2.455	-0.405	-2.16	-0.793	-1.644	-0.174	-0.19	-0.174	-0.104
	97.5%	2.85	5.844	2.488	4.696	1.876	3.787	0.277	0.386	0.297	0.104
P	$\widehat{\mu}$	0.003	0.002	0.007	0.005	0.005	0.022	-0.041	-0.014	-0.024	< .001
	$\widehat{M}$	< .001	-0.006	0.001	-0.017	-0.001	-0.026	-0.082	-0.128	-0.082	< .001
	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.004	0.002	0.021	0.044	0.216	0.078	< .001
	2.5%	-0.006	-0.026	-0.012	-0.074	-0.053	-0.153	-0.348	-0.633	-0.436	-0.026
	97.5%	0.029	0.063	0.071	0.16	0.125	0.353	0.554	1.286	0.741	0.026
Skew		2.902	1.253	2.77	1.199	3.305	1.13	1.255	1.337	1.242	-0.087
Kurtosis		11.762	0.469	9.692	0.399	18.94	0.205	1.758	1.494	1.761	0.1

Condition 2; Scenario 3; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.257	0.551	0.259	0.445	0.078	0.437	-0.018	-0.121	-0.004	-0.123	-0.009	-0.094	-0.006	-0.01
	$\widehat{M}$	0.042	-0.415	0.06	-0.308	0.005	-0.207	-0.03	-0.16	-0.03	-0.205	-0.021	-0.134	-0.005	-0.006
	$\widehat{\sigma}^2$	0.682	6.193	0.462	3.797	0.306	2.748	0.015	0.05	0.022	0.08	0.013	0.05	0.01	0.02
	2.5%	-0.74	-2.34	-0.544	-1.913	-0.861	-1.563	-0.239	-0.476	-0.223	-0.531	-0.201	-0.475	-0.224	-0.315
	97.5%	2.571	6.135	2.181	4.703	1.696	3.982	0.296	0.337	0.386	0.447	0.275	0.346	0.19	0.27
P	$\widehat{\mu}$	0.003	0.006	0.007	0.015	0.005	0.041	-0.035	-0.242	-0.015	-0.411	-0.022	-0.235	-0.002	-0.003
	$\widehat{M}$	< .001	-0.004	0.002	-0.01	< .001	-0.207	-0.06	-0.32	-0.101	-0.686	-0.053	-0.333	-0.001	-0.001
	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.004	0.001	0.024	0.06	0.201	0.239	0.891	0.08	0.315	0.001	0.001
	2.5%	-0.007	-0.025	-0.016	-0.065	-0.057	-0.146	-0.477	-0.951	-0.743	-1.77	-0.504	-1.188	-0.056	-0.079
	97.5%	0.026	0.066	0.062	0.16	0.113	0.372	0.592	0.673	1.287	1.492	0.687	0.864	0.048	0.068
Skew		1.742	1.044	2.182	0.983	2.033	0.947	0.101	0.444	0.351	0.599	0.358	0.38	-0.59	-0.205
Kurtosis		4.627	-0.186	6.432	-0.262	9.278	-0.378	4.945	-0.567	5.479	-0.686	3.539	-0.635	4.239	0.902

Condition 2; Scenario 3; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.253	0.588	0.263	0.453	0.048	0.484	-0.008	0.015	0.003	-0.001
	$\widehat{M}$	0.045	-0.397	0.056	-0.342	-0.005	-0.186	-0.039	-0.037	-0.031	-0.002
	$\widehat{\sigma}^2$	0.61	6.174	0.47	3.844	0.29	2.853	0.014	0.028	0.016	0.003
	2.5%	-0.642	-2.331	-0.539	-1.907	-0.976	-1.564	-0.174	-0.19	-0.179	-0.103
	97.5%	2.422	6.006	2.145	4.696	1.705	4.091	0.306	0.417	0.323	0.104
P	$\widehat{\mu}$	0.003	0.006	0.007	0.015	0.003	0.045	-0.015	0.052	0.008	< .001
	$\widehat{M}$	< .001	-0.004	0.002	-0.012	< .001	-0.018	-0.079	-0.123	-0.077	< .001
	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.004	0.001	0.025	0.058	0.306	0.102	< .001
	2.5%	-0.006	-0.025	-0.015	-0.065	-0.065	-0.146	-0.348	-0.633	-0.448	-0.026
	97.5%	0.024	0.065	0.061	0.16	0.114	0.382	0.612	1.392	0.809	0.026
Skew		1.641	0.975	1.783	0.948	1.167	0.025	1.169	1.108	1.172	-0.089
Kurtosis		3.29	-0.366	5.117	-0.324	5.494	-0.436	0.876	0.295	1.073	0.112

Condition 2; Scenario 3; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.158	1.34	0.129	1.022	0.036	0.882	0.04	0.068	0.036	–
	$\widehat{M}$	0.117	0.656	0.091	0.486	0.038	0.426	0.025	0.05	0.021	–
	$\widehat{\sigma}^2$	0.151	4.283	0.103	2.815	0.077	1.831	0.005	0.008	0.006	–
	2.5%	-0.497	-1.149	-0.36	-1.078	-0.644	-0.793	-0.067	-0.071	-0.088	–
	97.5%	1.208	5.919	0.946	4.738	0.616	3.82	0.201	0.273	0.214	–
P	$\widehat{\mu}$	0.002	0.014	0.004	0.035	0.002	0.082	0.08	0.226	0.091	–
	$\widehat{M}$	0.001	0.007	0.003	0.017	0.003	0.04	0.05	0.166	0.053	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.003	< .001	0.016	0.02	0.093	0.039	–
	2.5%	-0.005	-0.012	-0.01	-0.037	-0.043	-0.074	-0.133	-0.235	-0.22	–
	97.5%	0.012	0.064	0.027	0.161	0.041	0.357	0.402	0.91	0.536	–
Skew		0.956	1.095	0.92	1.055	-0.371	0.991	0.864	0.824	0.742	–
Kurtosis		2.916	-0.011	4.502	-0.045	6.014	-0.145	0.338	0.224	0.345	–

TABLE C-7

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 3 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 3; Scenario 1; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.281	-0.122	0.265	-0.095	0.148	-0.075	-0.128	-0.06	-0.142	-0.062	-0.108	-0.108	0.005	0.006
	$\widehat{M}$	0.29	-0.156	0.294	-0.118	0.141	-0.104	-0.133	-0.061	-0.16	-0.069	-0.115	-0.115	0.003	0.011
	$\widehat{\sigma}^2$	0.533	0.436	0.306	0.336	0.252	0.234	0.023	0.015	0.031	0.021	0.023	0.023	0.063	0.054
	2.5%	-1.199	-1.25	-0.953	-1.145	-0.826	-0.986	-0.412	-0.285	-0.435	-0.32	-0.386	-0.386	-0.475	-0.454
	97.5%	1.638	1.336	1.29	1.174	1.123	0.974	0.196	0.206	0.258	0.244	0.22	0.22	0.524	0.466
P	$\widehat{\mu}$	0.003	-0.001	0.008	-0.003	0.01	-0.007	-0.257	-0.12	-0.474	-0.205	-0.269	-0.269	0.001	0.001
	$\widehat{M}$	0.003	-0.002	0.008	-0.004	0.009	-0.104	-0.265	-0.123	-0.532	-0.229	-0.287	-0.287	< .001	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.09	0.059	0.343	0.234	0.142	0.142	0.001	0.001
	2.5%	-0.012	-0.013	-0.027	-0.039	-0.055	-0.092	-0.824	-0.571	-1.45	-1.067	-0.965	-0.965	-0.059	-0.057
	97.5%	0.016	0.014	0.037	0.04	0.075	0.091	0.393	0.412	0.861	0.816	0.551	0.551	0.065	0.058
Skew		-0.422	0.846	-0.432	0.512	0.082	0.399	0.244	0.332	0.76	0.589	0.219	0.219	0.07	-0.044
Kurtosis		3.233	3.641	2.298	1.664	2.658	1.61	0.512	0.733	1.132	1.186	0.331	0.331	-0.127	0.051

Condition 3; Scenario 1; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.294	-0.125	0.272	-0.096	0.152	-0.076	-0.08	-0.087	-0.071	0.003
	$\widehat{M}$	0.3	-0.162	0.301	-0.117	0.145	-0.106	-0.089	-0.103	-0.077	-0.001
	$\widehat{\sigma}^2$	0.546	0.433	0.311	0.335	0.255	0.233	0.013	0.02	0.016	0.029
	2.5%	-1.206	-1.246	-0.958	-1.142	-0.834	-0.986	-0.278	-0.31	-0.299	-0.329
	97.5%	1.708	1.328	1.3	1.175	1.14	0.97	0.2	0.229	0.189	0.329
P	$\widehat{\mu}$	0.003	-0.001	0.008	-0.003	0.01	-0.007	-0.161	-0.29	-0.176	< .001
	$\widehat{M}$	0.003	-0.002	0.009	-0.004	0.01	-0.01	-0.179	-0.343	-0.194	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.054	0.217	0.098	< .001
	2.5%	-0.012	-0.013	-0.027	-0.039	-0.056	-0.092	-0.555	-1.034	-0.747	-0.041
	97.5%	0.017	0.014	0.037	0.04	0.076	0.091	0.399	0.762	0.473	0.041
Skew		-0.402	0.858	-0.421	0.518	0.095	0.404	0.674	0.99	0.344	0.061
Kurtosis		3.124	3.679	2.279	1.684	2.684	1.622	1.26	2.029	0.371	0.262

Condition 3; Scenario 1; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.277	-0.157	0.245	-0.109	0.128	-0.084	-0.129	-0.063	-0.143	-0.065	-0.102	-0.053	0.013	0.006
	$\widehat{M}$	0.286	-0.174	0.268	-0.123	0.123	-0.112	-0.134	-0.067	-0.16	-0.074	-0.108	-0.052	0.023	0.013
	$\widehat{\sigma}^2$	0.495	0.399	0.283	0.318	0.223	0.223	0.022	0.014	0.03	0.02	0.022	0.017	0.058	0.055
	2.5%	-1.089	-1.25	-0.835	-1.142	-0.783	-0.989	-0.407	-0.284	-0.429	-0.321	-0.385	-0.319	-0.455	-0.454
	97.5%	1.605	1.252	1.209	1.145	1.029	0.914	0.196	0.203	0.254	0.23	0.212	0.205	0.488	0.482
P	$\widehat{\mu}$	0.003	-0.002	0.007	-0.004	0.009	-0.008	-0.258	-0.126	-0.478	-0.217	-0.255	-0.132	0.002	0.001
	$\widehat{M}$	0.003	-0.002	0.008	-0.004	0.008	-0.112	-0.269	-0.133	-0.532	-0.247	-0.272	-0.13	0.003	0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.089	0.057	0.337	0.227	0.14	0.109	0.001	0.001
	2.5%	-0.011	-0.013	-0.024	-0.039	-0.052	-0.092	-0.814	-0.568	-1.43	-1.072	-0.963	-0.797	-0.057	-0.057
	97.5%	0.016	0.013	0.035	0.039	0.069	0.085	0.393	0.406	0.847	0.765	0.529	0.513	0.061	0.06
Skew		-0.46	0.889	-0.488	0.479	-0.074	0.374	0.277	0.382	0.731	0.544	0.15	0.031	-0.033	-0.049
Kurtosis		3.489	4.279	2.476	1.565	1.955	1.256	0.631	0.754	1.074	1.082	0.226	0.041	-0.027	-0.002

Condition 3; Scenario 1; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.293	-0.15	0.261	-0.108	0.141	-0.084	-0.083	-0.091	-0.07	0.003
	$\widehat{M}$	0.295	-0.175	0.279	-0.126	0.127	-0.111	-0.091	-0.105	-0.076	-0.001
	$\widehat{\sigma}^2$	0.517	0.409	0.297	0.318	0.229	0.218	0.013	0.018	0.015	0.029
	2.5%	-1.115	-1.242	-0.845	-1.133	-0.782	-0.986	-0.278	-0.31	-0.299	-0.328
	97.5%	1.642	1.309	1.277	1.122	1.112	0.91	0.184	0.224	0.178	0.329
P	$\widehat{\mu}$	0.003	-0.002	0.007	-0.004	0.009	-0.008	-0.166	-0.302	-0.176	< .001
	$\widehat{M}$	0.003	-0.002	0.008	-0.004	0.009	-0.01	-0.181	-0.35	-0.19	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.053	0.205	0.095	< .001
	2.5%	-0.011	-0.013	-0.024	-0.039	-0.052	-0.092	-0.556	-1.034	-0.747	-0.041
	97.5%	0.016	0.014	0.036	0.038	0.074	0.085	0.368	0.747	0.445	0.041
Skew		-0.408	0.867	-0.404	0.474	0.012	0.002	0.673	0.903	0.266	0.062
Kurtosis		3.216	3.928	2.327	1.544	1.788	1.074	1.359	1.784	0.202	0.264

Condition 3; Scenario 1; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.15	0.203	0.107	0.138	0.053	0.131	0.034	0.063	0.024	–
	$\widehat{M}$	0.165	0.152	0.118	0.108	0.057	0.098	0.032	0.057	0.022	–
	$\widehat{\sigma}^2$	0.421	0.376	0.254	0.288	0.213	0.204	0.007	0.01	0.009	–
	2.5%	-1.185	-0.875	-0.934	-0.835	-0.829	-0.724	-0.117	-0.119	-0.15	–
	97.5%	1.338	1.566	1.004	1.301	0.905	1.09	0.205	0.279	0.215	–
P	$\widehat{\mu}$	0.002	0.002	0.003	0.005	0.004	0.012	0.068	0.21	0.059	–
	$\widehat{M}$	0.002	0.002	0.003	0.004	0.004	0.009	0.065	0.189	0.055	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.026	0.111	0.056	–
	2.5%	-0.012	-0.009	-0.027	-0.028	-0.055	-0.068	-0.234	-0.398	-0.376	–
	97.5%	0.013	0.017	0.029	0.044	0.06	0.102	0.41	0.93	0.538	–
Skew		-0.675	0.827	-0.624	0.507	-0.092	0.476	0.276	0.581	0.139	–
Kurtosis		3.298	2.987	2.17	1.214	2.014	1.43	0.703	1.228	0.23	–

TABLE C-8

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 3 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 3; Scenario 2; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.111	-0.121	0.144	-0.165	-0.006	-0.008	-0.067	-0.109	-0.05	-0.083	-0.036	-0.036	< .001	-0.002
	$\widehat{M}$	0.058	-0.328	0.119	-0.318	-0.014	-0.185	-0.073	-0.119	-0.068	-0.103	-0.042	-0.042	-0.002	-0.016
	$\widehat{\sigma}^2$	0.438	1.885	0.291	1.43	0.188	1.066	0.018	0.036	0.029	0.054	0.02	0.02	0.043	0.12
	2.5%	-1.031	-2.226	-0.781	-2.013	-0.824	-1.734	-0.309	-0.467	-0.344	-0.477	-0.291	-0.291	-0.4	-0.661
	97.5%	1.601	3.445	1.192	3.062	0.87	2.719	0.234	0.292	0.356	0.434	0.267	0.267	0.415	0.689
P	$\widehat{\mu}$	0.001	-0.001	0.004	-0.006	< .001	-0.001	-0.134	-0.218	-0.167	-0.275	-0.089	-0.089	< .001	< .001
	$\widehat{M}$	0.001	-0.004	0.003	-0.011	-0.001	-0.185	-0.146	-0.238	-0.225	-0.344	-0.105	-0.105	< .001	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.001	0.009	0.07	0.143	0.328	0.6	0.123	0.123	0.001	0.002
	2.5%	-0.01	-0.024	-0.022	-0.069	-0.055	-0.162	-0.618	-0.933	-1.148	-1.592	-0.727	-0.727	-0.05	-0.082
	97.5%	0.016	0.037	0.034	0.104	0.058	0.254	0.469	0.585	1.187	1.448	0.668	0.668	0.052	0.086
Skew		0.966	1.114	1.744	1.071	0.718	0.911	0.414	0.168	0.4	0.336	0.486	0.486	0.062	0.173
Kurtosis		3.641	2.149	18.158	2.371	6.258	1.365	0.712	0.098	1.837	0.07	0.883	0.883	0.273	0.175

Condition 3; Scenario 2; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.109	-0.117	0.143	-0.163	-0.007	-0.006	-0.069	-0.05	-0.04	0.001
	$\widehat{M}$	0.051	-0.328	0.117	-0.309	-0.017	-0.178	-0.086	-0.079	-0.057	-0.001
	$\widehat{\sigma}^2$	0.45	1.864	0.294	1.42	0.189	1.057	0.018	0.03	0.021	0.031
	2.5%	-1.011	-2.222	-0.773	-2.008	-0.824	-1.724	-0.29	-0.31	-0.286	-0.342
	97.5%	1.633	3.438	1.242	3.049	0.873	2.71	0.266	0.38	0.294	0.353
P	$\widehat{\mu}$	0.001	-0.001	0.004	-0.006	< .001	-0.001	-0.137	-0.166	-0.1	< .001
	$\widehat{M}$	0.001	-0.004	0.003	-0.01	-0.001	-0.017	-0.173	-0.261	-0.141	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.001	0.009	0.072	0.331	0.131	< .001
	2.5%	-0.01	-0.024	-0.022	-0.068	-0.055	-0.161	-0.58	-1.034	-0.714	-0.043
	97.5%	0.016	0.037	0.035	0.104	0.058	0.253	0.531	1.267	0.736	0.044
Skew		1.013	1.121	1.723	1.073	0.759	0.911	0.693	0.866	0.661	0.107
Kurtosis		3.691	2.188	17.369	2.386	6.429	1.375	0.819	0.773	0.644	< .001

Condition 3; Scenario 2; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.11	-0.126	0.144	-0.149	-0.007	-0.008	-0.069	-0.108	-0.053	-0.086	-0.037	-0.08	-0.008	-0.003
	$\widehat{M}$	0.059	-0.32	0.121	-0.297	-0.011	-0.178	-0.074	-0.119	-0.072	-0.108	-0.045	-0.1	-0.021	-0.016
	$\widehat{\sigma}^2$	0.392	1.793	0.26	1.332	0.166	1.034	0.017	0.035	0.028	0.055	0.019	0.044	0.106	0.12
	2.5%	-0.989	-2.203	-0.767	-1.933	-0.78	-1.662	-0.306	-0.46	-0.341	-0.497	-0.288	-0.474	-0.643	-0.645
	97.5%	1.54	3.389	1.138	2.867	0.809	2.715	0.224	0.292	0.353	0.435	0.278	0.348	0.655	0.717
P	$\widehat{\mu}$	0.001	-0.001	0.004	-0.005	< .001	-0.001	-0.138	-0.216	-0.175	-0.287	-0.091	-0.201	-0.001	< .001
	$\widehat{M}$	0.001	-0.003	0.003	-0.01	-0.001	-0.178	-0.148	-0.239	-0.24	-0.361	-0.113	-0.25	-0.003	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.001	0.009	0.069	0.14	0.311	0.612	0.121	0.272	0.002	0.002
	2.5%	-0.01	-0.024	-0.022	-0.066	-0.052	-0.155	-0.613	-0.92	-1.136	-1.655	-0.721	-1.184	-0.08	-0.081
	97.5%	0.015	0.036	0.033	0.098	0.054	0.253	0.449	0.585	1.176	1.45	0.695	0.871	0.082	0.09
Skew		0.656	1.129	1.546	1.103	0.064	0.92	0.416	0.194	0.645	0.367	0.52	0.227	0.191	0.184
Kurtosis		2.622	2.309	17.78	2.423	3.641	1.419	0.713	0.133	0.857	0.07	0.936	-0.27	0.582	0.189

Condition 3; Scenario 2; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.094	-0.128	0.133	-0.16	-0.016	-0.015	-0.071	-0.053	-0.041	0.002
	$\widehat{M}$	0.048	-0.33	0.116	-0.302	-0.013	-0.183	-0.088	-0.081	-0.059	-0.001
	$\widehat{\sigma}^2$	0.387	1.781	0.234	1.334	0.171	1.037	0.018	0.03	0.021	0.031
	2.5%	-1.007	-2.176	-0.761	-1.96	-0.824	-1.646	-0.29	-0.308	-0.286	-0.346
	97.5%	1.494	3.415	1.14	2.885	0.791	2.71	0.253	0.386	0.311	0.353
P	$\widehat{\mu}$	0.001	-0.001	0.004	-0.005	-0.001	-0.001	-0.142	-0.177	-0.103	< .001
	$\widehat{M}$	< .001	-0.004	0.003	-0.01	-0.001	-0.017	-0.176	-0.269	-0.148	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.001	0.009	0.07	0.331	0.134	< .001
	2.5%	-0.01	-0.023	-0.022	-0.067	-0.055	-0.153	-0.58	-1.026	-0.714	-0.043
	97.5%	0.015	0.037	0.033	0.098	0.053	0.253	0.506	1.287	0.778	0.044
Skew		0.534	1.161	0.231	1.103	0.066	0.009	0.707	0.92	0.726	0.106
Kurtosis		2.219	2.523	2.498	2.502	4.093	1.486	0.906	0.889	0.771	0.001

Condition 3; Scenario 2; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.165	0.621	0.134	0.408	0.018	0.436	0.043	0.086	0.042	–
	$\widehat{M}$	0.157	0.451	0.128	0.262	0.023	0.302	0.038	0.074	0.033	–
	$\widehat{\sigma}^2$	0.287	1.532	0.202	1.149	0.145	0.851	0.008	0.013	0.011	–
	2.5%	-0.853	-1.19	-0.738	-1.263	-0.756	-1.004	-0.125	-0.114	-0.15	–
	97.5%	1.275	3.968	0.994	3.269	0.766	2.871	0.243	0.346	0.268	–
P	$\widehat{\mu}$	0.002	0.007	0.004	0.014	0.001	0.041	0.087	0.288	0.104	–
	$\widehat{M}$	0.002	0.005	0.004	0.009	0.002	0.028	0.075	0.248	0.082	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.007	0.031	0.146	0.068	–
	2.5%	-0.009	-0.013	-0.021	-0.043	-0.05	-0.094	-0.25	-0.379	-0.376	–
	97.5%	0.013	0.043	0.028	0.111	0.051	0.268	0.485	1.153	0.669	–
Skew		0.13	1.088	0.715	1.068	-0.211	0.909	0.395	0.541	0.41	–
Kurtosis		1.417	1.793	11.067	2.042	2.533	1.255	0.466	0.584	0.476	–

TABLE C-9

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 3 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 3; Scenario 3; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.457	1.117	0.36	0.87	0.11	0.864	-0.022	-0.042	0.003	-0.013	-0.001	-0.001	0.002	< .001
	$\widehat{M}$	0.165	0.554	0.154	0.465	0.013	0.617	-0.032	-0.022	-0.013	0.012	-0.005	-0.005	0.003	-0.009
	$\widehat{\sigma}^2$	1.258	7.24	0.765	5.032	0.576	3.554	0.031	0.07	0.046	0.106	0.03	0.03	0.048	0.179
	2.5%	-0.99	-3.302	-0.83	-2.703	-1.192	-2.097	-0.328	-0.587	-0.382	-0.631	-0.347	-0.347	-0.409	-0.853
	97.5%	3.089	5.958	2.601	4.967	2.078	4.217	0.33	0.357	0.485	0.521	0.376	0.376	0.453	0.925
P	$\widehat{\mu}$	0.005	0.012	0.01	0.03	0.007	0.081	-0.044	-0.083	0.012	-0.044	-0.002	-0.002	< .001	< .001
	$\widehat{M}$	0.002	0.006	0.004	0.016	0.001	0.617	-0.064	-0.044	-0.044	0.04	-0.013	-0.013	< .001	-0.001
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.006	0.003	0.031	0.122	0.279	0.514	1.179	0.187	0.187	0.001	0.003
	2.5%	-0.01	-0.036	-0.024	-0.092	-0.079	-0.196	-0.657	-1.174	-1.273	-2.103	-0.867	-0.867	-0.051	-0.107
	97.5%	0.031	0.064	0.074	0.169	0.138	0.394	0.66	0.714	1.617	1.737	0.939	0.939	0.057	0.116
Skew		1.981	0.274	1.658	0.232	1.31	0.155	-1.08	-0.587	-0.235	-0.289	-0.196	-0.196	0.362	-0.09
Kurtosis		7.637	-0.927	6.269	-0.845	6.474	-0.875	9.407	0.216	3.113	-0.591	2.597	2.597	1.308	2.395

Condition 3; Scenario 3; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.466	1.116	0.365	0.87	0.113	0.864	0.004	0.035	0.021	< .001
	$\widehat{M}$	0.163	0.552	0.157	0.461	0.011	0.623	-0.01	0.011	0.007	< .001
	$\widehat{\sigma}^2$	1.285	7.17	0.777	4.999	0.581	3.526	0.024	0.04	0.027	0.03
	2.5%	-0.992	-3.277	-0.831	-2.698	-1.184	-2.084	-0.268	-0.299	-0.265	-0.337
	97.5%	3.102	5.949	2.609	4.944	2.104	4.213	0.326	0.482	0.362	0.342
P	$\widehat{\mu}$	0.005	0.012	0.01	0.03	0.008	0.081	0.007	0.115	0.052	< .001
	$\widehat{M}$	0.002	0.006	0.004	0.016	0.001	0.058	-0.021	0.038	0.018	< .001
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.006	0.003	0.031	0.096	0.445	0.17	< .001
	2.5%	-0.01	-0.035	-0.024	-0.092	-0.079	-0.195	-0.536	-0.995	-0.662	-0.042
	97.5%	0.031	0.064	0.075	0.168	0.14	0.393	0.652	1.605	0.905	0.043
Skew		1.968	0.278	1.676	0.234	1.339	0.157	0.284	0.473	0.249	0.076
Kurtosis		7.405	-0.917	6.288	-0.835	6.519	-0.865	-0.26	-0.262	-0.214	-0.031

Condition 3; Scenario 3; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.44	1.363	0.376	1.091	0.085	1.037	-0.017	-0.029	0.015	0.007	0.006	-0.005	-0.003	0.002
	$\widehat{M}$	0.189	0.867	0.191	0.754	0.024	0.823	-0.032	< .001	-0.015	0.038	-0.005	0.025	-0.013	-0.01
	$\widehat{\sigma}^2$	1.007	7.296	0.685	4.863	0.524	3.529	0.028	0.07	0.044	0.107	0.032	0.078	0.134	0.16
	2.5%	-1.152	-2.966	-0.933	-2.523	-1.33	-1.926	-0.316	-0.575	-0.35	-0.621	-0.348	-0.564	-0.705	-0.759
	97.5%	2.833	6.26	2.385	5.046	1.821	4.376	0.328	0.362	0.495	0.525	0.386	0.43	0.759	0.87
P	$\widehat{\mu}$	0.004	0.015	0.011	0.037	0.006	0.097	-0.034	-0.058	0.051	0.025	0.016	-0.014	< .001	< .001
	$\widehat{M}$	0.002	0.009	0.005	0.026	0.002	0.823	-0.063	0.001	-0.049	0.129	-0.013	0.063	-0.002	-0.001
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.006	0.002	0.031	0.113	0.28	0.494	1.193	0.197	0.489	0.002	0.003
	2.5%	-0.012	-0.032	-0.027	-0.086	-0.089	-0.18	-0.631	-1.15	-1.167	-2.07	-0.87	-1.409	-0.088	-0.095
	97.5%	0.028	0.067	0.068	0.172	0.121	0.408	0.656	0.723	1.651	1.751	0.966	1.075	0.095	0.109
Skew		1.074	0.207	1.213	0.162	0.806	0.177	-0.39	-0.613	0.315	-0.32	0.109	-0.413	-0.022	0.038
Kurtosis		2.666	-0.974	3.065	-0.941	4.321	-0.971	4.963	0.164	0.537	-0.673	0.555	-0.535	3.023	2.016

Condition 3; Scenario 3; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.456	1.43	0.378	1.147	0.086	1.086	0.013	0.051	0.032	< .001
	$\widehat{M}$	0.2	0.976	0.203	0.797	0.017	0.909	-0.007	0.019	0.015	0.001
	$\widehat{\sigma}^2$	1.06	7.233	0.695	4.842	0.524	3.508	0.027	0.046	0.031	0.03
	2.5%	-1.149	-2.963	-0.923	-2.429	-1.384	-1.946	-0.272	-0.299	-0.269	-0.34
	97.5%	2.885	6.175	2.404	5.121	1.827	4.335	0.339	0.494	0.393	0.341
P	$\widehat{\mu}$	0.005	0.015	0.011	0.039	0.006	0.101	0.025	0.169	0.081	< .001
	$\widehat{M}$	0.002	0.01	0.006	0.028	0.001	0.085	-0.014	0.065	0.036	< .001
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.006	0.002	0.031	0.107	0.517	0.191	< .001
	2.5%	-0.011	-0.032	-0.026	-0.083	-0.092	-0.182	-0.543	-0.995	-0.673	-0.042
	97.5%	0.029	0.066	0.069	0.175	0.122	0.405	0.679	1.646	0.983	0.043
Skew		1.278	0.176	1.159	0.155	0.588	0.031	0.284	0.413	0.222	0.071
Kurtosis		4.96	-1.043	3.021	-0.926	4.104	-0.971	-0.445	-0.587	-0.417	-0.039

Condition 3; Scenario 3; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.418	2.024	0.285	1.567	0.104	1.383	0.07	0.118	0.065	–
	$\widehat{M}$	0.28	1.655	0.205	1.256	0.079	1.208	0.062	0.105	0.063	–
	$\widehat{\sigma}^2$	0.58	5.559	0.365	3.858	0.316	2.693	0.009	0.015	0.013	–
	2.5%	-0.838	-1.858	-0.753	-1.741	-1.05	-1.323	-0.102	-0.098	-0.145	–
	97.5%	2.003	6.12	1.657	5.019	1.374	4.208	0.264	0.389	0.29	–
P	$\widehat{\mu}$	0.004	0.022	0.008	0.053	0.007	0.129	0.139	0.394	0.162	–
	$\widehat{M}$	0.003	0.018	0.006	0.043	0.005	0.113	0.124	0.35	0.157	–
	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.004	0.001	0.023	0.037	0.171	0.078	–
	2.5%	-0.008	-0.02	-0.022	-0.059	-0.07	-0.123	-0.204	-0.325	-0.362	–
	97.5%	0.02	0.066	0.047	0.171	0.092	0.393	0.527	1.298	0.726	–
Skew		1.448	0.157	0.95	0.088	0.48	0.03	0.2	0.403	0.138	–
Kurtosis		9.131	-0.938	5.398	-0.775	4.615	-0.717	-0.173	-0.088	-0.156	–

TABLE C-10

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 4 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 4; Scenario 1; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.313	-0.197	0.295	-0.151	0.167	-0.122	-0.166	-0.086	-0.186	-0.105	-0.136	-0.136	0.017	-0.008
	$\widehat{M}$	0.297	-0.203	0.282	-0.165	0.164	-0.127	-0.166	-0.089	-0.187	-0.108	-0.138	-0.138	0.019	-0.007
	$\widehat{\sigma}^2$	0.169	0.127	0.104	0.104	0.072	0.077	0.007	0.005	0.01	0.007	0.007	0.007	0.026	0.022
	2.5%	-0.419	-0.898	-0.3	-0.732	-0.344	-0.62	-0.326	-0.213	-0.375	-0.252	-0.292	-0.292	-0.281	-0.295
	97.5%	1.202	0.502	0.96	0.516	0.701	0.469	0.001	0.054	0.014	0.062	0.028	0.028	0.338	0.288
P	$\widehat{\mu}$	0.003	-0.002	0.008	-0.005	0.011	-0.011	-0.332	-0.172	-0.619	-0.35	-0.34	-0.34	0.002	-0.001
	$\widehat{M}$	0.003	-0.002	0.008	-0.006	0.011	-0.127	-0.332	-0.179	-0.624	-0.361	-0.345	-0.345	0.002	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.028	0.019	0.115	0.076	0.045	0.045	< .001	< .001
	2.5%	-0.004	-0.01	-0.009	-0.025	-0.023	-0.058	-0.651	-0.427	-1.249	-0.84	-0.731	-0.731	-0.035	-0.037
	97.5%	0.012	0.005	0.027	0.018	0.047	0.044	0.001	0.107	0.048	0.205	0.069	0.069	0.042	0.036
Skew		0.019	-0.124	0.051	0.173	-0.028	0.219	0.136	0.173	0.279	0.274	0.191	0.191	0.067	-0.053
Kurtosis		1.865	2.881	1.575	0.531	1.206	0.504	0.233	0.245	0.519	0.09	0.115	0.115	-0.008	-0.149

Condition 4; Scenario 1; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.326	-0.201	0.302	-0.153	0.172	-0.123	-0.114	-0.133	-0.101	0.001
	$\widehat{M}$	0.313	-0.209	0.287	-0.167	0.17	-0.129	-0.118	-0.136	-0.105	-0.001
	$\widehat{\sigma}^2$	0.175	0.126	0.106	0.104	0.073	0.076	0.004	0.006	0.005	0.012
	2.5%	-0.403	-0.897	-0.3	-0.735	-0.351	-0.624	-0.228	-0.269	-0.23	-0.214
	97.5%	1.219	0.495	0.972	0.513	0.706	0.467	0.018	0.036	0.052	0.22
P	$\widehat{\mu}$	0.003	-0.002	0.009	-0.005	0.011	-0.012	-0.227	-0.442	-0.252	< .001
	$\widehat{M}$	0.003	-0.002	0.008	-0.006	0.011	-0.012	-0.236	-0.455	-0.263	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.015	0.066	0.032	< .001
	2.5%	-0.004	-0.01	-0.009	-0.025	-0.023	-0.058	-0.456	-0.898	-0.575	-0.027
	97.5%	0.012	0.005	0.028	0.017	0.047	0.044	0.036	0.119	0.131	0.027
Skew		0.029	-0.113	0.06	0.175	-0.028	0.215	0.419	0.489	0.376	0.127
Kurtosis		1.886	2.789	1.615	0.522	1.213	0.503	0.572	1.016	0.247	0.059

Condition 4; Scenario 1; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.31	-0.22	0.278	-0.157	0.153	-0.125	-0.166	-0.087	-0.184	-0.106	-0.129	-0.08	0.008	-0.008
	$\widehat{M}$	0.291	-0.226	0.263	-0.17	0.149	-0.131	-0.167	-0.091	-0.186	-0.109	-0.132	-0.081	0.01	-0.007
	$\widehat{\sigma}^2$	0.157	0.112	0.095	0.1	0.066	0.076	0.007	0.005	0.01	0.007	0.007	0.006	0.024	0.022
	2.5%	-0.408	-0.891	-0.313	-0.729	-0.347	-0.62	-0.326	-0.216	-0.372	-0.252	-0.292	-0.229	-0.284	-0.297
	97.5%	1.156	0.444	0.894	0.463	0.658	0.469	-0.001	0.051	0.013	0.062	0.037	0.081	0.296	0.288
P	$\widehat{\mu}$	0.003	-0.002	0.008	-0.005	0.01	-0.012	-0.332	-0.174	-0.615	-0.353	-0.323	-0.2	0.001	-0.001
	$\widehat{M}$	0.003	-0.002	0.007	-0.006	0.01	-0.131	-0.334	-0.181	-0.619	-0.363	-0.33	-0.202	0.001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.027	0.019	0.113	0.076	0.045	0.039	< .001	< .001
	2.5%	-0.004	-0.01	-0.009	-0.025	-0.023	-0.058	-0.651	-0.432	-1.241	-0.84	-0.729	-0.573	-0.036	-0.037
	97.5%	0.012	0.005	0.026	0.016	0.044	0.044	-0.001	0.102	0.045	0.205	0.091	0.203	0.037	0.036
Skew		0.21	0.142	0.085	0.218	0.066	0.2	0.096	0.162	0.178	0.259	0.113	0.174	0.023	-0.062
Kurtosis		0.897	0.368	0.778	0.396	0.292	0.438	0.159	0.226	0.08	0.071	-0.15	0.167	-0.075	-0.14

Condition 4; Scenario 1; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.326	-0.219	0.289	-0.156	0.162	-0.126	-0.115	-0.133	-0.099	0.001
	$\widehat{M}$	0.311	-0.224	0.272	-0.17	0.156	-0.129	-0.119	-0.137	-0.101	-0.001
	$\widehat{\sigma}^2$	0.165	0.114	0.1	0.101	0.069	0.075	0.004	0.006	0.005	0.012
	2.5%	-0.396	-0.897	-0.301	-0.731	-0.344	-0.624	-0.229	-0.269	-0.227	-0.214
	97.5%	1.198	0.467	0.924	0.488	0.686	0.457	0.014	0.034	0.05	0.22
P	$\widehat{\mu}$	0.003	-0.002	0.008	-0.005	0.011	-0.012	-0.23	-0.445	-0.247	< .001
	$\widehat{M}$	0.003	-0.002	0.008	-0.006	0.01	-0.012	-0.238	-0.455	-0.254	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.015	0.063	0.031	< .001
	2.5%	-0.004	-0.01	-0.009	-0.025	-0.023	-0.058	-0.459	-0.898	-0.567	-0.027
	97.5%	0.012	0.005	0.026	0.017	0.046	0.043	0.027	0.112	0.126	0.027
Skew		0.23	0.128	0.138	0.233	0.123	0.001	0.379	0.298	0.266	0.126
Kurtosis		0.87	0.27	0.86	0.384	0.321	0.21	0.486	0.116	-0.106	0.059

Condition 4; Scenario 1; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.156	0.169	0.111	0.115	0.055	0.107	0.027	0.048	0.014	–
	$\widehat{M}$	0.146	0.165	0.096	0.102	0.066	0.103	0.025	0.047	0.013	–
	$\widehat{\sigma}^2$	0.142	0.111	0.088	0.092	0.064	0.069	0.002	0.003	0.003	–
	2.5%	-0.519	-0.472	-0.458	-0.423	-0.411	-0.379	-0.059	-0.068	-0.088	–
	97.5%	0.904	0.865	0.688	0.739	0.55	0.641	0.12	0.161	0.129	–
P	$\widehat{\mu}$	0.002	0.002	0.003	0.004	0.004	0.01	0.054	0.161	0.036	–
	$\widehat{M}$	0.001	0.002	0.003	0.004	0.004	0.01	0.051	0.157	0.033	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.008	0.039	0.019	–
	2.5%	-0.005	-0.005	-0.013	-0.014	-0.027	-0.035	-0.118	-0.227	-0.221	–
	97.5%	0.009	0.009	0.02	0.025	0.037	0.06	0.241	0.538	0.323	–
Skew		-0.228	0.328	-0.157	0.309	-0.139	0.283	0.224	0.094	0.213	–
Kurtosis		2.243	0.901	1.302	0.14	1.186	0.662	0.403	0.443	0.179	–

TABLE C-11

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 4 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 4; Scenario 2; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.004	-0.524	0.062	-0.474	-0.019	-0.302	-0.073	-0.176	-0.07	-0.175	-0.058	-0.058	0.008	-0.007
	$\widehat{M}$	-0.029	-0.629	0.042	-0.542	-0.028	-0.342	-0.07	-0.181	-0.074	-0.193	-0.059	-0.059	0.009	-0.005
	$\widehat{\sigma}^2$	0.187	0.736	0.097	0.521	0.068	0.35	0.006	0.015	0.01	0.021	0.005	0.005	0.016	0.049
	2.5%	-0.601	-1.706	-0.454	-1.54	-0.464	-1.19	-0.214	-0.385	-0.23	-0.41	-0.204	-0.204	-0.231	-0.445
	97.5%	0.756	1.885	0.677	1.563	0.44	1.191	0.076	0.156	0.117	0.18	0.091	0.091	0.244	0.411
P	$\widehat{\mu}$	< .001	-0.006	0.002	-0.016	-0.001	-0.028	-0.147	-0.352	-0.234	-0.584	-0.145	-0.145	0.001	-0.001
	$\widehat{M}$	< .001	-0.007	0.001	-0.018	-0.002	-0.342	-0.139	-0.363	-0.246	-0.643	-0.148	-0.148	0.001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	< .001	0.003	0.023	0.061	0.114	0.236	0.034	0.034	< .001	0.001
	2.5%	-0.006	-0.018	-0.013	-0.052	-0.031	-0.111	-0.428	-0.771	-0.766	-1.367	-0.511	-0.511	-0.029	-0.056
	97.5%	0.008	0.02	0.019	0.053	0.029	0.111	0.152	0.311	0.39	0.599	0.226	0.226	0.031	0.051
Skew		3.737	2.415	1.635	2.056	1.991	1.56	0.267	0.93	0.602	1.088	0.323	0.323	0.14	-0.016
Kurtosis		38.766	11.024	11.155	8.629	13.775	5.614	5.983	2.617	9.256	2.618	2.051	2.051	0.144	-0.019

Condition 4; Scenario 2; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.004	-0.515	0.058	-0.469	-0.022	-0.298	-0.095	-0.094	-0.077	0.007
	$\widehat{M}$	-0.041	-0.617	0.037	-0.537	-0.032	-0.338	-0.098	-0.104	-0.083	0.009
	$\widehat{\sigma}^2$	0.192	0.727	0.099	0.517	0.069	0.347	0.006	0.01	0.006	0.012
	2.5%	-0.614	-1.688	-0.457	-1.537	-0.465	-1.18	-0.217	-0.246	-0.216	-0.19
	97.5%	0.769	1.863	0.68	1.569	0.435	1.185	0.072	0.145	0.114	0.209
P	$\widehat{\mu}$	< .001	-0.006	0.002	-0.016	-0.001	-0.028	-0.19	-0.312	-0.192	0.001
	$\widehat{M}$	< .001	-0.007	0.001	-0.018	-0.002	-0.032	-0.197	-0.347	-0.207	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	< .001	0.003	0.024	0.109	0.039	< .001
	2.5%	-0.006	-0.018	-0.013	-0.052	-0.031	-0.11	-0.434	-0.82	-0.54	-0.024
	97.5%	0.008	0.02	0.019	0.053	0.029	0.111	0.145	0.485	0.285	0.026
Skew		3.803	2.415	1.698	2.053	2.065	1.555	1.554	1.622	1.066	0.062
Kurtosis		38.747	11.055	11.43	8.632	14.264	5.613	6.272	5.984	3.53	-0.153

Condition 4; Scenario 2; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.003	-0.534	0.054	-0.46	-0.019	-0.309	-0.072	-0.175	-0.073	-0.178	-0.058	-0.149	-0.004	-0.008
	$\widehat{M}$	-0.024	-0.631	0.042	-0.526	-0.027	-0.343	-0.069	-0.179	-0.078	-0.195	-0.06	-0.152	-0.001	-0.006
	$\widehat{\sigma}^2$	0.119	0.65	0.077	0.453	0.059	0.325	0.005	0.015	0.009	0.021	0.005	0.017	0.04	0.048
	2.5%	-0.586	-1.683	-0.45	-1.472	-0.452	-1.188	-0.208	-0.381	-0.229	-0.413	-0.201	-0.398	-0.435	-0.448
	97.5%	0.725	1.245	0.597	1.178	0.422	1.075	0.07	0.116	0.102	0.18	0.086	0.146	0.383	0.411
P	$\widehat{\mu}$	< .001	-0.006	0.002	-0.016	-0.001	-0.029	-0.145	-0.35	-0.242	-0.593	-0.144	-0.374	-0.001	-0.001
	$\widehat{M}$	< .001	-0.007	0.001	-0.018	-0.002	-0.343	-0.138	-0.357	-0.259	-0.648	-0.149	-0.38	< .001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	< .001	0.003	0.021	0.059	0.095	0.238	0.033	0.104	0.001	0.001
	2.5%	-0.006	-0.018	-0.013	-0.05	-0.03	-0.111	-0.415	-0.762	-0.762	-1.377	-0.502	-0.995	-0.054	-0.056
	97.5%	0.007	0.013	0.017	0.04	0.028	0.1	0.14	0.232	0.339	0.599	0.214	0.364	0.048	0.051
Skew		0.621	2.335	0.53	2.01	1.177	1.442	0.788	0.972	1.287	1.152	0.44	0.554	-0.136	-0.022
Kurtosis		5.647	11.469	4.2	8.776	8.499	5.407	4.155	2.78	5.753	2.943	2.071	1.26	0.435	-0.009

Condition 4; Scenario 2; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.012	-0.529	0.05	-0.462	-0.026	-0.308	-0.096	-0.097	-0.078	0.007
	$\widehat{M}$	-0.038	-0.617	0.037	-0.524	-0.031	-0.341	-0.099	-0.107	-0.084	0.009
	$\widehat{\sigma}^2$	0.133	0.643	0.084	0.451	0.056	0.336	0.006	0.01	0.006	0.012
	2.5%	-0.611	-1.691	-0.457	-1.465	-0.458	-1.192	-0.217	-0.247	-0.217	-0.19
	97.5%	0.75	1.473	0.624	1.175	0.414	1.032	0.069	0.121	0.098	0.209
P	$\widehat{\mu}$	< .001	-0.006	0.001	-0.016	-0.002	-0.029	-0.192	-0.323	-0.194	0.001
	$\widehat{M}$	< .001	-0.007	0.001	-0.018	-0.002	-0.032	-0.198	-0.358	-0.208	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	< .001	0.003	0.024	0.107	0.039	< .001
	2.5%	-0.006	-0.018	-0.013	-0.05	-0.031	-0.111	-0.433	-0.823	-0.541	-0.024
	97.5%	0.007	0.016	0.018	0.04	0.028	0.096	0.137	0.405	0.246	0.026
Skew		1.158	2.279	0.914	1.993	0.91	0.003	1.577	1.693	1.167	0.062
Kurtosis		8.779	11.092	6.959	8.754	6.892	5.771	6.669	6.568	4.144	-0.152

Condition 4; Scenario 2; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.081	0.392	0.071	0.237	0.012	0.258	0.039	0.075	0.029	–
	$\widehat{M}$	0.077	0.294	0.069	0.178	0.013	0.205	0.036	0.07	0.027	–
	$\widehat{\sigma}^2$	0.089	0.633	0.062	0.436	0.045	0.294	0.003	0.004	0.003	–
	2.5%	-0.46	-0.708	-0.391	-0.752	-0.399	-0.569	-0.054	-0.04	-0.072	–
	97.5%	0.651	2.652	0.568	2.189	0.431	1.71	0.155	0.233	0.157	–
P	$\widehat{\mu}$	0.001	0.004	0.002	0.008	0.001	0.024	0.078	0.25	0.073	–
	$\widehat{M}$	0.001	0.003	0.002	0.006	0.001	0.019	0.071	0.233	0.068	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	< .001	0.003	0.011	0.047	0.02	–
	2.5%	-0.005	-0.008	-0.011	-0.026	-0.027	-0.053	-0.107	-0.133	-0.181	–
	97.5%	0.007	0.029	0.016	0.075	0.029	0.16	0.309	0.776	0.393	–
Skew		-0.24	2.295	-0.045	1.985	-0.011	1.505	0.993	0.994	0.782	–
Kurtosis		3.206	9.294	2.559	7.54	2.115	4.886	3.407	3.131	2.228	–

TABLE C-12

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 4 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 4; Scenario 3; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.391	0.606	0.313	0.483	0.125	0.454	-0.036	-0.124	-0.019	-0.109	-0.018	-0.018	0.01	-0.029
	$\widehat{M}$	0.035	-0.445	0.069	-0.389	-0.024	-0.189	-0.032	-0.155	-0.032	-0.169	-0.023	-0.023	< .001	-0.021
	$\widehat{\sigma}^2$	1.218	6.486	0.724	4.626	0.493	2.962	0.024	0.056	0.037	0.081	0.022	0.022	0.024	0.075
	2.5%	-0.771	-2.576	-0.492	-2.279	-0.888	-1.898	-0.332	-0.544	-0.477	-0.543	-0.269	-0.269	-0.248	-0.634
	97.5%	3.613	6.021	2.966	4.658	2.003	3.915	0.267	0.298	0.438	0.456	0.291	0.291	0.302	0.499
P	$\widehat{\mu}$	0.004	0.007	0.009	0.016	0.008	0.042	-0.072	-0.249	-0.063	-0.362	-0.044	-0.044	0.001	-0.004
	$\widehat{M}$	< .001	-0.005	0.002	-0.013	-0.002	-0.189	-0.064	-0.31	-0.107	-0.565	-0.057	-0.057	< .001	-0.003
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.005	0.002	0.026	0.094	0.225	0.413	0.904	0.135	0.135	< .001	0.001
	2.5%	-0.008	-0.028	-0.014	-0.078	-0.059	-0.177	-0.665	-1.089	-1.59	-1.81	-0.674	-0.674	-0.031	-0.079
	97.5%	0.036	0.065	0.085	0.159	0.133	0.365	0.535	0.597	1.462	1.519	0.729	0.729	0.038	0.062
Skew		2.053	0.807	2.904	0.712	1.949	0.699	-2.343	0.076	-0.526	0.452	-1.457	-1.457	1.326	-0.045
Kurtosis		5.523	-0.629	12.766	-0.849	7.701	-0.674	16.164	-0.782	5.527	-0.752	11.338	11.338	11.287	1.185

Condition 4; Scenario 3; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.398	0.611	0.317	0.486	0.128	0.456	-0.013	0.005	0.003	0.002
	$\widehat{M}$	0.029	-0.422	0.066	-0.376	-0.025	-0.186	-0.034	-0.041	-0.028	-0.001
	$\widehat{\sigma}^2$	1.254	6.409	0.739	4.588	0.5	2.934	0.014	0.028	0.016	0.012
	2.5%	-0.732	-2.553	-0.495	-2.269	-0.886	-1.88	-0.201	-0.224	-0.198	-0.189
	97.5%	3.673	6.012	2.994	4.655	2.016	3.896	0.26	0.43	0.321	0.228
P	$\widehat{\mu}$	0.004	0.007	0.009	0.017	0.009	0.043	-0.027	0.017	0.008	< .001
	$\widehat{M}$	< .001	-0.005	0.002	-0.013	-0.002	-0.017	-0.069	-0.136	-0.07	< .001
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.005	0.002	0.026	0.057	0.31	0.102	< .001
	2.5%	-0.007	-0.027	-0.014	-0.077	-0.059	-0.176	-0.401	-0.746	-0.496	-0.024
	97.5%	0.037	0.065	0.085	0.159	0.134	0.364	0.52	1.433	0.803	0.028
Skew		2.046	0.811	2.893	0.714	1.98	0.7	0.826	1.077	0.909	0.222
Kurtosis		5.364	-0.614	12.565	-0.842	7.75	-0.667	0.443	0.67	0.6	-0.334

Condition 4; Scenario 3; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.366	0.767	0.279	0.678	0.069	0.577	-0.025	-0.104	-0.007	-0.096	-0.005	-0.074	0.002	-0.019
	$\widehat{M}$	0.068	-0.328	0.087	-0.214	-0.017	-0.1	-0.033	-0.129	-0.038	-0.171	-0.022	-0.098	< .001	-0.017
	$\widehat{\sigma}^2$	0.889	6.349	0.471	4.335	0.397	3.102	0.018	0.057	0.033	0.093	0.019	0.062	0.04	0.07
	2.5%	-0.772	-2.396	-0.488	-1.939	-1.17	-1.722	-0.259	-0.514	-0.365	-0.544	-0.235	-0.51	-0.418	-0.589
	97.5%	2.816	5.987	2.106	4.692	1.685	4.148	0.267	0.315	0.435	0.485	0.327	0.375	0.448	0.499
P	$\widehat{\mu}$	0.004	0.008	0.008	0.023	0.005	0.054	-0.051	-0.208	-0.024	-0.319	-0.011	-0.186	< .001	-0.002
	$\widehat{M}$	0.001	-0.004	0.003	-0.007	-0.001	-0.1	-0.066	-0.258	-0.128	-0.571	-0.056	-0.244	< .001	-0.002
	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.005	0.002	0.027	0.07	0.229	0.362	1.033	0.119	0.39	0.001	0.001
	2.5%	-0.008	-0.026	-0.014	-0.066	-0.078	-0.161	-0.518	-1.028	-1.218	-1.813	-0.588	-1.276	-0.052	-0.074
	97.5%	0.028	0.064	0.06	0.16	0.112	0.387	0.535	0.63	1.45	1.616	0.817	0.936	0.056	0.062
Skew		1.588	0.721	1.887	0.657	1.026	0.672	-0.717	0.077	0.518	0.418	0.266	0.141	0.116	-0.004
Kurtosis		4.016	-0.797	5.116	-0.952	6.004	-0.761	7.882	-0.895	1.668	-0.944	3.43	-0.874	1.795	1.213

Condition 4; Scenario 3; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.377	0.853	0.278	0.739	0.071	0.64	-0.003	0.015	0.012	0.002
	$\widehat{M}$	0.062	-0.267	0.083	-0.15	-0.007	-0.038	-0.035	-0.046	-0.025	-0.001
	$\widehat{\sigma}^2$	0.958	6.482	0.501	4.48	0.365	3.113	0.018	0.034	0.02	0.012
	2.5%	-0.849	-2.299	-0.678	-1.912	-1.189	-1.698	-0.203	-0.225	-0.203	-0.189
	97.5%	2.747	5.944	2.23	4.836	1.637	4.222	0.316	0.455	0.345	0.228
P	$\widehat{\mu}$	0.004	0.009	0.008	0.025	0.005	0.06	-0.006	0.051	0.029	< .001
	$\widehat{M}$	0.001	-0.003	0.002	-0.005	< .001	-0.004	-0.07	-0.155	-0.064	< .001
	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.005	0.002	0.027	0.072	0.375	0.123	< .001
	2.5%	-0.008	-0.025	-0.019	-0.065	-0.079	-0.159	-0.406	-0.749	-0.506	-0.024
	97.5%	0.027	0.064	0.064	0.165	0.109	0.394	0.632	1.518	0.863	0.028
Skew		1.424	0.676	1.561	0.626	0.703	0.027	0.835	0.978	0.829	0.224
Kurtosis		3.233	-0.875	3.728	-0.972	4.31	-0.849	0.126	0.107	0.169	-0.341

Condition 4; Scenario 3; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.263	1.858	0.176	1.434	0.072	1.184	0.065	0.105	0.057	–
	$\widehat{M}$	0.202	0.987	0.129	0.767	0.054	0.725	0.052	0.084	0.04	–
	$\widehat{\sigma}^2$	0.225	4.793	0.132	3.359	0.143	2.096	0.005	0.009	0.007	–
	2.5%	-0.585	-1.063	-0.436	-1.098	-0.676	-0.873	-0.051	-0.034	-0.075	–
	97.5%	1.32	6.067	1.086	4.699	0.972	3.886	0.212	0.318	0.223	–
P	$\widehat{\mu}$	0.003	0.02	0.005	0.049	0.005	0.11	0.129	0.351	0.143	–
	$\widehat{M}$	0.002	0.011	0.004	0.026	0.004	0.068	0.104	0.28	0.1	–
	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.004	0.001	0.018	0.02	0.098	0.042	–
	2.5%	-0.006	-0.011	-0.012	-0.037	-0.045	-0.082	-0.102	-0.114	-0.187	–
	97.5%	0.013	0.065	0.031	0.16	0.065	0.363	0.425	1.062	0.558	–
Skew		0.246	0.637	0.68	0.574	-0.003	0.541	0.537	0.733	0.561	–
Kurtosis		2.502	-0.944	2.463	-1.045	6.448	-0.879	-0.116	0.131	0.032	–

TABLE C-13

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 5 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 5; Scenario 1; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.267	-0.14	0.246	-0.096	0.127	-0.065	-0.125	-0.071	-0.138	-0.073	-0.099	-0.099	0.006	0.001
	$\widehat{M}$	0.259	-0.193	0.242	-0.127	0.142	-0.086	-0.132	-0.079	-0.155	-0.084	-0.102	-0.102	0.008	-0.001
	$\widehat{\sigma}^2$	0.45	0.365	0.379	0.33	0.222	0.241	0.019	0.014	0.029	0.021	0.021	0.021	0.028	0.025
	2.5%	-1.081	-1.166	-0.755	-1.101	-0.679	-0.881	-0.373	-0.299	-0.419	-0.324	-0.38	-0.38	-0.336	-0.301
	97.5%	1.597	1.175	1.322	1.09	1.023	0.951	0.178	0.198	0.244	0.253	0.22	0.22	0.318	0.314
P	$\widehat{\mu}$	0.003	-0.002	0.007	-0.003	0.008	-0.006	-0.25	-0.143	-0.459	-0.245	-0.248	-0.248	0.001	< .001
	$\widehat{M}$	0.003	-0.002	0.007	-0.004	0.009	-0.086	-0.264	-0.158	-0.518	-0.279	-0.256	-0.256	0.002	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.077	0.057	0.318	0.236	0.132	0.132	0.002	0.002
	2.5%	-0.011	-0.013	-0.022	-0.037	-0.045	-0.082	-0.745	-0.597	-1.398	-1.08	-0.95	-0.95	-0.084	-0.075
	97.5%	0.016	0.013	0.038	0.037	0.068	0.089	0.356	0.396	0.815	0.843	0.549	0.549	0.08	0.078
Skew		-0.177	0.668	0.663	0.986	-0.509	1.078	0.555	0.381	0.621	0.635	0.4	0.4	-0.126	0.059
Kurtosis		2.336	1.935	18.467	5.236	5.137	5.86	1.159	0.796	0.998	1.307	0.969	0.969	-0.066	0.505

Condition 5; Scenario 1; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.277	-0.142	0.252	-0.097	0.13	-0.066	-0.089	-0.095	-0.069	0.003
	$\widehat{M}$	0.265	-0.198	0.245	-0.127	0.144	-0.088	-0.099	-0.111	-0.073	0.004
	$\widehat{\sigma}^2$	0.46	0.363	0.385	0.329	0.224	0.241	0.012	0.019	0.015	0.013
	2.5%	-1.081	-1.167	-0.759	-1.1	-0.679	-0.881	-0.279	-0.301	-0.293	-0.223
	97.5%	1.613	1.171	1.342	1.089	1.031	0.946	0.193	0.254	0.192	0.232
P	$\widehat{\mu}$	0.003	-0.002	0.007	-0.003	0.009	-0.006	-0.178	-0.316	-0.173	0.001
	$\widehat{M}$	0.003	-0.002	0.007	-0.004	0.01	-0.008	-0.197	-0.37	-0.184	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.049	0.211	0.094	0.001
	2.5%	-0.011	-0.013	-0.022	-0.037	-0.045	-0.082	-0.559	-1.003	-0.732	-0.056
	97.5%	0.016	0.013	0.038	0.037	0.069	0.088	0.387	0.845	0.479	0.058
Skew		-0.149	0.676	0.697	0.997	-0.493	1.088	0.844	1.042	0.601	0.068
Kurtosis		2.346	1.967	18.623	5.292	5.073	5.939	1.664	2.216	1.481	-0.056

Condition 5; Scenario 1; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.265	-0.172	0.232	-0.115	0.114	-0.082	-0.125	-0.074	-0.137	-0.077	-0.095	-0.055	0.005	0.001
	$\widehat{M}$	0.248	-0.21	0.225	-0.136	0.126	-0.098	-0.132	-0.08	-0.153	-0.085	-0.098	-0.052	-0.004	-0.001
	$\widehat{\sigma}^2$	0.413	0.336	0.332	0.299	0.192	0.214	0.019	0.014	0.027	0.02	0.02	0.017	0.027	0.025
	2.5%	-0.995	-1.168	-0.74	-1.087	-0.679	-0.897	-0.373	-0.299	-0.417	-0.327	-0.37	-0.295	-0.312	-0.301
	97.5%	1.575	1.044	1.257	1.011	1.003	0.838	0.17	0.186	0.226	0.234	0.216	0.208	0.333	0.316
P	$\widehat{\mu}$	0.003	-0.002	0.007	-0.004	0.008	-0.008	-0.249	-0.149	-0.457	-0.258	-0.237	-0.138	0.001	< .001
	$\widehat{M}$	0.002	-0.002	0.006	-0.005	0.008	-0.098	-0.263	-0.161	-0.51	-0.284	-0.245	-0.131	-0.001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.075	0.056	0.305	0.224	0.127	0.104	0.002	0.002
	2.5%	-0.01	-0.013	-0.021	-0.037	-0.045	-0.084	-0.745	-0.597	-1.39	-1.09	-0.925	-0.737	-0.078	-0.075
	97.5%	0.016	0.011	0.036	0.034	0.067	0.078	0.34	0.372	0.753	0.781	0.539	0.519	0.083	0.079
Skew		0.068	0.549	1.251	0.713	-0.282	0.665	0.495	0.321	0.516	0.514	0.309	0.147	0.133	0.046
Kurtosis		1.537	1.808	21.289	4.053	3.821	3.914	1.056	0.645	0.739	0.959	0.676	0.429	0.542	0.504

Condition 5; Scenario 1; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.281	-0.164	0.245	-0.108	0.122	-0.075	-0.091	-0.097	-0.069	0.003
	$\widehat{M}$	0.262	-0.211	0.231	-0.136	0.135	-0.091	-0.1	-0.113	-0.073	0.004
	$\widehat{\sigma}^2$	0.429	0.345	0.355	0.316	0.204	0.221	0.012	0.018	0.014	0.013
	2.5%	-0.98	-1.173	-0.723	-1.089	-0.675	-0.901	-0.279	-0.301	-0.29	-0.222
	97.5%	1.598	1.075	1.282	1.05	1.019	0.893	0.178	0.233	0.189	0.232
P	$\widehat{\mu}$	0.003	-0.002	0.007	-0.004	0.008	-0.007	-0.182	-0.323	-0.172	0.001
	$\widehat{M}$	0.003	-0.002	0.007	-0.005	0.009	-0.009	-0.201	-0.374	-0.184	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.048	0.201	0.09	0.001
	2.5%	-0.01	-0.013	-0.021	-0.037	-0.045	-0.084	-0.559	-1.003	-0.725	-0.056
	97.5%	0.016	0.012	0.037	0.036	0.068	0.083	0.356	0.779	0.474	0.058
Skew		0.054	0.608	1.1	0.94	-0.329	0.002	0.841	0.928	0.472	0.067
Kurtosis		1.721	1.832	20.242	5.368	4.09	4.246	1.769	1.857	1.09	-0.057

Condition 5; Scenario 1; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.123	0.167	0.084	0.128	0.027	0.126	0.023	0.048	0.021	–
	$\widehat{M}$	0.132	0.12	0.097	0.102	0.033	0.102	0.021	0.042	0.019	–
	$\widehat{\sigma}^2$	0.377	0.32	0.304	0.281	0.186	0.207	0.007	0.01	0.009	–
	2.5%	-1.138	-0.8	-0.863	-0.761	-0.779	-0.641	-0.134	-0.125	-0.164	–
	97.5%	1.312	1.417	1.083	1.239	0.851	1.102	0.201	0.273	0.21	–
P	$\widehat{\mu}$	0.001	0.002	0.002	0.004	0.002	0.012	0.045	0.161	0.052	–
	$\widehat{M}$	0.001	0.001	0.003	0.003	0.002	0.009	0.042	0.141	0.048	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.027	0.115	0.055	–
	2.5%	-0.011	-0.009	-0.025	-0.026	-0.052	-0.06	-0.269	-0.417	-0.41	–
	97.5%	0.013	0.015	0.031	0.042	0.057	0.103	0.401	0.909	0.525	–
Skew		-0.452	0.652	-0.202	1.082	-0.789	1.125	0.332	0.58	0.348	–
Kurtosis		1.983	1.914	11.016	5.09	5.197	5.479	1.002	1.403	1.02	–

TABLE C-14

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 5 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 5; Scenario 2; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.131	-0.11	0.114	-0.131	0.014	-0.024	-0.052	-0.111	-0.046	-0.103	-0.035	-0.035	0.003	-0.008
	$\widehat{M}$	0.077	-0.38	0.102	-0.337	0.004	-0.193	-0.058	-0.131	-0.061	-0.126	-0.044	-0.044	0.005	-0.013
	$\widehat{\sigma}^2$	0.524	2.078	0.291	1.578	0.22	0.99	0.015	0.034	0.027	0.053	0.018	0.018	0.021	0.055
	2.5%	-0.974	-2.144	-0.813	-1.93	-0.769	-1.569	-0.282	-0.441	-0.329	-0.5	-0.267	-0.267	-0.277	-0.458
	97.5%	1.706	3.757	1.231	3.399	0.978	2.778	0.216	0.283	0.375	0.429	0.275	0.275	0.284	0.475
P	$\widehat{\mu}$	0.001	-0.001	0.003	-0.004	0.001	-0.002	-0.105	-0.223	-0.154	-0.345	-0.087	-0.087	0.001	-0.002
	$\widehat{M}$	0.001	-0.004	0.003	-0.011	< .001	-0.193	-0.117	-0.261	-0.205	-0.419	-0.11	-0.11	0.001	-0.003
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.001	0.009	0.058	0.136	0.3	0.587	0.11	0.11	0.001	0.003
	2.5%	-0.01	-0.023	-0.023	-0.066	-0.051	-0.146	-0.564	-0.882	-1.098	-1.665	-0.667	-0.667	-0.069	-0.115
	97.5%	0.017	0.04	0.035	0.116	0.065	0.259	0.431	0.565	1.249	1.43	0.689	0.689	0.071	0.119
Skew		0.74	1.49	0.445	1.169	-0.123	1.013	0.502	0.347	0.627	0.482	0.6	0.6	-0.117	0.128
Kurtosis		9.217	3.758	6.418	2.712	11.449	1.584	1.363	0.079	1.358	0.05	1.582	1.582	0.304	0.446

Condition 5; Scenario 2; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.129	-0.106	0.112	-0.129	0.013	-0.022	-0.06	-0.052	-0.039	0.002
	$\widehat{M}$	0.073	-0.373	0.097	-0.336	0.003	-0.189	-0.075	-0.081	-0.059	< .001
	$\widehat{\sigma}^2$	0.534	2.06	0.294	1.569	0.221	0.982	0.015	0.028	0.019	0.014
	2.5%	-0.985	-2.114	-0.817	-1.919	-0.77	-1.563	-0.259	-0.303	-0.27	-0.22
	97.5%	1.739	3.749	1.232	3.395	0.982	2.775	0.239	0.394	0.3	0.231
P	$\widehat{\mu}$	0.001	-0.001	0.003	-0.004	0.001	-0.002	-0.121	-0.175	-0.099	0.001
	$\widehat{M}$	0.001	-0.004	0.003	-0.011	< .001	-0.018	-0.149	-0.271	-0.147	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.001	0.009	0.062	0.312	0.118	0.001
	2.5%	-0.01	-0.023	-0.023	-0.065	-0.051	-0.146	-0.517	-1.01	-0.676	-0.055
	97.5%	0.017	0.04	0.035	0.116	0.065	0.259	0.478	1.315	0.751	0.058
Skew		0.797	1.497	0.484	1.171	-0.063	1.013	0.859	0.98	0.819	0.065
Kurtosis		9.107	3.799	6.394	2.731	11.469	1.596	1.322	1.16	1.088	-0.068

Condition 5; Scenario 2; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.114	-0.138	0.101	-0.134	0.032	-0.051	-0.053	-0.114	-0.053	-0.109	-0.038	-0.084	-0.002	-0.008
	$\widehat{M}$	0.077	-0.372	0.096	-0.325	0.018	-0.204	-0.059	-0.132	-0.066	-0.132	-0.046	-0.101	-0.009	-0.014
	$\widehat{\sigma}^2$	0.422	1.885	0.237	1.422	0.177	0.902	0.014	0.034	0.025	0.052	0.016	0.04	0.049	0.055
	2.5%	-0.886	-2.036	-0.753	-1.845	-0.72	-1.569	-0.274	-0.441	-0.327	-0.505	-0.267	-0.467	-0.439	-0.462
	97.5%	1.561	3.636	1.123	3.177	0.939	2.623	0.207	0.281	0.357	0.422	0.265	0.332	0.473	0.48
P	$\widehat{\mu}$	0.001	-0.001	0.003	-0.005	0.002	-0.005	-0.105	-0.227	-0.175	-0.363	-0.096	-0.211	-0.001	-0.002
	$\widehat{M}$	0.001	-0.004	0.003	-0.011	0.001	-0.204	-0.118	-0.264	-0.219	-0.438	-0.115	-0.254	-0.002	-0.004
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.001	0.008	0.058	0.134	0.283	0.582	0.102	0.252	0.003	0.003
	2.5%	-0.009	-0.022	-0.021	-0.063	-0.048	-0.146	-0.547	-0.882	-1.089	-1.684	-0.667	-1.167	-0.11	-0.116
	97.5%	0.016	0.039	0.032	0.108	0.063	0.245	0.415	0.563	1.189	1.407	0.663	0.829	0.118	0.12
Skew		-0.023	1.392	-0.217	1.203	1.012	0.95	0.523	0.342	0.636	0.479	0.471	0.131	0.179	0.139
Kurtosis		7.752	3.485	4.207	2.874	7.148	1.573	1.44	0.063	1.362	0.085	1.411	-0.107	0.57	0.395

Condition 5; Scenario 2; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.106	-0.158	0.094	-0.147	0.023	-0.056	-0.063	-0.06	-0.044	0.002
	$\widehat{M}$	0.074	-0.379	0.096	-0.321	0.01	-0.197	-0.077	-0.085	-0.06	0.001
	$\widehat{\sigma}^2$	0.422	1.806	0.244	1.386	0.168	0.885	0.015	0.026	0.018	0.014
	2.5%	-0.914	-2.1	-0.788	-1.878	-0.733	-1.563	-0.26	-0.303	-0.27	-0.22
	97.5%	1.583	3.175	1.14	3.176	0.873	2.624	0.255	0.39	0.276	0.233
P	$\widehat{\mu}$	0.001	-0.002	0.003	-0.005	0.002	-0.005	-0.125	-0.2	-0.109	0.001
	$\widehat{M}$	0.001	-0.004	0.003	-0.011	0.001	-0.018	-0.155	-0.284	-0.149	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.001	0.008	0.06	0.294	0.11	0.001
	2.5%	-0.009	-0.023	-0.023	-0.064	-0.049	-0.146	-0.521	-1.01	-0.676	-0.055
	97.5%	0.016	0.034	0.033	0.108	0.058	0.245	0.509	1.3	0.691	0.058
Skew		0.071	1.392	-0.174	1.179	1.021	0.008	0.897	0.996	0.772	0.066
Kurtosis		7.685	3.701	4.242	3.032	7.572	1.674	1.548	1.35	1.15	-0.067

Condition 5; Scenario 2; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.15	0.587	0.094	0.404	0.023	0.4	0.038	0.074	0.037	–
	$\widehat{M}$	0.132	0.33	0.101	0.191	0.026	0.236	0.035	0.06	0.025	–
	$\widehat{\sigma}^2$	0.325	1.714	0.205	1.273	0.163	0.811	0.007	0.013	0.01	–
	2.5%	-0.874	-1.272	-0.8	-1.206	-0.74	-1.016	-0.116	-0.12	-0.14	–
	97.5%	1.312	4.097	0.978	3.496	0.721	2.884	0.235	0.349	0.267	–
P	$\widehat{\mu}$	0.002	0.006	0.003	0.014	0.002	0.037	0.077	0.246	0.092	–
	$\widehat{M}$	0.001	0.004	0.003	0.007	0.002	0.022	0.069	0.2	0.063	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.007	0.03	0.146	0.064	–
	2.5%	-0.009	-0.014	-0.023	-0.041	-0.049	-0.095	-0.231	-0.4	-0.349	–
	97.5%	0.013	0.044	0.028	0.119	0.048	0.269	0.47	1.164	0.667	–
Skew		-0.659	1.436	-0.732	1.25	-0.641	0.995	0.525	0.657	0.58	–
Kurtosis		7.386	3.166	4.187	2.327	10.043	1.3	1.089	0.761	0.817	–

TABLE C-15

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 5 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 5; Scenario 3; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.458	1.191	0.371	0.898	0.128	0.877	-0.024	-0.059	0.009	-0.02	-0.003	-0.003	0.003	-0.023
	$\widehat{M}$	0.195	0.518	0.184	0.463	0.028	0.461	-0.032	-0.029	-0.015	0.003	-0.013	-0.013	-0.002	-0.027
	$\widehat{\sigma}^2$	1.1	7.087	0.754	5.004	0.578	3.495	0.028	0.074	0.042	0.107	0.028	0.028	0.022	0.086
	2.5%	-0.947	-2.839	-0.798	-2.758	-1.122	-2.03	-0.316	-0.632	-0.34	-0.651	-0.293	-0.293	-0.27	-0.635
	97.5%	3.195	6.193	2.736	4.999	2.273	4.128	0.321	0.35	0.482	0.51	0.362	0.362	0.291	0.64
P	$\widehat{\mu}$	0.005	0.013	0.011	0.031	0.009	0.082	-0.049	-0.118	0.03	-0.066	-0.006	-0.006	0.001	-0.006
	$\widehat{M}$	0.002	0.005	0.005	0.016	0.002	0.461	-0.064	-0.059	-0.05	0.01	-0.033	-0.033	< .001	-0.007
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.006	0.003	0.03	0.114	0.296	0.462	1.185	0.178	0.178	0.001	0.005
	2.5%	-0.009	-0.031	-0.023	-0.094	-0.075	-0.189	-0.632	-1.264	-1.133	-2.171	-0.732	-0.732	-0.068	-0.159
	97.5%	0.032	0.067	0.078	0.17	0.151	0.385	0.643	0.7	1.607	1.701	0.907	0.907	0.073	0.16
Skew		1.613	0.355	1.309	0.275	1.587	0.227	-1.222	-0.548	-0.22	-0.252	-0.251	-0.251	0.08	0.293
Kurtosis		5.673	-0.933	5.562	-0.904	9.497	-1.024	10.265	-0.124	3.702	-0.761	3.777	3.777	0.984	2.349

Condition 5; Scenario 3; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.466	1.19	0.375	0.897	0.131	0.877	0.001	0.039	0.022	-0.001
	$\widehat{M}$	0.188	0.513	0.182	0.463	0.027	0.465	-0.021	-0.001	0.002	-0.005
	$\widehat{\sigma}^2$	1.126	7.027	0.766	4.974	0.583	3.469	0.023	0.04	0.026	0.014
	2.5%	-0.915	-2.826	-0.801	-2.75	-1.098	-2.026	-0.256	-0.283	-0.252	-0.234
	97.5%	3.232	6.186	2.758	4.997	2.303	4.124	0.327	0.469	0.36	0.235
P	$\widehat{\mu}$	0.005	0.013	0.011	0.031	0.009	0.082	0.001	0.129	0.055	< .001
	$\widehat{M}$	0.002	0.005	0.005	0.016	0.002	0.043	-0.042	-0.003	0.004	-0.001
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.006	0.003	0.03	0.091	0.439	0.165	0.001
	2.5%	-0.009	-0.03	-0.023	-0.094	-0.073	-0.189	-0.512	-0.945	-0.63	-0.058
	97.5%	0.032	0.067	0.079	0.17	0.154	0.385	0.653	1.562	0.899	0.059
Skew		1.622	0.36	1.337	0.277	1.612	0.228	0.371	0.477	0.368	0.094
Kurtosis		5.639	-0.923	5.585	-0.896	9.549	-1.017	-0.209	-0.395	-0.363	0.146

Condition 5; Scenario 3; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.432	1.4	0.352	1.116	0.11	1.016	-0.011	-0.037	0.019	-0.003	0.003	-0.001	-0.015	-0.019
	$\widehat{M}$	0.233	0.895	0.21	0.766	0.038	0.694	-0.027	-0.007	-0.014	0.027	-0.009	0.024	-0.018	-0.022
	$\widehat{\sigma}^2$	0.876	6.973	0.639	4.748	0.49	3.472	0.024	0.073	0.04	0.111	0.029	0.076	0.061	0.078
	2.5%	-1.153	-2.639	-1.087	-2.381	-1.257	-1.897	-0.286	-0.605	-0.317	-0.653	-0.293	-0.561	-0.531	-0.576
	97.5%	2.768	6.251	2.256	5.026	1.903	4.283	0.328	0.365	0.482	0.523	0.393	0.428	0.525	0.596
P	$\widehat{\mu}$	0.004	0.015	0.01	0.038	0.007	0.095	-0.023	-0.074	0.062	-0.009	0.009	-0.002	-0.004	-0.005
	$\widehat{M}$	0.002	0.01	0.006	0.026	0.003	0.694	-0.054	-0.015	-0.049	0.089	-0.024	0.059	-0.005	-0.005
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.006	0.002	0.03	0.097	0.294	0.446	1.236	0.178	0.474	0.004	0.005
	2.5%	-0.012	-0.028	-0.031	-0.081	-0.084	-0.177	-0.571	-1.209	-1.056	-2.178	-0.732	-1.402	-0.133	-0.144
	97.5%	0.028	0.067	0.064	0.171	0.127	0.4	0.657	0.73	1.607	1.745	0.984	1.07	0.131	0.149
Skew		0.818	0.26	0.708	0.239	0.687	0.181	0.172	-0.562	0.325	-0.294	0.216	-0.426	-0.011	0.209
Kurtosis		1.888	-1.019	2.974	-0.994	4.363	-1.008	0.44	-0.195	1.258	-0.828	0.94	-0.509	2.399	1.848

Condition 5; Scenario 3; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.438	1.452	0.383	1.116	0.102	1.048	0.008	0.051	0.029	-0.001
	$\widehat{M}$	0.221	0.972	0.218	0.804	0.037	0.759	-0.018	0.002	0.004	-0.004
	$\widehat{\sigma}^2$	0.895	6.892	0.62	4.584	0.487	3.426	0.025	0.045	0.03	0.014
	2.5%	-1.125	-2.541	-0.899	-2.337	-1.226	-1.852	-0.26	-0.29	-0.255	-0.235
	97.5%	2.825	6.269	2.256	4.997	1.881	4.299	0.33	0.484	0.377	0.236
P	$\widehat{\mu}$	0.004	0.016	0.011	0.038	0.007	0.098	0.017	0.17	0.073	< .001
	$\widehat{M}$	0.002	0.01	0.006	0.027	0.002	0.071	-0.036	0.007	0.008	-0.001
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.005	0.002	0.03	0.101	0.499	0.185	0.001
	2.5%	-0.011	-0.027	-0.026	-0.08	-0.082	-0.173	-0.52	-0.966	-0.638	-0.059
	97.5%	0.028	0.067	0.064	0.17	0.125	0.401	0.66	1.615	0.943	0.059
Skew		0.967	0.279	0.824	0.23	0.632	0.03	0.301	0.399	0.359	0.094
Kurtosis		2.104	-1.008	2.963	-1.007	4.824	-1.075	-0.478	-0.68	-0.527	0.151

Condition 5; Scenario 3; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.379	2.036	0.276	1.544	0.098	1.36	0.061	0.109	0.06	–
	$\widehat{M}$	0.299	1.556	0.222	1.239	0.065	1.073	0.051	0.091	0.045	–
	$\widehat{\sigma}^2$	0.461	5.54	0.344	3.892	0.292	2.657	0.01	0.017	0.013	–
	2.5%	-0.846	-1.64	-0.737	-1.734	-0.97	-1.281	-0.111	-0.113	-0.138	–
	97.5%	1.978	6.266	1.521	5.023	1.502	4.186	0.263	0.401	0.297	–
P	$\widehat{\mu}$	0.004	0.022	0.008	0.053	0.007	0.127	0.122	0.363	0.149	–
	$\widehat{M}$	0.003	0.017	0.006	0.042	0.004	0.1	0.102	0.304	0.113	–
	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.005	0.001	0.023	0.039	0.19	0.082	–
	2.5%	-0.008	-0.018	-0.021	-0.059	-0.065	-0.119	-0.222	-0.378	-0.345	–
	97.5%	0.02	0.067	0.043	0.171	0.1	0.391	0.526	1.335	0.743	–
Skew		0.544	0.257	0.126	0.186	0.143	0.159	0.266	0.343	0.287	–
Kurtosis		1.892	-1.011	4	-0.948	4.959	-1.029	-0.264	-0.337	-0.373	–

TABLE C-16

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 6 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 6; Scenario 1; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	0.287	-0.183	0.281	-0.159	0.165	-0.128	-0.151	-0.086	-0.168	-0.099	-0.126	-0.126	0.01	-0.007
	$\widehat{M}$	0.3	-0.204	0.262	-0.164	0.161	-0.136	-0.153	-0.084	-0.172	-0.104	-0.126	-0.126	0.013	-0.011
B	$\widehat{\sigma}^2$	0.152	0.143	0.103	0.093	0.07	0.071	0.006	0.004	0.01	0.007	0.007	0.007	0.011	0.01
	2.5%	-0.467	-0.891	-0.294	-0.766	-0.322	-0.616	-0.298	-0.213	-0.344	-0.252	-0.288	-0.288	-0.196	-0.199
	97.5%	1.018	0.558	0.917	0.461	0.697	0.42	-0.003	0.037	0.041	0.069	0.034	0.034	0.216	0.185
	$\widehat{\mu}$	0.003	-0.002	0.008	-0.005	0.011	-0.012	-0.302	-0.173	-0.559	-0.332	-0.316	-0.316	0.003	-0.002
	$\widehat{M}$	0.003	-0.002	0.007	-0.006	0.011	-0.136	-0.306	-0.169	-0.575	-0.347	-0.316	-0.316	0.003	-0.003
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.024	0.018	0.108	0.08	0.045	0.045	0.001	0.001
	2.5%	-0.005	-0.01	-0.008	-0.026	-0.021	-0.057	-0.595	-0.425	-1.146	-0.839	-0.72	-0.72	-0.049	-0.05
	97.5%	0.01	0.006	0.026	0.016	0.046	0.039	-0.005	0.074	0.135	0.229	0.085	0.085	0.054	0.046
Skew		-0.622	1.068	0.054	0.263	0.498	0.268	0.252	0.207	0.479	0.62	0.131	0.131	-0.056	-0.01
Kurtosis		6.618	8.73	1.097	0.95	3.125	0.618	0.994	1.548	1.291	2.421	0.164	0.164	-0.209	-0.069

Condition 6; Scenario 1; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.297	-0.186	0.286	-0.161	0.169	-0.13	-0.11	-0.125	-0.098	-0.001
	$\widehat{M}$	0.305	-0.207	0.269	-0.166	0.166	-0.137	-0.114	-0.128	-0.097	-0.001
B	$\widehat{\sigma}^2$	0.155	0.142	0.104	0.093	0.071	0.071	0.003	0.006	0.005	0.005
	2.5%	-0.456	-0.892	-0.294	-0.767	-0.32	-0.617	-0.219	-0.258	-0.229	-0.14
	97.5%	1.029	0.555	0.929	0.459	0.703	0.417	0.006	0.032	0.041	0.142
	$\widehat{\mu}$	0.003	-0.002	0.008	-0.005	0.011	-0.012	-0.22	-0.416	-0.245	< .001
	$\widehat{M}$	0.003	-0.002	0.008	-0.006	0.011	-0.013	-0.228	-0.427	-0.242	< .001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.014	0.066	0.03	< .001
	2.5%	-0.005	-0.01	-0.008	-0.026	-0.021	-0.058	-0.438	-0.861	-0.573	-0.035
	97.5%	0.01	0.006	0.027	0.016	0.047	0.039	0.011	0.108	0.102	0.036
Skew		-0.587	1.079	0.065	0.263	0.505	0.265	0.556	0.823	0.261	0.017
Kurtosis		6.425	8.833	1.094	0.947	3.199	0.609	2.731	3.107	0.316	0.029

Condition 6; Scenario 1; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	0.28	-0.219	0.258	-0.17	0.146	-0.135	-0.15	-0.088	-0.166	-0.102	-0.119	-0.08	-0.001	-0.007
	$\widehat{M}$	0.289	-0.225	0.247	-0.173	0.144	-0.14	-0.153	-0.085	-0.169	-0.104	-0.119	-0.078	-0.005	-0.011
B	$\widehat{\sigma}^2$	0.132	0.111	0.092	0.086	0.061	0.068	0.006	0.004	0.009	0.007	0.007	0.006	0.011	0.01
	2.5%	-0.447	-0.912	-0.304	-0.766	-0.33	-0.616	-0.292	-0.214	-0.341	-0.253	-0.286	-0.231	-0.198	-0.198
	97.5%	1.004	0.443	0.863	0.419	0.653	0.382	-0.003	0.033	0.041	0.067	0.043	0.076	0.196	0.189
	$\widehat{\mu}$	0.003	-0.002	0.007	-0.006	0.01	-0.013	-0.3	-0.176	-0.553	-0.339	-0.297	-0.2	< .001	-0.002
	$\widehat{M}$	0.003	-0.002	0.007	-0.006	0.01	-0.14	-0.306	-0.171	-0.562	-0.348	-0.297	-0.196	-0.001	-0.003
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.023	0.017	0.105	0.076	0.045	0.037	0.001	0.001
	2.5%	-0.004	-0.01	-0.009	-0.026	-0.022	-0.057	-0.585	-0.427	-1.136	-0.845	-0.714	-0.577	-0.049	-0.049
	97.5%	0.01	0.005	0.025	0.014	0.044	0.036	-0.005	0.067	0.137	0.224	0.106	0.19	0.049	0.047
Skew		-0.008	-0.157	0.112	0.01	0.162	0.202	0.065	-0.105	0.348	0.327	0.143	0.037	-0.021	0.002
Kurtosis		0.312	0.342	-0.073	0.047	0.68	0.367	-0.154	-0.248	0.433	0.454	0.055	0.038	-0.112	-0.046

Condition 6; Scenario 1; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.296	-0.212	0.271	-0.167	0.155	-0.132	-0.112	-0.126	-0.095	-0.001
	$\widehat{M}$	0.301	-0.227	0.257	-0.171	0.149	-0.14	-0.114	-0.129	-0.095	-0.001
B	$\widehat{\sigma}^2$	0.14	0.118	0.097	0.09	0.064	0.07	0.003	0.006	0.005	0.005
	2.5%	-0.445	-0.893	-0.302	-0.767	-0.322	-0.617	-0.218	-0.26	-0.226	-0.14
	97.5%	1.027	0.473	0.887	0.438	0.683	0.399	0.005	0.032	0.044	0.142
	$\widehat{\mu}$	0.003	-0.002	0.008	-0.006	0.01	-0.012	-0.223	-0.42	-0.238	< .001
	$\widehat{M}$	0.003	-0.002	0.007	-0.006	0.01	-0.013	-0.229	-0.429	-0.238	< .001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.013	0.063	0.03	< .001
	2.5%	-0.004	-0.01	-0.009	-0.026	-0.021	-0.058	-0.437	-0.865	-0.566	-0.035
	97.5%	0.01	0.005	0.025	0.015	0.046	0.037	0.01	0.108	0.111	0.036
Skew		0.066	0.053	0.172	0.13	0.284	0.001	0.152	0.528	0.227	0.017
Kurtosis		0.473	0.686	-0.045	0.466	1.044	0.589	-0.028	0.917	0.223	0.028

Condition 6; Scenario 1; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.117	0.16	0.099	0.095	0.046	0.089	0.021	0.043	0.012	–
	$\widehat{M}$	0.142	0.142	0.09	0.087	0.048	0.077	0.021	0.043	0.011	–
B	$\widehat{\sigma}^2$	0.129	0.133	0.088	0.086	0.059	0.064	0.002	0.004	0.003	–
	2.5%	-0.6	-0.536	-0.447	-0.434	-0.411	-0.369	-0.065	-0.069	-0.09	–
	97.5%	0.812	0.889	0.667	0.685	0.52	0.613	0.109	0.161	0.118	–
	$\widehat{\mu}$	0.001	0.002	0.003	0.003	0.003	0.008	0.043	0.143	0.031	–
	$\widehat{M}$	0.001	0.002	0.003	0.003	0.003	0.007	0.042	0.144	0.028	–
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.008	0.04	0.019	–
	2.5%	-0.006	-0.006	-0.013	-0.015	-0.027	-0.034	-0.13	-0.229	-0.225	–
	97.5%	0.008	0.01	0.019	0.023	0.035	0.057	0.218	0.536	0.296	–
Skew		-0.779	1.096	-0.158	0.344	0.04	0.338	0.207	0.32	0.075	–
Kurtosis		6.473	8.372	1.007	1.072	1.074	0.586	1.113	0.977	0.001	–

TABLE C-17

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 6 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 6; Scenario 2; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.016	-0.534	0.059	-0.454	-0.009	-0.305	-0.071	-0.166	-0.07	-0.187	-0.052	-0.052	0.005	-0.006
	$\widehat{M}$	-0.013	-0.636	0.049	-0.524	-0.015	-0.374	-0.07	-0.168	-0.074	-0.199	-0.051	-0.051	0.007	-0.006
	$\widehat{\sigma}^2$	0.134	0.625	0.092	0.449	0.057	0.326	0.005	0.013	0.007	0.02	0.006	0.006	0.007	0.021
	2.5%	-0.59	-1.586	-0.393	-1.51	-0.399	-1.09	-0.202	-0.352	-0.21	-0.42	-0.18	-0.18	-0.164	-0.28
	97.5%	0.767	1.015	0.616	0.992	0.37	1.092	0.07	0.107	0.106	0.166	0.089	0.089	0.167	0.276
P	$\widehat{\mu}$	< .001	-0.006	0.002	-0.015	-0.001	-0.028	-0.143	-0.331	-0.233	-0.623	-0.13	-0.13	0.001	-0.001
	$\widehat{M}$	< .001	-0.007	0.001	-0.018	-0.001	-0.374	-0.14	-0.335	-0.248	-0.661	-0.128	-0.128	0.002	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	< .001	0.003	0.021	0.052	0.082	0.221	0.037	0.037	< .001	0.001
	2.5%	-0.006	-0.017	-0.011	-0.051	-0.027	-0.102	-0.404	-0.705	-0.7	-1.399	-0.449	-0.449	-0.041	-0.07
	97.5%	0.008	0.011	0.018	0.034	0.025	0.101	0.14	0.214	0.355	0.555	0.224	0.224	0.042	0.069
Skew		2.005	2.826	-0.692	2.254	0.982	2.445	0.477	0.79	1.204	1.077	1.207	1.207	0.078	-0.064
Kurtosis		15.942	14.458	22.491	12.513	22.347	11.222	3.447	1.901	6.514	3.121	8.102	8.102	1.072	0.007

Condition 6; Scenario 2; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.008	-0.526	0.055	-0.45	-0.012	-0.301	-0.092	-0.097	-0.074	0.005
	$\widehat{M}$	-0.023	-0.629	0.044	-0.519	-0.018	-0.37	-0.098	-0.104	-0.078	0.004
	$\widehat{\sigma}^2$	0.137	0.619	0.093	0.446	0.057	0.324	0.005	0.008	0.007	0.005
	2.5%	-0.599	-1.574	-0.401	-1.499	-0.402	-1.086	-0.205	-0.239	-0.207	-0.136
	97.5%	0.77	1.012	0.619	0.982	0.372	1.091	0.063	0.119	0.123	0.146
P	$\widehat{\mu}$	< .001	-0.006	0.002	-0.015	-0.001	-0.028	-0.184	-0.322	-0.185	0.001
	$\widehat{M}$	< .001	-0.007	0.001	-0.018	-0.001	-0.035	-0.197	-0.348	-0.196	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	< .001	0.003	0.021	0.093	0.042	< .001
	2.5%	-0.006	-0.017	-0.011	-0.051	-0.027	-0.101	-0.41	-0.796	-0.518	-0.034
	97.5%	0.008	0.011	0.018	0.033	0.025	0.101	0.127	0.399	0.307	0.037
Skew		2.076	2.825	-0.606	2.256	1.085	2.443	1.332	1.688	1.648	0.027
Kurtosis		16.097	14.491	22.115	12.565	22.839	11.237	5.271	6.861	8.17	-0.04

Condition 6; Scenario 2; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.023	-0.536	0.063	-0.436	-0.004	-0.309	-0.072	-0.165	-0.075	-0.191	-0.054	-0.154	-0.004	-0.006
	$\widehat{M}$	-0.002	-0.622	0.049	-0.5	-0.006	-0.37	-0.07	-0.166	-0.078	-0.2	-0.052	-0.156	-0.006	-0.007
	$\widehat{\sigma}^2$	0.113	0.567	0.086	0.389	0.052	0.308	0.005	0.012	0.007	0.019	0.005	0.017	0.018	0.022
	2.5%	-0.528	-1.558	-0.375	-1.383	-0.369	-1.071	-0.202	-0.35	-0.21	-0.421	-0.18	-0.393	-0.261	-0.28
	97.5%	0.743	0.894	0.599	0.908	0.37	0.976	0.062	0.083	0.083	0.113	0.084	0.155	0.259	0.276
P	$\widehat{\mu}$	< .001	-0.006	0.002	-0.015	< .001	-0.029	-0.145	-0.329	-0.25	-0.635	-0.134	-0.384	-0.001	-0.001
	$\widehat{M}$	< .001	-0.007	0.001	-0.017	< .001	-0.37	-0.141	-0.332	-0.259	-0.665	-0.13	-0.39	-0.002	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.003	0.02	0.05	0.072	0.213	0.034	0.108	0.001	0.001
	2.5%	-0.005	-0.017	-0.011	-0.047	-0.025	-0.1	-0.404	-0.701	-0.7	-1.404	-0.449	-0.983	-0.065	-0.07
	97.5%	0.007	0.01	0.017	0.031	0.025	0.091	0.125	0.167	0.278	0.377	0.21	0.389	0.065	0.069
Skew		1.331	2.681	-0.63	2.169	0.504	2.423	0.471	0.768	0.96	1.048	0.99	0.675	-0.086	-0.075
Kurtosis		7.112	13.814	24.313	12.41	22.186	11.542	3.617	1.873	5.625	3.136	7.152	2.323	0.408	0.045

Condition 6; Scenario 2; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.011	-0.515	0.056	-0.425	-0.01	-0.294	-0.093	-0.098	-0.074	0.005
	$\widehat{M}$	-0.014	-0.61	0.043	-0.502	-0.013	-0.369	-0.099	-0.106	-0.079	0.004
	$\widehat{\sigma}^2$	0.129	0.622	0.088	0.433	0.052	0.334	0.006	0.009	0.007	0.005
	2.5%	-0.551	-1.541	-0.386	-1.375	-0.381	-1.067	-0.205	-0.239	-0.207	-0.134
	97.5%	0.781	1.445	0.606	1.168	0.372	1.333	0.062	0.154	0.123	0.146
P	$\widehat{\mu}$	< .001	-0.006	0.002	-0.014	-0.001	-0.027	-0.185	-0.327	-0.185	0.001
	$\widehat{M}$	< .001	-0.007	0.001	-0.017	-0.001	-0.035	-0.197	-0.353	-0.198	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	< .001	0.003	0.022	0.1	0.043	< .001
	2.5%	-0.006	-0.017	-0.011	-0.047	-0.025	-0.1	-0.41	-0.796	-0.517	-0.034
	97.5%	0.008	0.016	0.017	0.04	0.025	0.124	0.124	0.515	0.308	0.037
Skew		0.584	2.998	-0.556	2.417	-0.095	0.003	1.591	2.171	1.674	0.027
Kurtosis		9.917	16.275	23.311	12.994	15.82	11.403	6.624	9.862	7.658	-0.04

Condition 6; Scenario 2; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.078	0.308	0.065	0.201	0.015	0.213	0.029	0.057	0.022	–
	$\widehat{M}$	0.066	0.227	0.064	0.12	0.016	0.15	0.026	0.056	0.02	–
	$\widehat{\sigma}^2$	0.085	0.545	0.066	0.382	0.041	0.273	0.003	0.004	0.004	–
	2.5%	-0.494	-0.683	-0.359	-0.712	-0.363	-0.562	-0.063	-0.053	-0.091	–
	97.5%	0.619	1.734	0.537	1.636	0.362	1.726	0.139	0.206	0.163	–
P	$\widehat{\mu}$	0.001	0.003	0.002	0.007	0.001	0.02	0.057	0.189	0.056	–
	$\widehat{M}$	0.001	0.002	0.002	0.004	0.001	0.014	0.051	0.188	0.05	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.002	0.011	0.045	0.024	–
	2.5%	-0.005	-0.007	-0.01	-0.024	-0.024	-0.052	-0.125	-0.178	-0.228	–
	97.5%	0.006	0.019	0.015	0.056	0.024	0.161	0.278	0.688	0.407	–
Skew		-0.054	2.677	-1.643	2.257	-1.198	2.245	0.813	0.886	1.054	–
Kurtosis		2.174	12.573	22.577	10.892	11.265	9.532	2.535	2.734	4.541	–

TABLE C-18

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 6 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 6; Scenario 3; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	0.34	0.486	0.282	0.394	0.071	0.464	-0.027	-0.138	-0.004	-0.105	-0.002	-0.002	0.003	-0.007
	$\widehat{M}$	0.019	-0.534	0.052	-0.439	-0.044	-0.236	-0.029	-0.183	-0.026	-0.165	-0.011	-0.011	0.005	-0.007
B	$\widehat{\sigma}^2$	0.927	6.356	0.678	4.476	0.479	3.004	0.022	0.057	0.029	0.083	0.02	0.02	0.008	0.04
	2.5%	-0.636	-2.581	-0.489	-2.176	-1.004	-1.643	-0.317	-0.543	-0.288	-0.56	-0.238	-0.238	-0.169	-0.407
	97.5%	3.022	5.902	2.788	4.843	2.194	3.951	0.302	0.318	0.406	0.464	0.316	0.316	0.183	0.426
	$\widehat{\mu}$	0.003	0.005	0.008	0.013	0.005	0.043	-0.053	-0.275	-0.013	-0.35	-0.005	-0.005	0.001	-0.002
	$\widehat{M}$	< .001	-0.006	0.001	-0.015	-0.003	-0.236	-0.058	-0.365	-0.085	-0.55	-0.028	-0.028	0.001	-0.002
P	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.005	0.002	0.026	0.086	0.227	0.318	0.921	0.128	0.128	< .001	0.002
	2.5%	-0.006	-0.028	-0.014	-0.074	-0.067	-0.153	-0.634	-1.085	-0.959	-1.866	-0.597	-0.597	-0.042	-0.102
	97.5%	0.03	0.064	0.08	0.165	0.146	0.369	0.604	0.636	1.354	1.545	0.791	0.791	0.046	0.106
Skew		2.016	0.958	2.358	0.909	2.953	0.79	-2.228	0.331	-0.806	0.427	-1.899	-1.899	-0.056	0.038
Kurtosis		5.416	-0.354	8.886	-0.488	18.014	-0.566	21.887	-0.733	9.177	-0.768	20.487	20.487	1.343	0.812

Condition 6; Scenario 3; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.345	0.491	0.285	0.397	0.073	0.466	-0.01	0.017	0.01	0.002
	$\widehat{M}$	0.011	-0.524	0.048	-0.434	-0.045	-0.231	-0.036	-0.031	-0.021	0.005
B	$\widehat{\sigma}^2$	0.956	6.289	0.69	4.443	0.484	2.979	0.015	0.026	0.016	0.005
	2.5%	-0.625	-2.553	-0.491	-2.17	-0.998	-1.634	-0.182	-0.191	-0.181	-0.138
	97.5%	3.085	5.898	2.808	4.838	2.198	3.95	0.297	0.42	0.33	0.154
	$\widehat{\mu}$	0.003	0.005	0.008	0.014	0.005	0.044	-0.02	0.058	0.026	< .001
	$\widehat{M}$	< .001	-0.006	0.001	-0.015	-0.003	-0.021	-0.073	-0.103	-0.051	0.001
P	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.005	0.002	0.026	0.059	0.284	0.098	< .001
	2.5%	-0.006	-0.027	-0.014	-0.074	-0.067	-0.153	-0.364	-0.637	-0.454	-0.034
	97.5%	0.031	0.063	0.08	0.165	0.147	0.369	0.593	1.401	0.825	0.039
Skew		2.011	0.962	2.357	0.911	2.978	0.792	1.034	1.076	0.938	0.122
Kurtosis		5.265	-0.338	8.767	-0.479	18.009	-0.558	0.709	0.452	0.721	0.084

Condition 6; Scenario 3; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	0.334	0.663	0.253	0.59	0.031	0.592	-0.016	-0.116	0.005	-0.091	0.006	-0.077	-0.004	-0.006
	$\widehat{M}$	0.054	-0.372	0.072	-0.304	-0.03	-0.146	-0.032	-0.165	-0.03	-0.16	-0.012	-0.122	-0.003	-0.007
B	$\widehat{\sigma}^2$	0.741	6.38	0.51	4.375	0.344	3.087	0.017	0.058	0.028	0.091	0.017	0.058	0.02	0.037
	2.5%	-0.649	-2.268	-0.703	-1.963	-1.1	-1.556	-0.223	-0.514	-0.237	-0.56	-0.221	-0.475	-0.316	-0.392
	97.5%	2.53	5.917	2.051	4.95	1.703	4.082	0.313	0.337	0.41	0.474	0.351	0.388	0.281	0.401
	$\widehat{\mu}$	0.003	0.007	0.007	0.02	0.002	0.055	-0.033	-0.232	0.017	-0.304	0.016	-0.192	-0.001	-0.001
	$\widehat{M}$	0.001	-0.004	0.002	-0.01	-0.002	-0.146	-0.064	-0.33	-0.099	-0.534	-0.031	-0.306	-0.001	-0.002
P	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.005	0.002	0.027	0.067	0.233	0.313	1.013	0.103	0.365	0.001	0.002
	2.5%	-0.006	-0.024	-0.02	-0.067	-0.073	-0.145	-0.446	-1.029	-0.792	-1.867	-0.552	-1.187	-0.079	-0.098
	97.5%	0.025	0.064	0.059	0.169	0.113	0.381	0.626	0.674	1.368	1.579	0.878	0.971	0.07	0.1
Skew		1.564	0.867	1.606	0.843	1.099	0.748	0.402	0.323	0.006	0.399	0.575	0.261	0.037	0.019
Kurtosis		2.954	-0.55	4.921	-0.63	7.942	-0.719	3.224	-0.841	5.606	-0.914	2.612	-0.858	2.363	0.985

Condition 6; Scenario 3; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.337	0.812	0.284	0.673	0.036	0.682	< .001	0.032	0.02	0.002
	$\widehat{M}$	0.052	-0.285	0.083	-0.24	-0.028	-0.03	-0.035	-0.032	-0.018	0.005
B	$\widehat{\sigma}^2$	0.742	6.663	0.54	4.421	0.362	3.2	0.018	0.032	0.019	0.005
	2.5%	-0.679	-2.253	-0.605	-2.004	-1.187	-1.527	-0.186	-0.195	-0.182	-0.138
	97.5%	2.554	5.983	2.197	4.927	1.634	4.238	0.318	0.447	0.357	0.155
	$\widehat{\mu}$	0.003	0.009	0.008	0.023	0.002	0.064	0.001	0.107	0.051	< .001
	$\widehat{M}$	0.001	-0.003	0.002	-0.008	-0.002	-0.003	-0.07	-0.106	-0.045	0.001
P	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.005	0.002	0.028	0.071	0.36	0.119	< .001
	2.5%	-0.007	-0.024	-0.017	-0.068	-0.079	-0.143	-0.371	-0.649	-0.455	-0.035
	97.5%	0.026	0.064	0.063	0.168	0.109	0.396	0.637	1.489	0.892	0.039
Skew		1.376	0.765	1.419	0.721	1.03	0.028	0.904	0.923	0.905	0.122
Kurtosis		2.37	-0.793	4.328	-0.844	6.02	-0.854	0.072	-0.156	0.277	0.085

Condition 6; Scenario 3; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.233	1.7	0.159	1.323	0.031	1.152	0.06	0.1	0.057	–
	$\widehat{M}$	0.158	0.871	0.115	0.635	0.023	0.603	0.045	0.074	0.038	–
B	$\widehat{\sigma}^2$	0.208	4.761	0.163	3.317	0.123	2.139	0.005	0.009	0.007	–
	2.5%	-0.484	-1.045	-0.501	-0.953	-0.738	-0.745	-0.053	-0.034	-0.072	–
	97.5%	1.398	6.016	1.162	4.902	0.862	3.961	0.227	0.317	0.234	–
	$\widehat{\mu}$	0.002	0.018	0.005	0.045	0.002	0.108	0.119	0.334	0.142	–
	$\widehat{M}$	0.002	0.009	0.003	0.021	0.002	0.056	0.089	0.248	0.096	–
P	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.004	0.001	0.019	0.022	0.102	0.042	–
	2.5%	-0.005	-0.011	-0.014	-0.032	-0.049	-0.07	-0.106	-0.113	-0.18	–
	97.5%	0.014	0.065	0.033	0.167	0.057	0.37	0.453	1.058	0.586	–
Skew		0.569	0.783	0.381	0.768	-0.163	0.675	0.739	0.767	0.619	–
Kurtosis		3.204	-0.729	4.768	-0.771	6.192	-0.77	0.053	-0.158	-0.069	–

TABLE C-19

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 7 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 7; Scenario 1; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.403	-0.039	0.35	-0.012	0.171	-0.018	-0.165	-0.05	-0.183	-0.06	-0.126	-0.126	-0.002	-0.011
	$\widehat{M}$	0.415	-0.065	0.366	-0.042	0.178	-0.042	-0.177	-0.056	-0.2	-0.071	-0.134	-0.134	0.002	-0.018
	$\widehat{\sigma}^2$	0.819	0.516	0.443	0.441	0.345	0.288	0.038	0.018	0.057	0.027	0.033	0.033	0.122	0.103
	2.5%	-1.333	-1.356	-1.019	-1.216	-1.075	-0.965	-0.543	-0.302	-0.601	-0.344	-0.444	-0.444	-0.686	-0.61
	97.5%	2.232	1.53	1.602	1.364	1.282	1.136	0.264	0.244	0.346	0.302	0.274	0.274	0.705	0.616
P	$\widehat{\mu}$	0.004	< .001	0.01	< .001	0.011	-0.002	-0.331	-0.099	-0.611	-0.199	-0.315	-0.315	< .001	-0.001
	$\widehat{M}$	0.004	-0.001	0.01	-0.001	0.012	-0.042	-0.355	-0.113	-0.665	-0.237	-0.336	-0.336	< .001	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.002	0.003	0.154	0.073	0.631	0.304	0.206	0.206	0.002	0.002
	2.5%	-0.013	-0.015	-0.029	-0.041	-0.072	-0.09	-1.086	-0.605	-2.003	-1.148	-1.11	-1.11	-0.086	-0.076
	97.5%	0.022	0.016	0.046	0.046	0.085	0.106	0.529	0.487	1.154	1.005	0.686	0.686	0.088	0.077
Skew		0.013	0.468	-0.177	-0.275	-0.082	0.222	0.198	0.303	0.461	0.361	0.318	0.318	0.079	0.324
Kurtosis		3.114	2.087	2.232	6.958	2.184	1.42	0.271	0.184	0.229	1.248	0.088	0.088	-0.038	1.122

Condition 7; Scenario 1; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.423	-0.045	0.36	-0.015	0.175	-0.02	-0.078	-0.09	-0.067	-0.019
	$\widehat{M}$	0.43	-0.072	0.372	-0.041	0.185	-0.044	-0.096	-0.117	-0.084	-0.013
	$\widehat{\sigma}^2$	0.843	0.51	0.452	0.438	0.351	0.288	0.018	0.029	0.02	0.051
	2.5%	-1.334	-1.329	-1.036	-1.218	-1.09	-0.975	-0.302	-0.368	-0.294	-0.442
	97.5%	2.268	1.528	1.611	1.359	1.286	1.144	0.243	0.296	0.261	0.409
P	$\widehat{\mu}$	0.004	< .001	0.01	-0.001	0.012	-0.002	-0.157	-0.301	-0.166	-0.002
	$\widehat{M}$	0.004	-0.001	0.011	-0.001	0.012	-0.004	-0.191	-0.391	-0.21	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.002	0.003	0.074	0.322	0.123	0.001
	2.5%	-0.013	-0.014	-0.03	-0.042	-0.073	-0.091	-0.603	-1.225	-0.735	-0.055
	97.5%	0.023	0.016	0.046	0.046	0.086	0.107	0.486	0.988	0.652	0.051
Skew		0.029	0.496	-0.166	-0.265	-0.073	0.229	0.623	0.733	0.631	0.013
Kurtosis		3.052	2.117	2.203	6.91	2.178	1.402	0.467	0.705	0.334	0.039

Condition 7; Scenario 1; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.411	-0.067	0.34	-0.022	0.156	-0.023	-0.167	-0.054	-0.184	-0.062	-0.121	-0.045	-0.004	-0.014
	$\widehat{M}$	0.409	-0.09	0.352	-0.047	0.167	-0.05	-0.176	-0.059	-0.2	-0.074	-0.126	-0.049	-0.005	-0.022
	$\widehat{\sigma}^2$	0.73	0.47	0.392	0.398	0.318	0.276	0.038	0.018	0.056	0.026	0.033	0.02	0.103	0.099
	2.5%	-1.266	-1.341	-0.947	-1.227	-1.041	-0.959	-0.543	-0.304	-0.599	-0.342	-0.438	-0.308	-0.616	-0.605
	97.5%	2.212	1.374	1.549	1.311	1.22	1.136	0.25	0.239	0.323	0.301	0.277	0.249	0.621	0.605
P	$\widehat{\mu}$	0.004	-0.001	0.01	-0.001	0.01	-0.002	-0.333	-0.107	-0.614	-0.207	-0.302	-0.111	< .001	-0.002
	$\widehat{M}$	0.004	-0.001	0.01	-0.002	0.011	-0.05	-0.353	-0.118	-0.665	-0.248	-0.316	-0.122	-0.001	-0.003
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.151	0.071	0.621	0.29	0.206	0.127	0.002	0.002
	2.5%	-0.013	-0.014	-0.027	-0.042	-0.069	-0.09	-1.086	-0.608	-1.997	-1.141	-1.095	-0.77	-0.077	-0.076
	97.5%	0.022	0.015	0.044	0.045	0.081	0.106	0.5	0.479	1.079	1.004	0.694	0.622	0.078	0.076
Skew		0.372	0.328	-0.16	0.395	0.015	0.248	0.165	0.27	0.448	0.498	0.298	0.252	0.111	0.158
Kurtosis		1.754	1.309	1.605	1.689	1.845	0.869	0.264	0.125	0.22	0.397	0.111	0.143	0.116	0.044

Condition 7; Scenario 1; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.436	-0.063	0.36	-0.02	0.17	-0.023	-0.082	-0.094	-0.067	-0.019
	$\widehat{M}$	0.429	-0.089	0.367	-0.047	0.174	-0.047	-0.098	-0.12	-0.084	-0.013
	$\widehat{\sigma}^2$	0.76	0.473	0.413	0.396	0.328	0.276	0.018	0.028	0.019	0.051
	2.5%	-1.27	-1.321	-0.94	-1.218	-1.03	-0.963	-0.302	-0.368	-0.294	-0.442
	97.5%	2.263	1.408	1.611	1.321	1.275	1.126	0.213	0.287	0.252	0.409
P	$\widehat{\mu}$	0.004	-0.001	0.01	-0.001	0.011	-0.002	-0.164	-0.312	-0.167	-0.002
	$\widehat{M}$	0.004	-0.001	0.01	-0.002	0.012	-0.004	-0.195	-0.402	-0.209	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.071	0.313	0.12	0.001
	2.5%	-0.013	-0.014	-0.027	-0.042	-0.069	-0.09	-0.603	-1.225	-0.735	-0.055
	97.5%	0.023	0.015	0.046	0.045	0.085	0.105	0.428	0.957	0.63	0.051
Skew		0.393	0.357	-0.081	0.416	0.057	0.002	0.595	0.706	0.612	0.013
Kurtosis		1.82	1.413	1.651	1.603	1.879	0.876	0.437	0.619	0.319	0.037

Condition 7; Scenario 1; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.267	0.347	0.167	0.253	0.074	0.224	0.049	0.082	0.032	–
	$\widehat{M}$	0.297	0.309	0.179	0.218	0.1	0.183	0.045	0.073	0.025	–
	$\widehat{\sigma}^2$	0.618	0.425	0.346	0.357	0.28	0.24	0.007	0.012	0.01	–
	2.5%	-1.315	-0.75	-1.082	-0.798	-1.03	-0.612	-0.107	-0.118	-0.154	–
	97.5%	1.601	1.804	1.265	1.554	1.003	1.316	0.241	0.309	0.255	–
P	$\widehat{\mu}$	0.003	0.004	0.005	0.009	0.005	0.021	0.097	0.273	0.08	–
	$\widehat{M}$	0.003	0.003	0.005	0.007	0.007	0.017	0.09	0.244	0.062	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.029	0.136	0.062	–
	2.5%	-0.013	-0.008	-0.031	-0.027	-0.069	-0.057	-0.213	-0.392	-0.386	–
	97.5%	0.016	0.019	0.036	0.053	0.067	0.123	0.481	1.03	0.638	–
Skew		-0.474	0.717	-0.531	0.368	-0.345	0.451	0.294	0.328	0.365	–
Kurtosis		2.742	1.899	2.082	2.372	1.608	0.951	0.412	0.226	0.143	–

TABLE C-20

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 7 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 7; Scenario 2; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.18	0.075	0.211	-0.001	-0.004	0.179	-0.066	-0.087	-0.045	-0.055	-0.034	-0.034	0.015	-0.015
	$\widehat{M}$	0.065	-0.227	0.13	-0.266	-0.039	-0.043	-0.074	-0.101	-0.065	-0.085	-0.048	-0.048	0.01	-0.04
	$\widehat{\sigma}^2$	0.75	2.71	0.42	1.984	0.354	1.322	0.03	0.048	0.044	0.07	0.028	0.028	0.078	0.202
	2.5%	-1.231	-2.63	-0.843	-2.252	-1.03	-1.572	-0.387	-0.493	-0.423	-0.543	-0.342	-0.342	-0.529	-0.87
	97.5%	2.309	4.226	1.704	3.485	1.247	2.968	0.333	0.354	0.404	0.459	0.339	0.339	0.565	0.991
P	$\widehat{\mu}$	0.002	0.001	0.006	< .001	< .001	0.017	-0.131	-0.174	-0.15	-0.182	-0.086	-0.086	0.002	-0.002
	$\widehat{M}$	0.001	-0.002	0.004	-0.009	-0.003	-0.043	-0.149	-0.201	-0.216	-0.281	-0.121	-0.121	0.001	-0.005
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.002	0.012	0.121	0.192	0.489	0.78	0.177	0.177	0.001	0.003
	2.5%	-0.012	-0.028	-0.024	-0.077	-0.069	-0.147	-0.774	-0.986	-1.409	-1.809	-0.856	-0.856	-0.066	-0.109
	97.5%	0.023	0.046	0.049	0.119	0.083	0.277	0.667	0.708	1.349	1.532	0.848	0.848	0.071	0.124
Skew		1.266	0.92	1.751	0.887	2.476	0.754	0.286	0.06	0.247	0.146	-0.068	-0.068	0.132	0.325
Kurtosis		4.205	1.188	10.666	0.92	31.265	0.533	0.784	-0.104	0.703	-0.53	2.894	2.894	0.521	0.741

Condition 7; Scenario 2; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.182	0.077	0.212	< .001	-0.004	0.18	-0.059	-0.033	-0.03	0.007
	$\widehat{M}$	0.065	-0.235	0.128	-0.262	-0.043	-0.041	-0.085	-0.065	-0.056	0.005
	$\widehat{\sigma}^2$	0.775	2.673	0.43	1.964	0.355	1.308	0.028	0.042	0.028	0.054
	2.5%	-1.243	-2.668	-0.836	-2.222	-1.012	-1.563	-0.331	-0.388	-0.319	-0.44
	97.5%	2.357	4.221	1.742	3.477	1.26	2.957	0.329	0.41	0.346	0.48
P	$\widehat{\mu}$	0.002	0.001	0.006	< .001	< .001	0.017	-0.117	-0.112	-0.074	0.001
	$\widehat{M}$	0.001	-0.003	0.004	-0.009	-0.003	-0.004	-0.169	-0.217	-0.139	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.002	0.011	0.111	0.47	0.174	0.001
	2.5%	-0.012	-0.029	-0.024	-0.076	-0.067	-0.146	-0.662	-1.294	-0.797	-0.055
	97.5%	0.024	0.045	0.05	0.118	0.084	0.276	0.658	1.365	0.865	0.06
Skew		1.308	0.931	1.784	0.895	2.46	0.759	0.597	0.503	0.484	0.058
Kurtosis		4.226	1.225	10.72	0.947	30.46	0.552	0.198	-0.149	0.06	-0.101

Condition 7; Scenario 2; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.18	0.051	0.199	0.009	-0.011	0.167	-0.068	-0.091	-0.049	-0.061	-0.036	-0.067	-0.009	-0.014
	$\widehat{M}$	0.074	-0.262	0.131	-0.256	-0.036	-0.054	-0.079	-0.108	-0.071	-0.092	-0.05	-0.084	-0.036	-0.042
	$\widehat{\sigma}^2$	0.696	2.594	0.335	1.873	0.267	1.26	0.029	0.048	0.043	0.07	0.026	0.053	0.188	0.205
	2.5%	-1.162	-2.595	-0.799	-2.163	-0.984	-1.538	-0.387	-0.493	-0.421	-0.545	-0.342	-0.495	-0.824	-0.88
	97.5%	2.146	3.999	1.619	3.444	1.162	2.87	0.328	0.357	0.411	0.464	0.337	0.383	0.983	1.002
P	$\widehat{\mu}$	0.002	0.001	0.006	< .001	-0.001	0.016	-0.137	-0.183	-0.165	-0.204	-0.091	-0.168	-0.001	-0.002
	$\widehat{M}$	0.001	-0.003	0.004	-0.009	-0.002	-0.054	-0.158	-0.217	-0.236	-0.307	-0.123	-0.21	-0.005	-0.005
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.001	0.011	0.116	0.19	0.482	0.783	0.164	0.333	0.003	0.003
	2.5%	-0.012	-0.028	-0.023	-0.074	-0.066	-0.143	-0.774	-0.987	-1.404	-1.817	-0.856	-1.238	-0.103	-0.11
	97.5%	0.021	0.043	0.046	0.117	0.077	0.268	0.656	0.714	1.37	1.546	0.844	0.957	0.123	0.125
Skew		1.116	0.94	0.753	0.929	0.025	0.819	0.374	0.078	0.307	0.178	0.29	0.079	0.34	0.317
Kurtosis		4.291	1.315	3.114	0.944	4.417	0.585	0.647	0.025	0.77	-0.514	0.725	-0.398	0.845	0.677

Condition 7; Scenario 2; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.175	0.032	0.201	-0.012	-0.017	0.151	-0.063	-0.04	-0.034	0.007
	$\widehat{M}$	0.071	-0.271	0.129	-0.261	-0.043	-0.053	-0.086	-0.069	-0.058	0.005
	$\widehat{\sigma}^2$	0.726	2.527	0.362	1.826	0.253	1.236	0.027	0.041	0.027	0.055
	2.5%	-1.206	-2.619	-0.816	-2.152	-0.969	-1.544	-0.332	-0.388	-0.32	-0.44
	97.5%	2.244	3.853	1.667	3.396	1.138	2.845	0.332	0.409	0.341	0.484
P	$\widehat{\mu}$	0.002	< .001	0.006	< .001	-0.001	0.014	-0.126	-0.132	-0.086	0.001
	$\widehat{M}$	0.001	-0.003	0.004	-0.009	-0.003	-0.005	-0.171	-0.231	-0.144	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.001	0.011	0.107	0.46	0.169	0.001
	2.5%	-0.012	-0.028	-0.023	-0.073	-0.065	-0.144	-0.665	-1.294	-0.801	-0.055
	97.5%	0.022	0.041	0.048	0.116	0.076	0.266	0.664	1.364	0.852	0.061
Skew		1.198	0.937	0.899	0.955	0.104	0.011	0.648	0.565	0.484	0.058
Kurtosis		4.337	1.443	4.5	1.118	4.021	0.619	0.394	0.017	0.126	-0.106

Condition 7; Scenario 2; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.243	0.94	0.189	0.66	0.029	0.691	0.071	0.125	0.06	–
	$\widehat{M}$	0.201	0.69	0.164	0.425	0.024	0.508	0.06	0.116	0.053	–
	$\widehat{\sigma}^2$	0.421	2.099	0.25	1.517	0.246	1.011	0.009	0.014	0.012	–
	2.5%	-0.905	-1.28	-0.789	-1.212	-0.921	-0.855	-0.105	-0.094	-0.142	–
	97.5%	1.662	4.435	1.259	3.731	0.98	3.013	0.277	0.356	0.284	–
P	$\widehat{\mu}$	0.002	0.01	0.005	0.023	0.002	0.065	0.143	0.418	0.151	–
	$\widehat{M}$	0.002	0.007	0.005	0.014	0.002	0.047	0.119	0.387	0.133	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.001	0.009	0.035	0.151	0.072	–
	2.5%	-0.009	-0.014	-0.023	-0.041	-0.061	-0.08	-0.211	-0.314	-0.356	–
	97.5%	0.017	0.048	0.036	0.127	0.065	0.281	0.553	1.187	0.709	–
Skew		0.403	0.869	0.526	0.849	2.03	0.731	0.368	0.208	0.189	–
Kurtosis		2.156	0.865	3.993	0.723	30.938	0.334	0.105	0.085	0.13	–

TABLE C-21

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 7 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 7; Scenario 3; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\hat{\mu}$	0.735	1.668	0.459	1.4	0.228	1.234	-0.024	-0.008	0.008	0.04	0.01	0.01	-0.005	-0.028
	$\hat{M}$	0.341	1.824	0.221	1.395	0.048	1.43	-0.029	0.038	0.003	0.112	0.013	0.013	0.014	-0.034
	$\hat{\sigma}^2$	2.14	7.73	1.177	5.635	0.833	3.342	0.053	0.083	0.074	0.104	0.049	0.049	0.11	0.251
	2.5%	-1.245	-3.137	-1.189	-2.672	-1.184	-2	-0.494	-0.695	-0.638	-0.668	-0.456	-0.456	-0.677	-1.03
	97.5%	4.566	6.203	2.978	5.267	2.673	4.136	0.411	0.402	0.522	0.513	0.43	0.43	0.594	1.007
P	$\hat{\mu}$	0.007	0.018	0.013	0.048	0.015	0.115	-0.049	-0.015	0.027	0.134	0.024	0.024	-0.001	-0.003
	$\hat{M}$	0.003	0.02	0.006	0.048	0.003	1.43	-0.058	0.077	0.011	0.372	0.03	0.03	0.002	-0.004
	$\hat{\sigma}^2$	< .001	0.001	0.001	0.007	0.004	0.029	0.212	0.331	0.827	1.151	0.307	0.307	0.002	0.004
	2.5%	-0.012	-0.034	-0.034	-0.091	-0.079	-0.187	-0.987	-1.389	-2.127	-2.228	-1.141	-1.141	-0.085	-0.129
	97.5%	0.046	0.067	0.085	0.179	0.178	0.386	0.822	0.805	1.74	1.71	1.076	1.076	0.074	0.126
Skew		1.676	-0.142	1.783	-0.041	1.688	-0.195	-0.839	-0.8	-0.651	-0.65	-1.012	-1.012	-0.984	0.255
Kurtosis		5.666	-0.774	10.071	-1.072	5.982	-0.975	4.062	0.313	2.248	-0.145	4.591	4.591	5.848	1.941

Condition 7; Scenario 3; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\hat{\mu}$	0.75	1.66	0.467	1.395	0.233	1.23	0.019	0.062	0.046	-0.004
	$\hat{M}$	0.338	1.772	0.223	1.398	0.046	1.419	0.004	0.053	0.041	-0.005
	$\hat{\sigma}^2$	2.174	7.643	1.193	5.591	0.839	3.306	0.034	0.046	0.032	0.053
	2.5%	-1.238	-3.118	-1.179	-2.655	-1.172	-1.986	-0.306	-0.336	-0.279	-0.424
	97.5%	4.57	6.187	3.022	5.265	2.686	4.123	0.368	0.474	0.388	0.435
P	$\hat{\mu}$	0.008	0.018	0.013	0.048	0.016	0.115	0.039	0.205	0.115	-0.001
	$\hat{M}$	0.003	0.019	0.006	0.048	0.003	0.133	0.007	0.176	0.103	-0.001
	$\hat{\sigma}^2$	< .001	0.001	0.001	0.006	0.004	0.029	0.136	0.513	0.199	0.001
	2.5%	-0.012	-0.034	-0.034	-0.09	-0.078	-0.185	-0.612	-1.119	-0.697	-0.053
	97.5%	0.046	0.067	0.086	0.179	0.179	0.385	0.736	1.581	0.971	0.054
Skew		1.651	-0.132	1.768	-0.034	1.698	-0.191	0.116	0.067	0.038	< .001
Kurtosis		5.479	-0.765	9.796	-1.065	6.017	-0.965	-0.652	-0.535	-0.52	-0.24

Condition 7; Scenario 3; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\hat{\mu}$	0.669	1.765	0.425	1.491	0.16	1.282	-0.018	-0.003	0.018	0.051	0.015	0.031	-0.021	-0.026
	$\hat{M}$	0.333	1.962	0.235	1.53	0.057	1.47	-0.029	0.038	0.006	0.123	0.011	0.086	-0.026	-0.032
	$\hat{\sigma}^2$	1.737	7.233	0.841	5.097	0.605	3.243	0.047	0.082	0.067	0.105	0.039	0.074	0.195	0.226
	2.5%	-1.372	-3.021	-1.165	-2.432	-1.407	-1.98	-0.467	-0.674	-0.549	-0.655	-0.394	-0.558	-0.937	-0.994
	97.5%	3.728	6.332	2.535	5.225	2.111	4.254	0.399	0.402	0.508	0.522	0.426	0.441	0.909	0.948
P	$\hat{\mu}$	0.007	0.019	0.012	0.051	0.011	0.12	-0.035	-0.005	0.058	0.169	0.037	0.079	-0.003	-0.003
	$\hat{M}$	0.003	0.021	0.007	0.052	0.004	1.47	-0.058	0.077	0.019	0.411	0.028	0.216	-0.003	-0.004
	$\hat{\sigma}^2$	< .001	0.001	0.001	0.006	0.003	0.028	0.188	0.326	0.741	1.166	0.246	0.461	0.003	0.004
	2.5%	-0.014	-0.033	-0.033	-0.083	-0.094	-0.185	-0.935	-1.347	-1.831	-2.183	-0.985	-1.395	-0.117	-0.124
	97.5%	0.037	0.068	0.072	0.178	0.141	0.397	0.798	0.805	1.694	1.741	1.064	1.104	0.114	0.118
Skew		1.346	-0.087	0.789	-0.024	0.767	-0.164	-0.223	-0.731	-0.421	-0.624	-0.277	-0.669	0.083	0.119
Kurtosis		4.903	-0.958	2.502	-1.041	2.962	-0.958	0.587	0.109	1.547	-0.349	1.105	-0.003	1.104	1.247

Condition 7; Scenario 3; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\hat{\mu}$	0.676	1.767	0.413	1.512	0.157	1.303	0.022	0.067	0.048	-0.004
	$\hat{M}$	0.362	1.928	0.233	1.518	0.055	1.461	0.002	0.054	0.041	-0.005
	$\hat{\sigma}^2$	1.659	7.221	0.897	5.153	0.647	3.216	0.036	0.051	0.034	0.053
	2.5%	-1.349	-3.001	-1.188	-2.442	-1.456	-1.973	-0.306	-0.339	-0.28	-0.42
	97.5%	3.723	6.371	2.484	5.289	2.086	4.261	0.374	0.498	0.407	0.439
P	$\hat{\mu}$	0.007	0.019	0.012	0.052	0.01	0.122	0.044	0.223	0.12	< .001
	$\hat{M}$	0.004	0.021	0.007	0.052	0.004	0.137	0.005	0.183	0.102	-0.001
	$\hat{\sigma}^2$	< .001	0.001	0.001	0.006	0.003	0.028	0.144	0.564	0.213	0.001
	2.5%	-0.013	-0.032	-0.034	-0.083	-0.097	-0.184	-0.612	-1.129	-0.7	-0.052
	97.5%	0.037	0.069	0.071	0.18	0.139	0.398	0.749	1.661	1.017	0.055
Skew		0.982	-0.074	0.823	-0.03	0.661	0.028	0.121	0.088	0.047	0.002
Kurtosis		1.779	-0.954	4.093	-1.027	3.672	-0.916	-0.76	-0.66	-0.54	-0.245

Condition 7; Scenario 3; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\hat{\mu}$	0.636	2.636	0.359	2.107	0.194	1.779	0.095	0.156	0.088	–
	$\hat{M}$	0.463	2.815	0.282	2.186	0.108	2.002	0.091	0.155	0.09	–
	$\hat{\sigma}^2$	0.916	5.558	0.609	4.004	0.428	2.395	0.009	0.013	0.012	–
	2.5%	-0.936	-1.867	-1.061	-1.468	-0.975	-1.233	-0.077	-0.061	-0.12	–
	97.5%	2.792	6.389	1.946	5.268	1.757	4.235	0.29	0.395	0.302	–
P	$\hat{\mu}$	0.006	0.028	0.01	0.072	0.013	0.166	0.19	0.52	0.22	–
	$\hat{M}$	0.005	0.03	0.008	0.074	0.007	0.187	0.182	0.516	0.224	–
	$\hat{\sigma}^2$	< .001	0.001	< .001	0.005	0.002	0.021	0.037	0.149	0.075	–
	2.5%	-0.009	-0.02	-0.03	-0.05	-0.065	-0.115	-0.155	-0.203	-0.3	–
	97.5%	0.028	0.069	0.056	0.18	0.117	0.395	0.58	1.317	0.756	–
Skew		1.111	-0.283	1.763	-0.152	1.305	-0.322	0.15	0.112	0.053	–
Kurtosis		5.388	-0.656	20.04	-0.973	6.371	-0.794	-0.317	-0.322	-0.341	–

TABLE C-22

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 8 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 8; Scenario 1; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.462	-0.121	0.415	-0.07	0.219	-0.064	-0.228	-0.08	-0.232	-0.088	-0.161	-0.161	0.03	-0.009
	$\widehat{M}$	0.434	-0.118	0.407	-0.083	0.22	-0.076	-0.232	-0.082	-0.241	-0.089	-0.163	-0.163	0.032	-0.009
	$\widehat{\sigma}^2$	0.264	0.176	0.167	0.154	0.104	0.109	0.014	0.006	0.02	0.009	0.012	0.012	0.046	0.037
	2.5%	-0.505	-0.902	-0.319	-0.749	-0.384	-0.649	-0.443	-0.229	-0.49	-0.263	-0.371	-0.371	-0.405	-0.383
	97.5%	1.472	0.721	1.216	0.734	0.82	0.614	0.014	0.077	0.072	0.116	0.061	0.061	0.437	0.361
P	$\widehat{\mu}$	0.005	-0.001	0.012	-0.002	0.015	-0.006	-0.455	-0.16	-0.772	-0.293	-0.402	-0.402	0.004	-0.001
	$\widehat{M}$	0.004	-0.001	0.012	-0.003	0.015	-0.076	-0.463	-0.165	-0.802	-0.298	-0.408	-0.408	0.004	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.054	0.024	0.218	0.103	0.076	0.076	0.001	0.001
	2.5%	-0.005	-0.01	-0.009	-0.026	-0.026	-0.061	-0.885	-0.457	-1.633	-0.876	-0.927	-0.927	-0.051	-0.048
	97.5%	0.015	0.008	0.035	0.025	0.055	0.057	0.028	0.154	0.24	0.386	0.152	0.152	0.055	0.045
Skew		0.131	0.12	-0.293	0.461	-0.434	0.31	0.439	0.382	0.459	0.251	0.267	0.267	-0.079	0.037
Kurtosis		1.809	0.794	3.758	2.715	4.454	2.273	1.046	1.428	0.999	1.325	0.577	0.577	-0.012	-0.215

Condition 8; Scenario 1; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.487	-0.129	0.428	-0.075	0.226	-0.067	-0.121	-0.129	-0.095	-0.003
	$\widehat{M}$	0.465	-0.129	0.414	-0.089	0.225	-0.078	-0.126	-0.136	-0.097	< .001
	$\widehat{\sigma}^2$	0.274	0.173	0.171	0.153	0.106	0.109	0.006	0.01	0.007	0.02
	2.5%	-0.502	-0.893	-0.314	-0.751	-0.39	-0.651	-0.252	-0.293	-0.25	-0.282
	97.5%	1.504	0.697	1.247	0.72	0.832	0.611	0.046	0.083	0.082	0.268
P	$\widehat{\mu}$	0.005	-0.001	0.012	-0.003	0.015	-0.006	-0.242	-0.429	-0.238	< .001
	$\widehat{M}$	0.005	-0.001	0.012	-0.003	0.015	-0.007	-0.252	-0.453	-0.243	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.024	0.106	0.045	< .001
	2.5%	-0.005	-0.01	-0.009	-0.026	-0.026	-0.061	-0.505	-0.978	-0.625	-0.035
	97.5%	0.015	0.008	0.036	0.025	0.055	0.057	0.092	0.277	0.204	0.034
Skew		0.135	0.133	-0.283	0.479	-0.444	0.319	0.824	0.719	0.39	-0.027
Kurtosis		1.865	0.754	3.727	2.761	4.445	2.299	2.589	1.653	0.614	0.076

Condition 8; Scenario 1; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.459	-0.143	0.398	-0.076	0.202	-0.066	-0.227	-0.081	-0.23	-0.087	-0.152	-0.065	0.013	-0.009
	$\widehat{M}$	0.434	-0.134	0.384	-0.087	0.205	-0.078	-0.23	-0.082	-0.238	-0.09	-0.152	-0.065	0.013	-0.01
	$\widehat{\sigma}^2$	0.255	0.162	0.159	0.149	0.096	0.108	0.013	0.006	0.02	0.009	0.012	0.008	0.038	0.037
	2.5%	-0.505	-0.902	-0.321	-0.75	-0.384	-0.649	-0.438	-0.228	-0.49	-0.263	-0.368	-0.231	-0.373	-0.391
	97.5%	1.426	0.684	1.189	0.719	0.794	0.614	0.015	0.077	0.074	0.116	0.066	0.115	0.417	0.355
P	$\widehat{\mu}$	0.005	-0.002	0.011	-0.003	0.013	-0.006	-0.454	-0.161	-0.765	-0.289	-0.379	-0.161	0.002	-0.001
	$\widehat{M}$	0.004	-0.001	0.011	-0.003	0.014	-0.078	-0.46	-0.165	-0.796	-0.302	-0.382	-0.163	0.002	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.054	0.024	0.218	0.104	0.078	0.048	0.001	0.001
	2.5%	-0.005	-0.01	-0.009	-0.026	-0.026	-0.061	-0.877	-0.456	-1.633	-0.876	-0.92	-0.577	-0.047	-0.049
	97.5%	0.014	0.007	0.034	0.025	0.053	0.057	0.03	0.154	0.245	0.387	0.166	0.289	0.052	0.044
Skew		0.157	0.122	-0.297	0.533	-0.572	0.373	0.476	0.403	0.498	0.355	0.23	0.18	0.035	0.026
Kurtosis		1.498	0.477	3.889	2.524	4.246	1.996	1.093	1.405	1.09	1.016	0.542	0.089	-0.135	-0.238

Condition 8; Scenario 1; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.482	-0.143	0.417	-0.079	0.218	-0.071	-0.122	-0.13	-0.094	-0.003
	$\widehat{M}$	0.465	-0.137	0.408	-0.091	0.217	-0.081	-0.128	-0.136	-0.097	< .001
	$\widehat{\sigma}^2$	0.259	0.164	0.165	0.146	0.102	0.105	0.006	0.009	0.007	0.02
	2.5%	-0.502	-0.903	-0.317	-0.749	-0.385	-0.651	-0.252	-0.291	-0.25	-0.282
	97.5%	1.49	0.666	1.224	0.703	0.824	0.6	0.047	0.072	0.082	0.269
P	$\widehat{\mu}$	0.005	-0.002	0.012	-0.003	0.015	-0.007	-0.244	-0.433	-0.235	< .001
	$\widehat{M}$	0.005	-0.001	0.012	-0.003	0.014	-0.007	-0.255	-0.455	-0.243	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.023	0.103	0.045	< .001
	2.5%	-0.005	-0.01	-0.009	-0.026	-0.026	-0.061	-0.505	-0.972	-0.625	-0.035
	97.5%	0.015	0.007	0.035	0.024	0.055	0.056	0.094	0.239	0.205	0.034
Skew		0.03	0.091	-0.334	0.475	-0.527	0.001	0.851	0.638	0.344	-0.027
Kurtosis		1.117	0.477	3.79	2.473	4.168	1.774	2.735	1.382	0.497	0.076

Condition 8; Scenario 1; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.283	0.315	0.197	0.233	0.092	0.209	0.041	0.079	0.03	–
	$\widehat{M}$	0.279	0.296	0.198	0.203	0.104	0.195	0.041	0.076	0.029	–
	$\widehat{\sigma}^2$	0.192	0.152	0.131	0.129	0.084	0.094	0.003	0.004	0.004	–
	2.5%	-0.554	-0.381	-0.461	-0.379	-0.46	-0.368	-0.052	-0.037	-0.08	–
	97.5%	1.087	1.164	0.88	1.047	0.622	0.827	0.143	0.212	0.148	–
P	$\widehat{\mu}$	0.003	0.003	0.006	0.008	0.006	0.02	0.083	0.262	0.074	–
	$\widehat{M}$	0.003	0.003	0.006	0.007	0.007	0.018	0.083	0.255	0.074	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	< .001	0.001	0.01	0.046	0.022	–
	2.5%	-0.006	-0.004	-0.013	-0.013	-0.031	-0.034	-0.104	-0.124	-0.2	–
	97.5%	0.011	0.013	0.025	0.036	0.041	0.077	0.286	0.708	0.37	–
Skew		-0.397	0.339	-0.588	0.708	-0.743	0.542	0.327	0.342	0.225	–
Kurtosis		1.898	0.216	4.485	2.102	4.754	1.891	1.185	0.518	0.267	–

TABLE C-23

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 8 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 8; Scenario 2; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.109	-0.362	0.114	-0.359	-0.023	-0.162	-0.091	-0.172	-0.068	-0.141	-0.057	-0.057	0.019	-0.023
	$\widehat{M}$	-0.01	-0.573	0.068	-0.564	-0.059	-0.311	-0.091	-0.191	-0.08	-0.169	-0.062	-0.062	0.018	-0.032
	$\widehat{\sigma}^2$	0.489	1.3	0.178	1.07	0.135	0.74	0.013	0.024	0.02	0.035	0.012	0.012	0.03	0.079
	2.5%	-0.7	-1.887	-0.448	-1.67	-0.541	-1.348	-0.292	-0.413	-0.29	-0.427	-0.246	-0.246	-0.322	-0.563
	97.5%	2.202	3.176	1.054	3.06	0.681	2.512	0.129	0.235	0.269	0.339	0.226	0.226	0.334	0.535
P	$\widehat{\mu}$	0.001	-0.004	0.003	-0.012	-0.002	-0.015	-0.182	-0.345	-0.226	-0.469	-0.143	-0.143	0.002	-0.003
	$\widehat{M}$	< .001	-0.006	0.002	-0.019	-0.004	-0.311	-0.183	-0.382	-0.265	-0.563	-0.156	-0.156	0.002	-0.004
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.006	0.051	0.094	0.22	0.389	0.074	0.074	< .001	0.001
	2.5%	-0.007	-0.02	-0.013	-0.057	-0.036	-0.126	-0.584	-0.826	-0.967	-1.425	-0.616	-0.616	-0.04	-0.07
	97.5%	0.022	0.034	0.03	0.104	0.045	0.234	0.258	0.47	0.898	1.13	0.565	0.565	0.042	0.067
Skew		2.865	1.828	2.294	1.936	2.239	1.622	-0.421	0.968	0.824	0.957	0.59	0.59	-0.242	0.114
Kurtosis		12.003	4.206	16.443	4.883	17.99	3.604	10.733	1.495	5.259	1.329	4.168	4.168	0.477	-0.138

Condition 8; Scenario 2; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.103	-0.352	0.111	-0.353	-0.026	-0.158	-0.104	-0.08	-0.071	0.013
	$\widehat{M}$	-0.026	-0.558	0.064	-0.556	-0.062	-0.3	-0.117	-0.105	-0.082	0.015
	$\widehat{\sigma}^2$	0.51	1.276	0.184	1.057	0.137	0.731	0.011	0.019	0.014	0.022
	2.5%	-0.715	-1.847	-0.457	-1.654	-0.542	-1.338	-0.263	-0.281	-0.251	-0.271
	97.5%	2.264	3.138	1.106	3.053	0.694	2.496	0.209	0.321	0.266	0.313
P	$\widehat{\mu}$	0.001	-0.004	0.003	-0.012	-0.002	-0.015	-0.209	-0.267	-0.177	0.002
	$\widehat{M}$	< .001	-0.006	0.002	-0.019	-0.004	-0.028	-0.235	-0.35	-0.205	0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.006	0.046	0.216	0.085	< .001
	2.5%	-0.007	-0.02	-0.013	-0.056	-0.036	-0.125	-0.526	-0.935	-0.628	-0.034
	97.5%	0.023	0.034	0.032	0.104	0.046	0.233	0.418	1.069	0.665	0.039
Skew		2.863	1.834	2.365	1.942	2.317	1.624	1.44	1.492	1.141	-0.008
Kurtosis		11.791	4.253	16.555	4.926	18.354	3.63	3.379	3.453	2.332	0.037

Condition 8; Scenario 2; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.087	-0.392	0.099	-0.367	-0.032	-0.183	-0.089	-0.174	-0.069	-0.145	-0.057	-0.137	-0.013	-0.022
	$\widehat{M}$	-0.009	-0.573	0.067	-0.564	-0.054	-0.32	-0.091	-0.191	-0.085	-0.172	-0.062	-0.158	-0.017	-0.033
	$\widehat{\sigma}^2$	0.381	1.151	0.134	0.939	0.099	0.69	0.011	0.023	0.017	0.035	0.011	0.028	0.07	0.08
	2.5%	-0.683	-1.87	-0.448	-1.614	-0.514	-1.322	-0.263	-0.41	-0.273	-0.428	-0.243	-0.403	-0.529	-0.563
	97.5%	1.949	2.898	0.926	2.69	0.6	2.409	0.145	0.248	0.297	0.339	0.202	0.272	0.52	0.535
P	$\widehat{\mu}$	0.001	-0.004	0.003	-0.013	-0.002	-0.017	-0.179	-0.349	-0.23	-0.483	-0.144	-0.341	-0.002	-0.003
	$\widehat{M}$	< .001	-0.006	0.002	-0.019	-0.004	-0.32	-0.183	-0.382	-0.283	-0.572	-0.156	-0.394	-0.002	-0.004
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	< .001	0.006	0.044	0.091	0.193	0.385	0.068	0.173	0.001	0.001
	2.5%	-0.007	-0.02	-0.013	-0.055	-0.034	-0.123	-0.526	-0.821	-0.912	-1.427	-0.608	-1.009	-0.066	-0.07
	97.5%	0.019	0.031	0.026	0.092	0.04	0.225	0.29	0.496	0.991	1.13	0.505	0.68	0.065	0.067
Skew		2.362	1.807	0.897	1.989	0.794	1.647	0.752	1.02	1.302	0.995	0.921	0.731	0.033	0.101
Kurtosis		9.034	4.512	7.488	5.322	10.755	3.883	3.971	1.756	4.449	1.518	2.869	0.809	0.071	-0.22

Condition 8; Scenario 2; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.081	-0.396	0.101	-0.379	-0.033	-0.187	-0.107	-0.086	-0.075	0.013
	$\widehat{M}$	-0.028	-0.572	0.06	-0.562	-0.06	-0.318	-0.118	-0.107	-0.083	0.016
	$\widehat{\sigma}^2$	0.407	1.085	0.146	0.904	0.104	0.671	0.011	0.018	0.013	0.022
	2.5%	-0.699	-1.849	-0.446	-1.641	-0.513	-1.338	-0.263	-0.282	-0.251	-0.271
	97.5%	2.018	2.808	0.954	2.678	0.625	2.382	0.191	0.295	0.224	0.313
P	$\widehat{\mu}$	0.001	-0.004	0.003	-0.013	-0.002	-0.017	-0.214	-0.287	-0.187	0.002
	$\widehat{M}$	< .001	-0.006	0.002	-0.019	-0.004	-0.03	-0.236	-0.357	-0.208	0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	< .001	0.006	0.043	0.199	0.08	< .001
	2.5%	-0.007	-0.02	-0.013	-0.056	-0.034	-0.125	-0.526	-0.939	-0.628	-0.034
	97.5%	0.02	0.03	0.027	0.091	0.042	0.222	0.382	0.982	0.56	0.039
Skew		2.563	1.69	1.992	1.934	1.436	0.006	1.463	1.525	1.117	-0.008
Kurtosis		9.789	3.77	11.089	5.131	14.755	3.902	3.801	3.873	2.505	0.038

Condition 8; Scenario 2; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.165	0.741	0.101	0.486	0.01	0.501	0.064	0.118	0.052	–
	$\widehat{M}$	0.134	0.573	0.095	0.295	0.008	0.382	0.058	0.107	0.046	–
	$\widehat{\sigma}^2$	0.149	1.051	0.081	0.828	0.073	0.562	0.003	0.006	0.005	–
	2.5%	-0.514	-0.633	-0.384	-0.702	-0.461	-0.535	-0.037	-0.002	-0.077	–
	97.5%	1.025	3.836	0.665	3.416	0.494	2.787	0.211	0.31	0.235	–
P	$\widehat{\mu}$	0.002	0.008	0.003	0.017	0.001	0.047	0.127	0.393	0.129	–
	$\widehat{M}$	0.001	0.006	0.003	0.01	0.001	0.036	0.117	0.358	0.117	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	< .001	0.005	0.014	0.064	0.033	–
	2.5%	-0.005	-0.007	-0.011	-0.024	-0.031	-0.05	-0.074	-0.007	-0.193	–
	97.5%	0.01	0.041	0.019	0.116	0.033	0.26	0.422	1.033	0.587	–
Skew		0.578	1.651	-0.187	1.778	-0.057	1.459	0.783	0.916	0.671	–
Kurtosis		1.597	3.271	3.481	3.915	9.047	2.687	1.259	1.429	0.977	–

TABLE C-24

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 8 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 8; Scenario 3; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	0.578	0.822	0.412	0.626	0.119	0.747	-0.048	-0.123	-0.015	-0.052	-0.019	-0.019	0.004	-0.019
	$\widehat{M}$	0.071	-0.035	0.117	-0.179	-0.062	0.2	-0.043	-0.142	-0.024	-0.07	-0.024	-0.024	-0.001	-0.021
B	$\widehat{\sigma}^2$	1.69	6.571	0.89	5.114	0.661	3.322	0.038	0.07	0.056	0.095	0.036	0.036	0.041	0.145
	2.5%	-0.716	-2.778	-0.54	-2.445	-0.947	-1.86	-0.419	-0.598	-0.605	-0.605	-0.393	-0.393	-0.4	-0.777
	97.5%	4.277	5.742	3.118	4.724	2.637	3.899	0.31	0.327	0.464	0.474	0.358	0.358	0.389	0.743
	$\widehat{\mu}$	0.006	0.009	0.012	0.021	0.008	0.07	-0.095	-0.246	-0.051	-0.173	-0.049	-0.049	< .001	-0.002
	$\widehat{M}$	0.001	< .001	0.003	-0.006	-0.004	0.2	-0.086	-0.284	-0.08	-0.234	-0.058	-0.058	< .001	-0.003
P	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.006	0.003	0.029	0.153	0.278	0.619	1.052	0.225	0.225	0.001	0.002
	2.5%	-0.007	-0.03	-0.015	-0.083	-0.063	-0.174	-0.84	-1.197	-2.017	-2.017	-0.982	-0.982	-0.05	-0.097
	97.5%	0.043	0.062	0.089	0.161	0.176	0.364	0.619	0.654	1.547	1.579	0.896	0.896	0.049	0.093
	Skew	1.867	0.502	2.703	0.472	2.748	0.344	-2.283	0.007	-0.903	-0.006	-1.975	-1.975	0.323	0.283
	Kurtosis	4.295	-0.912	11.002	-1.068	11.001	-1.162	13.914	-0.917	4.716	-0.965	11.703	11.703	4.156	2.406

Condition 8; Scenario 3; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.589	0.825	0.418	0.627	0.123	0.747	-0.015	0.03	0.011	0.005
	$\widehat{M}$	0.067	-0.019	0.112	-0.171	-0.067	0.198	-0.043	-0.014	-0.018	0.003
B	$\widehat{\sigma}^2$	1.738	6.463	0.913	5.056	0.669	3.281	0.02	0.034	0.022	0.021
	2.5%	-0.712	-2.745	-0.524	-2.428	-0.938	-1.852	-0.243	-0.24	-0.236	-0.285
	97.5%	4.347	5.719	3.143	4.716	2.649	3.887	0.305	0.445	0.34	0.3
	$\widehat{\mu}$	0.006	0.009	0.012	0.021	0.008	0.07	-0.03	0.099	0.028	0.001
	$\widehat{M}$	0.001	< .001	0.003	-0.006	-0.004	0.018	-0.085	-0.049	-0.045	< .001
P	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.006	0.003	0.029	0.079	0.373	0.136	< .001
	2.5%	-0.007	-0.03	-0.015	-0.083	-0.063	-0.173	-0.485	-0.799	-0.591	-0.036
	97.5%	0.043	0.062	0.09	0.161	0.176	0.363	0.611	1.485	0.849	0.037
	Skew	1.828	0.511	2.694	0.477	2.767	0.348	0.603	0.696	0.568	0.073
	Kurtosis	3.954	-0.892	10.918	-1.055	11.049	-1.151	0.005	-0.212	-0.022	-0.037

Condition 8; Scenario 3; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	0.493	0.809	0.333	0.677	0.047	0.747	-0.034	-0.119	-0.001	-0.055	-0.009	-0.079	-0.009	-0.015
	$\widehat{M}$	0.068	-0.035	0.123	-0.145	-0.057	0.159	-0.042	-0.148	-0.028	-0.086	-0.026	-0.102	-0.015	-0.021
B	$\widehat{\sigma}^2$	1.249	6.211	0.509	4.614	0.409	3.228	0.023	0.07	0.044	0.102	0.025	0.076	0.092	0.14
	2.5%	-0.697	-2.674	-0.533	-2.238	-1.062	-1.77	-0.342	-0.583	-0.386	-0.621	-0.271	-0.576	-0.64	-0.753
	97.5%	3.535	5.61	2.078	4.681	1.97	3.998	0.312	0.34	0.475	0.488	0.352	0.38	0.613	0.727
	$\widehat{\mu}$	0.005	0.009	0.01	0.023	0.003	0.07	-0.068	-0.237	-0.003	-0.183	-0.021	-0.198	-0.001	-0.002
	$\widehat{M}$	0.001	< .001	0.004	-0.005	-0.004	0.159	-0.084	-0.297	-0.092	-0.286	-0.065	-0.254	-0.002	-0.003
P	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.005	0.002	0.028	0.093	0.279	0.494	1.138	0.156	0.478	0.001	0.002
	2.5%	-0.007	-0.029	-0.015	-0.076	-0.071	-0.165	-0.683	-1.167	-1.288	-2.071	-0.678	-1.44	-0.08	-0.094
	97.5%	0.035	0.06	0.059	0.16	0.131	0.373	0.625	0.679	1.585	1.628	0.88	0.951	0.077	0.091
	Skew	1.57	0.52	1.513	0.559	1.862	0.4	-0.173	0.048	-0.084	0.015	-0.62	-0.047	0.451	0.234
	Kurtosis	2.489	-0.893	4.051	-0.978	6.736	-1.126	3.202	-0.944	2.781	-0.992	7.488	-0.927	5.451	2.094

Condition 8; Scenario 3; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.504	0.868	0.338	0.721	0.061	0.787	-0.015	0.029	0.011	0.005
	$\widehat{M}$	0.072	0.014	0.122	-0.112	-0.053	0.229	-0.05	-0.029	-0.024	0.004
B	$\widehat{\sigma}^2$	1.352	6.269	0.524	4.627	0.442	3.225	0.022	0.037	0.023	0.021
	2.5%	-0.793	-2.563	-0.576	-2.236	-1.05	-1.783	-0.248	-0.245	-0.236	-0.283
	97.5%	3.599	5.787	2.187	4.686	2.046	4.02	0.321	0.461	0.35	0.301
	$\widehat{\mu}$	0.005	0.009	0.01	0.025	0.004	0.073	-0.03	0.096	0.029	0.001
	$\widehat{M}$	0.001	< .001	0.004	-0.004	-0.004	0.021	-0.099	-0.094	-0.061	< .001
P	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.005	0.002	0.028	0.089	0.417	0.144	< .001
	2.5%	-0.008	-0.028	-0.016	-0.076	-0.07	-0.166	-0.495	-0.819	-0.591	-0.035
	97.5%	0.036	0.062	0.062	0.16	0.136	0.375	0.642	1.537	0.874	0.038
	Skew	1.476	0.545	1.322	0.529	2.102	0.028	0.71	0.723	0.601	0.074
	Kurtosis	2.222	-0.857	2.605	-1.005	8.12	-1.142	0.038	-0.371	-0.084	-0.058

Condition 8; Scenario 3; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.412	2.264	0.248	1.725	0.068	1.547	0.086	0.143	0.073	–
	$\widehat{M}$	0.287	1.731	0.194	1.178	0.046	1.2	0.079	0.131	0.063	–
B	$\widehat{\sigma}^2$	0.324	4.453	0.16	3.385	0.165	2.141	0.005	0.008	0.007	–
	2.5%	-0.456	-0.985	-0.39	-0.986	-0.752	-0.701	-0.031	0.001	-0.064	–
	97.5%	1.806	5.952	1.2	4.769	1.119	3.945	0.225	0.328	0.239	–
	$\widehat{\mu}$	0.004	0.024	0.007	0.059	0.005	0.144	0.171	0.476	0.182	–
	$\widehat{M}$	0.003	0.019	0.006	0.04	0.003	0.112	0.158	0.434	0.158	–
P	$\widehat{\sigma}^2$	< .001	0.001	< .001	0.004	0.001	0.019	0.018	0.085	0.042	–
	2.5%	-0.005	-0.011	-0.011	-0.034	-0.05	-0.065	-0.063	0.003	-0.159	–
	97.5%	0.018	0.064	0.034	0.163	0.075	0.368	0.45	1.094	0.598	–
	Skew	0.896	0.278	0.638	0.284	1.11	0.172	0.369	0.416	0.394	–
	Kurtosis	1.872	-1.122	2.387	-1.228	6.676	-1.214	-0.167	-0.339	-0.111	–

TABLE C-25

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 9 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 9; Scenario 1; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.382	-0.264	0.38	-0.258	0.17	-0.104	-0.125	-0.082	-0.114	-0.07	-0.09	-0.09	0.005	0.002
	$\widehat{M}$	0.383	-0.293	0.351	-0.301	0.174	-0.148	-0.135	-0.084	-0.13	-0.087	-0.087	-0.087	0.007	0.001
	$\widehat{\sigma}^2$	1.048	0.924	0.719	0.649	0.563	0.464	0.033	0.026	0.055	0.038	0.039	0.039	0.016	0.015
	2.5%	-1.628	-2.13	-1.249	-1.787	-1.277	-1.392	-0.481	-0.382	-0.53	-0.394	-0.46	-0.46	-0.239	-0.23
	97.5%	2.488	1.694	2.289	1.393	1.721	1.351	0.249	0.254	0.383	0.373	0.289	0.289	0.248	0.241
P	$\widehat{\mu}$	0.004	-0.003	0.011	-0.008	0.011	-0.009	-0.25	-0.163	-0.38	-0.232	-0.225	-0.225	0.001	0.001
	$\widehat{M}$	0.004	-0.003	0.01	-0.01	0.012	-0.148	-0.27	-0.167	-0.431	-0.291	-0.218	-0.218	0.002	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.003	0.003	0.133	0.104	0.613	0.425	0.241	0.241	0.001	0.001
	2.5%	-0.016	-0.023	-0.036	-0.058	-0.085	-0.117	-0.961	-0.763	-1.767	-1.314	-1.149	-1.149	-0.06	-0.057
	97.5%	0.025	0.018	0.065	0.045	0.115	0.113	0.499	0.508	1.277	1.244	0.722	0.722	0.062	0.06
Skew		0.137	0.168	0.29	0.078	0.717	0.279	0.114	0.103	0.22	0.49	< .001	< .001	-0.292	0.166
Kurtosis		1.874	1.111	1.178	0.998	5.357	0.899	0.179	0.071	-0.026	-0.046	-0.23	-0.23	2.616	0.393

Condition 9; Scenario 1; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.389	-0.265	0.384	-0.258	0.172	-0.105	-0.087	-0.075	-0.065	0.002
	$\widehat{M}$	0.387	-0.294	0.355	-0.302	0.179	-0.148	-0.101	-0.101	-0.069	0.003
	$\widehat{\sigma}^2$	1.067	0.919	0.728	0.647	0.569	0.463	0.023	0.037	0.028	0.007
	2.5%	-1.619	-2.117	-1.237	-1.785	-1.272	-1.389	-0.335	-0.381	-0.375	-0.16
	97.5%	2.513	1.703	2.302	1.391	1.731	1.347	0.243	0.369	0.278	0.169
P	$\widehat{\mu}$	0.004	-0.003	0.011	-0.008	0.011	-0.009	-0.174	-0.251	-0.163	0.001
	$\widehat{M}$	0.004	-0.003	0.01	-0.01	0.012	-0.012	-0.202	-0.336	-0.171	0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.003	0.003	0.09	0.412	0.176	< .001
	2.5%	-0.016	-0.022	-0.035	-0.058	-0.085	-0.116	-0.67	-1.27	-0.938	-0.04
	97.5%	0.025	0.018	0.066	0.045	0.115	0.113	0.486	1.229	0.695	0.042
Skew		0.156	0.177	0.301	0.082	0.73	0.283	0.464	0.567	0.23	0.008
Kurtosis		1.868	1.126	1.179	1.003	5.37	0.903	0.016	-0.041	-0.259	-0.298

Condition 9; Scenario 1; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.35	-0.323	0.314	-0.266	0.142	-0.136	-0.12	-0.085	-0.115	-0.079	-0.085	-0.065	0.01	0.001
	$\widehat{M}$	0.356	-0.335	0.305	-0.315	0.167	-0.169	-0.133	-0.089	-0.131	-0.101	-0.082	-0.072	0.01	-0.002
	$\widehat{\sigma}^2$	0.909	0.821	0.607	0.572	0.464	0.434	0.032	0.026	0.053	0.039	0.037	0.031	0.015	0.015
	2.5%	-1.491	-2.045	-1.181	-1.676	-1.26	-1.38	-0.456	-0.382	-0.513	-0.398	-0.456	-0.394	-0.236	-0.231
	97.5%	2.268	1.49	1.955	1.355	1.509	1.238	0.267	0.254	0.383	0.373	0.295	0.3	0.253	0.238
P	$\widehat{\mu}$	0.004	-0.003	0.009	-0.009	0.009	-0.011	-0.24	-0.169	-0.385	-0.262	-0.212	-0.162	0.003	< .001
	$\widehat{M}$	0.004	-0.004	0.009	-0.01	0.011	-0.169	-0.266	-0.178	-0.434	-0.338	-0.205	-0.179	0.003	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.002	0.003	0.127	0.103	0.592	0.434	0.231	0.191	0.001	0.001
	2.5%	-0.015	-0.022	-0.034	-0.054	-0.084	-0.116	-0.912	-0.763	-1.711	-1.327	-1.14	-0.985	-0.059	-0.058
	97.5%	0.023	0.016	0.056	0.044	0.101	0.104	0.535	0.508	1.278	1.244	0.738	0.75	0.063	0.059
Skew		0.024	0.253	0.197	0.32	0.139	0.318	0.238	0.137	0.374	0.529	0.031	0.147	0.043	0.139
Kurtosis		1.986	1.527	1.393	0.988	2.102	0.718	-0.125	0.076	-0.184	-0.013	-0.239	-0.229	0.269	0.354

Condition 9; Scenario 1; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.35	-0.338	0.316	-0.278	0.143	-0.142	-0.091	-0.086	-0.069	0.002
	$\widehat{M}$	0.36	-0.354	0.307	-0.318	0.17	-0.173	-0.105	-0.113	-0.075	0.003
	$\widehat{\sigma}^2$	0.912	0.8	0.642	0.586	0.475	0.441	0.022	0.037	0.028	0.007
	2.5%	-1.528	-2.049	-1.282	-1.755	-1.237	-1.394	-0.338	-0.386	-0.386	-0.16
	97.5%	2.29	1.462	1.988	1.305	1.566	1.245	0.241	0.361	0.265	0.169
P	$\widehat{\mu}$	0.004	-0.004	0.009	-0.009	0.009	-0.012	-0.181	-0.285	-0.172	0.001
	$\widehat{M}$	0.004	-0.004	0.009	-0.01	0.011	-0.014	-0.21	-0.377	-0.186	0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.002	0.003	0.09	0.411	0.173	< .001
	2.5%	-0.015	-0.022	-0.037	-0.057	-0.082	-0.117	-0.677	-1.287	-0.964	-0.04
	97.5%	0.023	0.015	0.057	0.042	0.104	0.104	0.482	1.203	0.663	0.042
Skew		0.071	0.094	0.1	0.26	0.117	0.003	0.479	0.627	0.206	0.008
Kurtosis		1.401	1.041	1.291	1.008	2.355	0.85	0.032	0.108	-0.247	-0.299

Condition 9; Scenario 1; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.062	0.136	0.101	0.041	-0.009	0.113	0.025	0.052	0.021	–
	$\widehat{M}$	0.12	0.084	0.102	-0.013	0.014	0.078	0.027	0.047	0.023	–
	$\widehat{\sigma}^2$	0.703	0.647	0.486	0.456	0.381	0.353	0.01	0.016	0.014	–
	2.5%	-1.659	-1.223	-1.28	-1.24	-1.311	-0.976	-0.162	-0.178	-0.21	–
	97.5%	1.667	1.8	1.547	1.487	1.17	1.409	0.226	0.334	0.253	–
P	$\widehat{\mu}$	0.001	0.001	0.003	0.001	-0.001	0.009	0.051	0.172	0.053	–
	$\widehat{M}$	0.001	0.001	0.003	< .001	0.001	0.007	0.054	0.155	0.057	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.002	0.002	0.038	0.18	0.085	–
	2.5%	-0.017	-0.013	-0.037	-0.04	-0.087	-0.082	-0.324	-0.593	-0.525	–
	97.5%	0.017	0.019	0.044	0.048	0.078	0.118	0.452	1.113	0.632	–
Skew		-0.305	0.4	0.014	0.327	-0.022	0.434	0.165	0.312	0.033	–
Kurtosis		1.363	1.294	0.862	0.794	1.17	0.781	0.12	0.057	-0.113	–

TABLE C-26

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 9 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 9; Scenario 2; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.456	0.129	0.392	0.071	0.178	0.175	-0.071	-0.089	-0.051	-0.063	-0.048	-0.048	0.004	0.007
	$\widehat{M}$	0.366	-0.003	0.325	-0.036	0.159	0.106	-0.071	-0.107	-0.059	-0.088	-0.051	-0.051	-0.001	-0.007
	$\widehat{\sigma}^2$	0.868	3.045	0.696	2.126	0.47	1.35	0.03	0.05	0.047	0.074	0.032	0.032	0.013	0.029
	2.5%	-1.076	-3.259	-1.072	-2.632	-1.183	-1.943	-0.407	-0.515	-0.442	-0.545	-0.395	-0.395	-0.202	-0.3
	97.5%	2.506	3.732	2.33	3.052	1.707	2.489	0.286	0.324	0.437	0.467	0.313	0.313	0.224	0.358
P	$\widehat{\mu}$	0.005	0.001	0.011	0.002	0.012	0.015	-0.142	-0.177	-0.17	-0.209	-0.119	-0.119	0.001	0.002
	$\widehat{M}$	0.004	< .001	0.009	-0.001	0.011	0.106	-0.142	-0.214	-0.198	-0.294	-0.128	-0.128	< .001	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.002	0.002	0.009	0.122	0.201	0.518	0.819	0.201	0.201	0.001	0.002
	2.5%	-0.011	-0.035	-0.031	-0.086	-0.079	-0.163	-0.813	-1.03	-1.474	-1.815	-0.988	-0.988	-0.05	-0.075
	97.5%	0.025	0.04	0.067	0.099	0.114	0.208	0.571	0.649	1.455	1.559	0.784	0.784	0.056	0.089
Skew		0.808	0.132	0.777	0.155	0.544	0.191	0.037	-0.044	0.232	0.152	-0.207	-0.207	0.253	0.293
Kurtosis		3.843	0.344	4.64	0.243	4.331	-0.198	0.097	-0.637	0.383	-0.555	1.004	1.004	0.719	1.4

Condition 9; Scenario 2; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.459	0.13	0.393	0.071	0.179	0.175	-0.06	-0.038	-0.034	0.004
	$\widehat{M}$	0.363	0.001	0.324	-0.035	0.16	0.101	-0.07	-0.06	-0.039	0.003
	$\widehat{\sigma}^2$	0.881	3.028	0.702	2.118	0.473	1.344	0.029	0.044	0.028	0.007
	2.5%	-1.079	-3.256	-1.06	-2.62	-1.183	-1.939	-0.361	-0.382	-0.342	-0.163
	97.5%	2.531	3.727	2.342	3.049	1.711	2.485	0.282	0.44	0.301	0.178
P	$\widehat{\mu}$	0.005	0.001	0.011	0.002	0.012	0.015	-0.121	-0.125	-0.086	0.001
	$\widehat{M}$	0.004	< .001	0.009	-0.001	0.011	0.008	-0.139	-0.199	-0.097	0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.002	0.002	0.009	0.116	0.491	0.178	< .001
	2.5%	-0.011	-0.034	-0.03	-0.085	-0.079	-0.162	-0.722	-1.275	-0.855	-0.041
	97.5%	0.025	0.039	0.067	0.099	0.114	0.208	0.564	1.466	0.752	0.044
Skew		0.83	0.135	0.797	0.157	0.565	0.194	0.25	0.416	0.158	0.146
Kurtosis		3.852	0.354	4.689	0.249	4.4	-0.193	-0.417	-0.24	-0.509	0.226

Condition 9; Scenario 2; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.473	0.219	0.393	0.174	0.188	0.214	-0.072	-0.08	-0.055	-0.064	-0.045	-0.049	< .001	0.006
	$\widehat{M}$	0.394	0.083	0.347	0.072	0.173	0.142	-0.077	-0.09	-0.069	-0.093	-0.053	-0.038	-0.005	-0.007
	$\widehat{\sigma}^2$	0.722	2.764	0.579	1.915	0.401	1.279	0.03	0.051	0.045	0.074	0.031	0.052	0.023	0.028
	2.5%	-1.135	-2.997	-1.072	-2.233	-1.094	-1.82	-0.396	-0.513	-0.44	-0.545	-0.389	-0.493	-0.288	-0.294
	97.5%	2.333	3.726	2.112	3.003	1.57	2.489	0.289	0.338	0.428	0.467	0.312	0.349	0.318	0.353
P	$\widehat{\mu}$	0.005	0.002	0.011	0.006	0.013	0.018	-0.145	-0.161	-0.185	-0.215	-0.112	-0.122	< .001	0.001
	$\widehat{M}$	0.004	0.001	0.01	0.002	0.012	0.142	-0.154	-0.18	-0.229	-0.309	-0.132	-0.096	-0.001	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.002	0.009	0.122	0.205	0.495	0.82	0.194	0.326	0.001	0.002
	2.5%	-0.011	-0.032	-0.031	-0.073	-0.073	-0.152	-0.793	-1.026	-1.468	-1.815	-0.972	-1.232	-0.072	-0.074
	97.5%	0.023	0.039	0.06	0.098	0.105	0.208	0.579	0.676	1.425	1.559	0.781	0.871	0.079	0.088
Skew		0.388	0.209	0.43	0.261	0.333	0.3	0.105	-0.029	0.311	0.166	-0.089	-0.182	0.168	0.32
Kurtosis		0.848	0.124	2.741	0.475	2.678	-0.061	0.055	-0.724	0.17	-0.624	0.522	-0.68	1.403	1.253

Condition 9; Scenario 2; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.449	0.181	0.372	0.15	0.173	0.193	-0.061	-0.044	-0.034	0.004
	$\widehat{M}$	0.389	0.076	0.333	0.046	0.172	0.124	-0.072	-0.071	-0.041	0.003
	$\widehat{\sigma}^2$	0.758	2.777	0.529	1.88	0.384	1.309	0.03	0.045	0.03	0.007
	2.5%	-1.191	-2.99	-1.025	-2.277	-1.183	-1.918	-0.361	-0.391	-0.349	-0.163
	97.5%	2.321	3.728	2.078	3.049	1.532	2.53	0.287	0.438	0.318	0.178
P	$\widehat{\mu}$	0.004	0.002	0.011	0.005	0.012	0.016	-0.122	-0.147	-0.085	0.001
	$\widehat{M}$	0.004	0.001	0.009	0.002	0.011	0.01	-0.145	-0.236	-0.103	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.002	0.009	0.122	0.504	0.19	< .001
	2.5%	-0.012	-0.032	-0.029	-0.074	-0.079	-0.161	-0.722	-1.303	-0.873	-0.041
	97.5%	0.023	0.039	0.059	0.099	0.102	0.212	0.574	1.459	0.794	0.044
Skew		0.076	0.182	0.183	0.25	0.072	0.009	0.28	0.465	0.198	0.143
Kurtosis		2.074	0.314	1.712	0.433	1.949	-0.054	-0.479	-0.252	-0.518	0.224

Condition 9; Scenario 2; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.332	0.837	0.258	0.611	0.11	0.558	0.033	0.066	0.034	–
	$\widehat{M}$	0.317	0.716	0.255	0.487	0.126	0.496	0.032	0.06	0.032	–
	$\widehat{\sigma}^2$	0.5	2.088	0.386	1.435	0.295	0.928	0.011	0.018	0.013	–
	2.5%	-0.977	-1.914	-1.028	-1.496	-1.026	-1.169	-0.16	-0.17	-0.178	–
	97.5%	1.757	3.824	1.581	3.089	1.259	2.48	0.243	0.363	0.257	–
P	$\widehat{\mu}$	0.003	0.009	0.007	0.02	0.007	0.047	0.065	0.22	0.085	–
	$\widehat{M}$	0.003	0.008	0.007	0.016	0.008	0.042	0.065	0.201	0.081	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.001	0.007	0.044	0.199	0.079	–
	2.5%	-0.01	-0.02	-0.029	-0.049	-0.068	-0.098	-0.32	-0.568	-0.444	–
	97.5%	0.018	0.04	0.045	0.1	0.084	0.208	0.487	1.211	0.643	–
Skew		0.045	0.183	-0.264	0.243	-0.192	0.221	0.169	0.33	0.116	–
Kurtosis		2.31	0.271	2.797	0.152	2.138	-0.125	-0.112	0.034	-0.378	–

TABLE C-27

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 9 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 9; Scenario 3; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.826	1.344	0.665	1.032	0.355	0.906	-0.057	-0.05	-0.034	-0.018	-0.036	-0.036	0.005	-0.002
	$\widehat{M}$	0.67	1.469	0.495	1.116	0.23	0.958	-0.058	-0.02	-0.031	0.017	-0.035	-0.035	0.001	-0.009
	$\widehat{\sigma}^2$	1.334	5.038	1.064	3.295	0.788	2.233	0.04	0.06	0.063	0.09	0.042	0.042	0.014	0.034
	2.5%	-1.033	-3.485	-0.937	-2.618	-1.341	-2.068	-0.477	-0.561	-0.564	-0.593	-0.472	-0.472	-0.216	-0.377
	97.5%	3.639	4.938	3.194	3.94	2.422	3.405	0.332	0.325	0.444	0.478	0.351	0.351	0.249	0.378
P	$\widehat{\mu}$	0.008	0.014	0.019	0.034	0.024	0.076	-0.113	-0.099	-0.114	-0.059	-0.09	-0.09	0.001	< .001
	$\widehat{M}$	0.007	0.016	0.014	0.036	0.015	0.958	-0.117	-0.04	-0.102	0.057	-0.086	-0.086	< .001	-0.002
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.003	0.004	0.016	0.16	0.242	0.696	1.001	0.265	0.265	0.001	0.002
	2.5%	-0.01	-0.037	-0.027	-0.085	-0.089	-0.173	-0.954	-1.121	-1.88	-1.978	-1.181	-1.181	-0.054	-0.094
	97.5%	0.036	0.052	0.091	0.128	0.161	0.285	0.663	0.651	1.48	1.594	0.876	0.876	0.062	0.095
Skew		1.107	-0.496	1.093	-0.318	0.955	-0.277	-0.583	-0.756	-0.488	-0.415	-0.373	-0.373	0.484	0.37
Kurtosis		3.574	0.15	4.751	-0.185	4.088	-0.167	2.562	1.063	1.382	-0.245	0.809	0.809	2.481	3.588

Condition 9; Scenario 3; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.832	1.343	0.669	1.031	0.357	0.905	-0.028	-0.003	-0.015	0.002
	$\widehat{M}$	0.674	1.479	0.492	1.117	0.23	0.956	-0.035	-0.004	-0.011	0.001
	$\widehat{\sigma}^2$	1.351	5.014	1.071	3.284	0.792	2.222	0.028	0.049	0.036	0.008
	2.5%	-1.021	-3.482	-0.943	-2.611	-1.338	-2.065	-0.346	-0.432	-0.411	-0.159
	97.5%	3.69	4.937	3.207	3.938	2.433	3.411	0.302	0.429	0.337	0.18
P	$\widehat{\mu}$	0.008	0.014	0.019	0.034	0.024	0.076	-0.057	-0.011	-0.038	< .001
	$\widehat{M}$	0.007	0.016	0.014	0.036	0.015	0.08	-0.07	-0.011	-0.029	< .001
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.003	0.004	0.016	0.113	0.548	0.225	< .001
	2.5%	-0.01	-0.037	-0.027	-0.085	-0.089	-0.173	-0.692	-1.44	-1.028	-0.04
	97.5%	0.037	0.052	0.092	0.128	0.162	0.286	0.603	1.431	0.842	0.045
Skew		1.122	-0.493	1.106	-0.316	0.973	-0.275	0.106	0.047	-0.166	0.122
Kurtosis		3.602	0.156	4.77	-0.183	4.145	-0.162	-0.412	-0.581	-0.272	-0.092

Condition 9; Scenario 3; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.779	1.566	0.646	1.222	0.329	1.021	-0.048	-0.032	-0.024	0.001	-0.031	-0.022	-0.003	-0.002
	$\widehat{M}$	0.7	1.799	0.536	1.356	0.251	1.038	-0.052	-0.002	-0.035	0.028	-0.032	0.019	-0.007	-0.009
	$\widehat{\sigma}^2$	0.982	4.355	0.792	2.771	0.741	2.025	0.037	0.057	0.06	0.09	0.042	0.068	0.024	0.029
	2.5%	-1.094	-2.79	-0.929	-2.186	-1.448	-1.698	-0.445	-0.553	-0.508	-0.581	-0.463	-0.561	-0.309	-0.336
	97.5%	2.989	5.052	2.626	4.121	2.174	3.493	0.331	0.341	0.454	0.488	0.351	0.384	0.311	0.358
P	$\widehat{\mu}$	0.008	0.017	0.018	0.04	0.022	0.086	-0.097	-0.064	-0.079	0.004	-0.079	-0.056	-0.001	-0.001
	$\widehat{M}$	0.007	0.019	0.015	0.044	0.017	1.038	-0.103	-0.004	-0.117	0.093	-0.08	0.049	-0.002	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.003	0.014	0.147	0.229	0.67	1.004	0.265	0.425	0.001	0.002
	2.5%	-0.011	-0.03	-0.027	-0.071	-0.096	-0.142	-0.89	-1.106	-1.693	-1.937	-1.157	-1.402	-0.077	-0.084
	97.5%	0.03	0.053	0.075	0.134	0.145	0.293	0.662	0.683	1.512	1.627	0.877	0.96	0.078	0.09
Skew		0.323	-0.484	0.624	-0.22	0.584	-0.19	-0.214	-0.571	-0.15	-0.347	-0.278	-0.449	0.135	0.177
Kurtosis		0.97	0.378	3.892	-0.261	3.273	-0.174	0.615	-0.064	0.333	-0.592	0.263	-0.48	3.023	2.369

Condition 9; Scenario 3; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.799	1.583	0.644	1.25	0.329	1.021	-0.022	0.006	-0.011	0.002
	$\widehat{M}$	0.721	1.797	0.539	1.349	0.258	1.054	-0.032	0.002	-0.008	0.001
	$\widehat{\sigma}^2$	1.044	4.317	0.786	2.778	0.688	1.998	0.032	0.054	0.039	0.008
	2.5%	-1.158	-2.79	-0.964	-2.262	-1.397	-1.805	-0.362	-0.432	-0.41	-0.159
	97.5%	2.96	5.032	2.637	4.145	2.145	3.422	0.316	0.45	0.348	0.18
P	$\widehat{\mu}$	0.008	0.017	0.018	0.041	0.022	0.086	-0.044	0.019	-0.028	< .001
	$\widehat{M}$	0.007	0.019	0.015	0.044	0.017	0.088	-0.064	0.006	-0.019	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.003	0.014	0.128	0.604	0.243	< .001
	2.5%	-0.012	-0.03	-0.028	-0.074	-0.093	-0.151	-0.725	-1.44	-1.026	-0.04
	97.5%	0.03	0.053	0.075	0.135	0.143	0.287	0.632	1.499	0.87	0.045
Skew		0.371	-0.492	0.347	-0.242	0.154	0.014	0.043	0.053	-0.161	0.121
Kurtosis		2.352	0.386	2.163	-0.201	0.918	-0.082	-0.495	-0.698	-0.407	-0.095

Condition 9; Scenario 3; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.643	2.083	0.491	1.59	0.265	1.298	0.039	0.074	0.034	–
	$\widehat{M}$	0.583	2.258	0.454	1.695	0.227	1.357	0.034	0.077	0.034	–
	$\widehat{\sigma}^2$	0.62	3.272	0.535	2.151	0.452	1.469	0.01	0.018	0.015	–
	2.5%	-0.861	-1.874	-0.866	-1.503	-1.121	-1.137	-0.148	-0.178	-0.205	–
	97.5%	2.258	4.943	2.072	3.987	1.697	3.415	0.25	0.356	0.272	–
P	$\widehat{\mu}$	0.006	0.022	0.014	0.052	0.018	0.109	0.079	0.248	0.086	–
	$\widehat{M}$	0.006	0.024	0.013	0.055	0.015	0.114	0.069	0.257	0.085	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.002	0.01	0.041	0.202	0.094	–
	2.5%	-0.009	-0.02	-0.025	-0.049	-0.075	-0.095	-0.297	-0.594	-0.512	–
	97.5%	0.023	0.052	0.059	0.13	0.113	0.286	0.5	1.188	0.679	–
Skew		0.17	-0.601	0.259	-0.395	0.324	-0.33	0.139	0.081	-0.111	–
Kurtosis		1.552	0.569	4.031	0.019	1.882	0.1	0.001	-0.191	-0.001	–

TABLE C-28

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 10 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 10; Scenario 1; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.537	-0.346	0.443	-0.268	0.233	-0.156	-0.182	-0.123	-0.175	-0.122	-0.129	-0.129	0.004	< .001
	$\widehat{M}$	0.551	-0.357	0.431	-0.269	0.223	-0.157	-0.194	-0.132	-0.197	-0.139	-0.139	-0.139	0.007	-0.001
	$\widehat{\sigma}^2$	0.492	0.401	0.334	0.325	0.228	0.195	0.018	0.012	0.027	0.018	0.018	0.018	0.006	0.006
	2.5%	-0.994	-1.608	-0.741	-1.441	-0.758	-1.005	-0.407	-0.315	-0.438	-0.342	-0.388	-0.388	-0.147	-0.159
	97.5%	1.969	0.958	1.562	0.863	1.165	0.734	0.122	0.12	0.204	0.197	0.157	0.157	0.158	0.141
P	$\widehat{\mu}$	0.005	-0.004	0.013	-0.009	0.016	-0.013	-0.364	-0.246	-0.583	-0.407	-0.322	-0.322	0.001	< .001
	$\widehat{M}$	0.006	-0.004	0.012	-0.009	0.015	-0.157	-0.388	-0.265	-0.655	-0.462	-0.347	-0.347	0.002	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.001	0.072	0.05	0.297	0.2	0.112	0.112	< .001	< .001
	2.5%	-0.01	-0.017	-0.021	-0.047	-0.051	-0.084	-0.813	-0.63	-1.46	-1.139	-0.971	-0.971	-0.037	-0.04
	97.5%	0.02	0.01	0.045	0.028	0.078	0.061	0.244	0.241	0.681	0.656	0.391	0.391	0.039	0.035
Skew		-0.168	0.138	0.142	-0.278	-0.033	0.091	0.67	0.55	0.675	0.811	0.228	0.228	0.072	-0.05
Kurtosis		1.421	1.255	1.822	2.715	1.256	1.305	0.989	0.774	0.989	1.304	0.331	0.331	-0.004	0.237

Condition 10; Scenario 1; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.544	-0.347	0.447	-0.268	0.235	-0.157	-0.137	-0.134	-0.102	0.001
	$\widehat{M}$	0.558	-0.362	0.434	-0.271	0.226	-0.156	-0.156	-0.155	-0.106	-0.001
	$\widehat{\sigma}^2$	0.504	0.398	0.339	0.323	0.231	0.194	0.011	0.018	0.013	0.003
	2.5%	-1	-1.601	-0.744	-1.438	-0.752	-1.004	-0.299	-0.332	-0.3	-0.097
	97.5%	1.994	0.951	1.578	0.86	1.18	0.734	0.117	0.191	0.15	0.112
P	$\widehat{\mu}$	0.005	-0.004	0.013	-0.009	0.016	-0.013	-0.274	-0.447	-0.255	< .001
	$\widehat{M}$	0.006	-0.004	0.012	-0.009	0.015	-0.013	-0.311	-0.518	-0.266	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.001	0.046	0.2	0.078	< .001
	2.5%	-0.01	-0.017	-0.021	-0.047	-0.05	-0.084	-0.597	-1.108	-0.749	-0.024
	97.5%	0.02	0.01	0.045	0.028	0.079	0.061	0.235	0.639	0.376	0.028
Skew		-0.148	0.148	0.155	-0.272	-0.02	0.095	0.997	0.983	0.451	0.193
Kurtosis		1.401	1.266	1.817	2.715	1.255	1.309	1.492	1.399	0.517	0.035

Condition 10; Scenario 1; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.505	-0.419	0.377	-0.283	0.195	-0.188	-0.177	-0.127	-0.171	-0.132	-0.116	-0.097	0.002	-0.001
	$\widehat{M}$	0.516	-0.417	0.39	-0.288	0.198	-0.185	-0.192	-0.141	-0.189	-0.147	-0.119	-0.097	0.002	-0.003
	$\widehat{\sigma}^2$	0.388	0.308	0.251	0.25	0.165	0.181	0.016	0.012	0.023	0.018	0.015	0.013	0.006	0.006
	2.5%	-0.761	-1.548	-0.655	-1.259	-0.604	-1.022	-0.38	-0.313	-0.409	-0.345	-0.341	-0.305	-0.152	-0.154
	97.5%	1.675	0.688	1.348	0.742	0.969	0.655	0.114	0.12	0.184	0.158	0.144	0.135	0.168	0.143
P	$\widehat{\mu}$	0.005	-0.004	0.011	-0.009	0.013	-0.016	-0.353	-0.255	-0.569	-0.439	-0.29	-0.241	0.001	< .001
	$\widehat{M}$	0.005	-0.004	0.011	-0.009	0.013	-0.185	-0.384	-0.281	-0.628	-0.489	-0.297	-0.24	0.001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.001	0.066	0.049	0.252	0.195	0.096	0.083	< .001	< .001
	2.5%	-0.008	-0.016	-0.019	-0.041	-0.04	-0.086	-0.76	-0.626	-1.362	-1.151	-0.852	-0.763	-0.038	-0.038
	97.5%	0.017	0.007	0.039	0.024	0.065	0.055	0.229	0.241	0.614	0.529	0.359	0.337	0.042	0.036
Skew		-0.287	0.133	-0.261	0.137	-0.147	0.085	0.739	0.613	0.776	0.795	0.233	0.138	0.038	-0.033
Kurtosis		1.221	1.443	1.15	1.322	0.842	1.243	1.038	0.946	0.988	1.234	0.132	0.169	0.066	0.194

Condition 10; Scenario 1; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.536	-0.407	0.395	-0.273	0.206	-0.176	-0.141	-0.141	-0.1	0.001
	$\widehat{M}$	0.534	-0.409	0.397	-0.283	0.202	-0.171	-0.161	-0.162	-0.101	-0.001
	$\widehat{\sigma}^2$	0.424	0.332	0.276	0.28	0.185	0.183	0.011	0.017	0.012	0.003
	2.5%	-0.757	-1.54	-0.654	-1.27	-0.648	-0.975	-0.299	-0.337	-0.298	-0.097
	97.5%	1.882	0.748	1.467	0.778	1.043	0.664	0.126	0.169	0.141	0.112
P	$\widehat{\mu}$	0.005	-0.004	0.011	-0.009	0.014	-0.015	-0.281	-0.469	-0.249	< .001
	$\widehat{M}$	0.005	-0.004	0.011	-0.009	0.013	-0.014	-0.322	-0.54	-0.252	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.001	0.045	0.19	0.074	< .001
	2.5%	-0.008	-0.016	-0.019	-0.041	-0.043	-0.082	-0.598	-1.122	-0.745	-0.024
	97.5%	0.019	0.008	0.042	0.025	0.07	0.056	0.253	0.564	0.353	0.028
Skew		-0.085	0.191	-0.034	0.055	-0.023	0.001	1.006	0.965	0.328	0.195
Kurtosis		1.193	1.365	1.248	1.929	1.06	1.151	1.484	1.488	0.347	0.033

Condition 10; Scenario 1; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.103	0.154	0.086	0.102	-0.004	0.118	0.02	0.042	0.016	–
	$\widehat{M}$	0.143	0.123	0.097	0.089	0.013	0.109	0.018	0.039	0.015	–
	$\widehat{\sigma}^2$	0.3	0.254	0.207	0.196	0.15	0.13	0.004	0.007	0.005	–
	2.5%	-1.175	-0.755	-0.925	-0.728	-0.873	-0.539	-0.102	-0.103	-0.124	–
	97.5%	1.073	1.24	0.912	1.026	0.695	0.928	0.151	0.217	0.162	–
P	$\widehat{\mu}$	0.001	0.002	0.002	0.003	< .001	0.01	0.041	0.14	0.041	–
	$\widehat{M}$	0.001	0.001	0.003	0.003	0.001	0.009	0.036	0.13	0.037	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.001	0.017	0.073	0.032	–
	2.5%	-0.012	-0.008	-0.026	-0.024	-0.058	-0.045	-0.204	-0.344	-0.311	–
	97.5%	0.011	0.013	0.026	0.033	0.046	0.078	0.302	0.722	0.406	–
Skew		-0.667	0.505	-0.321	0.404	-0.381	0.387	0.315	0.479	0.137	–
Kurtosis		1.408	1.005	1.353	1.102	0.885	1.101	0.562	0.684	0.458	–

TABLE C-29

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 10 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 10; Scenario 2; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.445	-0.292	0.381	-0.251	0.177	-0.09	-0.107	-0.181	-0.093	-0.161	-0.078	-0.078	0.004	-0.002
	$\widehat{M}$	0.308	-0.534	0.272	-0.411	0.113	-0.23	-0.105	-0.202	-0.096	-0.197	-0.076	-0.076	0.003	0.001
	$\widehat{\sigma}^2$	0.566	1.8	0.394	1.155	0.243	0.786	0.014	0.028	0.025	0.045	0.015	0.015	0.005	0.011
	2.5%	-0.586	-2.391	-0.455	-1.909	-0.546	-1.453	-0.332	-0.464	-0.377	-0.508	-0.327	-0.327	-0.134	-0.207
	97.5%	2.585	3.027	1.918	2.387	1.494	2.05	0.15	0.195	0.268	0.325	0.174	0.174	0.132	0.199
P	$\widehat{\mu}$	0.004	-0.003	0.011	-0.008	0.012	-0.008	-0.214	-0.362	-0.311	-0.538	-0.196	-0.196	0.001	< .001
	$\widehat{M}$	0.003	-0.006	0.008	-0.013	0.007	-0.23	-0.21	-0.404	-0.32	-0.658	-0.191	-0.191	0.001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.006	0.055	0.114	0.275	0.496	0.097	0.097	< .001	0.001
	2.5%	-0.006	-0.025	-0.013	-0.062	-0.036	-0.122	-0.665	-0.928	-1.256	-1.692	-0.819	-0.819	-0.034	-0.052
	97.5%	0.026	0.032	0.055	0.078	0.1	0.172	0.301	0.389	0.895	1.085	0.434	0.434	0.033	0.05
Skew		1.844	0.835	2.033	0.806	1.315	0.772	-0.119	0.518	0.244	0.622	-0.094	-0.094	-0.131	-0.048
Kurtosis		5.602	0.75	9.738	0.806	7.068	0.596	3.184	0.065	2.55	0.031	1.438	1.438	0.374	0.497

Condition 10; Scenario 2; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.444	-0.288	0.381	-0.249	0.176	-0.088	-0.11	-0.095	-0.081	0.003
	$\widehat{M}$	0.302	-0.533	0.269	-0.406	0.11	-0.228	-0.123	-0.115	-0.089	0.002
	$\widehat{\sigma}^2$	0.58	1.785	0.401	1.149	0.246	0.781	0.013	0.024	0.016	0.003
	2.5%	-0.593	-2.38	-0.46	-1.903	-0.547	-1.447	-0.296	-0.333	-0.306	-0.105
	97.5%	2.625	3.024	1.928	2.383	1.504	2.045	0.179	0.29	0.194	0.104
P	$\widehat{\mu}$	0.004	-0.003	0.011	-0.008	0.012	-0.007	-0.219	-0.316	-0.202	0.001
	$\widehat{M}$	0.003	-0.006	0.008	-0.013	0.007	-0.019	-0.245	-0.383	-0.221	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.005	0.05	0.263	0.101	< .001
	2.5%	-0.006	-0.025	-0.013	-0.062	-0.036	-0.121	-0.593	-1.111	-0.764	-0.026
	97.5%	0.026	0.032	0.055	0.077	0.1	0.171	0.358	0.968	0.486	0.026
Skew		1.863	0.838	2.05	0.807	1.351	0.773	0.905	0.943	0.455	-0.043
Kurtosis		5.617	0.762	9.76	0.816	7.146	0.605	1.482	1.151	0.429	-0.265

Condition 10; Scenario 2; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.401	-0.321	0.343	-0.245	0.162	-0.121	-0.105	-0.179	-0.095	-0.17	-0.077	-0.137	-0.004	-0.002
	$\widehat{M}$	0.32	-0.528	0.266	-0.388	0.123	-0.247	-0.106	-0.2	-0.106	-0.206	-0.077	-0.15	-0.003	-0.002
	$\widehat{\sigma}^2$	0.38	1.525	0.277	0.973	0.187	0.712	0.012	0.027	0.023	0.046	0.015	0.032	0.009	0.011
	2.5%	-0.518	-2.193	-0.436	-1.783	-0.53	-1.379	-0.311	-0.462	-0.361	-0.512	-0.31	-0.455	-0.189	-0.207
	97.5%	1.829	2.806	1.596	2.25	1.162	2.033	0.161	0.198	0.269	0.346	0.172	0.239	0.182	0.192
P	$\widehat{\mu}$	0.004	-0.003	0.01	-0.008	0.011	-0.01	-0.209	-0.359	-0.318	-0.568	-0.194	-0.342	-0.001	-0.001
	$\widehat{M}$	0.003	-0.006	0.008	-0.013	0.008	-0.247	-0.212	-0.399	-0.354	-0.688	-0.194	-0.376	-0.001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.005	0.048	0.11	0.253	0.511	0.091	0.202	0.001	0.001
	2.5%	-0.005	-0.023	-0.012	-0.058	-0.035	-0.115	-0.622	-0.925	-1.202	-1.707	-0.775	-1.138	-0.047	-0.052
	97.5%	0.018	0.03	0.046	0.073	0.077	0.17	0.322	0.396	0.896	1.153	0.43	0.597	0.046	0.048
Skew		1.179	0.938	1.483	0.813	0.624	0.874	0.394	0.556	0.803	0.755	0.203	0.344	-0.043	-0.019
Kurtosis		5.095	1.421	5.975	1.126	5.459	1.017	2.226	0.196	1.823	0.266	1.221	-0.155	0.881	0.42

Condition 10; Scenario 2; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.39	-0.307	0.339	-0.254	0.144	-0.108	-0.11	-0.099	-0.082	0.003
	$\widehat{M}$	0.301	-0.524	0.262	-0.391	0.115	-0.249	-0.124	-0.127	-0.092	0.002
	$\widehat{\sigma}^2$	0.403	1.58	0.273	1.021	0.173	0.754	0.013	0.025	0.017	0.003
	2.5%	-0.589	-2.181	-0.451	-1.835	-0.547	-1.385	-0.296	-0.336	-0.306	-0.105
	97.5%	1.964	2.84	1.565	2.278	1.157	2.041	0.197	0.331	0.213	0.104
P	$\widehat{\mu}$	0.004	-0.003	0.01	-0.008	0.01	-0.009	-0.221	-0.329	-0.206	0.001
	$\widehat{M}$	0.003	-0.006	0.007	-0.013	0.008	-0.021	-0.248	-0.423	-0.231	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.005	0.053	0.282	0.104	< .001
	2.5%	-0.006	-0.023	-0.013	-0.06	-0.036	-0.116	-0.593	-1.12	-0.766	-0.026
	97.5%	0.02	0.03	0.045	0.074	0.077	0.171	0.394	1.103	0.533	0.026
Skew		1.172	0.879	1.133	0.787	0.252	0.005	1.04	1.065	0.532	-0.041
Kurtosis		4.093	1.122	3.16	0.992	4.622	0.899	1.885	1.306	0.467	-0.264

Condition 10; Scenario 2; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.265	0.744	0.215	0.531	0.087	0.469	0.026	0.053	0.02	–
	$\widehat{M}$	0.257	0.536	0.197	0.387	0.078	0.34	0.023	0.047	0.014	–
	$\widehat{\sigma}^2$	0.175	1.146	0.141	0.742	0.109	0.494	0.004	0.007	0.006	–
	2.5%	-0.519	-0.864	-0.438	-0.796	-0.537	-0.629	-0.084	-0.09	-0.121	–
	97.5%	1.155	3.218	0.957	2.534	0.778	2.098	0.17	0.243	0.175	–
P	$\widehat{\mu}$	0.003	0.008	0.006	0.017	0.006	0.039	0.052	0.177	0.05	–
	$\widehat{M}$	0.003	0.006	0.006	0.013	0.005	0.029	0.046	0.156	0.036	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	< .001	0.003	0.016	0.08	0.035	–
	2.5%	-0.005	-0.009	-0.012	-0.026	-0.036	-0.053	-0.168	-0.301	-0.302	–
	97.5%	0.012	0.034	0.027	0.082	0.052	0.176	0.339	0.811	0.437	–
Skew		0.168	0.799	0.379	0.8	-0.279	0.739	0.428	0.542	0.356	–
Kurtosis		0.953	0.389	2.304	0.448	4.529	0.428	0.544	0.674	0.359	–

TABLE C-30

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 10 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 10; Scenario 3; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.915	0.956	0.751	0.67	0.4	0.731	-0.102	-0.139	-0.07	-0.08	-0.065	-0.065	0.001	-0.011
	$\widehat{M}$	0.574	0.763	0.497	0.483	0.195	0.772	-0.099	-0.141	-0.059	-0.058	-0.048	-0.048	0.001	-0.009
	$\widehat{\sigma}^2$	1.442	4.748	0.967	3.005	0.756	1.861	0.034	0.056	0.046	0.068	0.036	0.036	0.006	0.013
	2.5%	-0.534	-3.023	-0.422	-2.515	-0.821	-1.981	-0.524	-0.575	-0.562	-0.572	-0.489	-0.489	-0.146	-0.264
	97.5%	4.149	4.677	3.334	3.541	2.729	2.973	0.266	0.276	0.366	0.386	0.283	0.283	0.155	0.215
P	$\widehat{\mu}$	0.009	0.01	0.021	0.022	0.027	0.061	-0.205	-0.279	-0.233	-0.266	-0.163	-0.163	< .001	-0.003
	$\widehat{M}$	0.006	0.008	0.014	0.016	0.013	0.772	-0.198	-0.283	-0.198	-0.196	-0.119	-0.119	< .001	-0.002
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.003	0.003	0.013	0.136	0.224	0.516	0.76	0.224	0.224	< .001	0.001
	2.5%	-0.005	-0.032	-0.012	-0.082	-0.055	-0.166	-1.049	-1.15	-1.874	-1.906	-1.224	-1.224	-0.037	-0.066
	97.5%	0.041	0.05	0.095	0.115	0.182	0.249	0.531	0.552	1.222	1.287	0.709	0.709	0.039	0.054
Skew		1.531	0.022	1.741	-0.082	1.873	-0.346	-0.804	-0.052	-0.811	-0.13	-1.346	-1.346	-0.284	-0.322
Kurtosis		3.114	-0.895	5.287	-0.769	6.339	0.273	3.798	-0.861	3.553	-0.68	7.279	7.279	3.379	1.842

Condition 10; Scenario 3; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.921	0.957	0.755	0.67	0.403	0.731	-0.07	-0.03	-0.036	-0.002
	$\widehat{M}$	0.575	0.748	0.495	0.482	0.195	0.772	-0.087	-0.041	-0.035	-0.002
	$\widehat{\sigma}^2$	1.466	4.717	0.978	2.991	0.763	1.85	0.022	0.029	0.022	0.003
	2.5%	-0.524	-3.017	-0.418	-2.501	-0.819	-1.977	-0.333	-0.337	-0.338	-0.105
	97.5%	4.176	4.674	3.357	3.54	2.734	2.972	0.247	0.352	0.278	0.099
P	$\widehat{\mu}$	0.009	0.01	0.022	0.022	0.027	0.061	-0.14	-0.102	-0.089	< .001
	$\widehat{M}$	0.006	0.008	0.014	0.016	0.013	0.065	-0.174	-0.136	-0.086	< .001
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.003	0.003	0.013	0.087	0.32	0.137	< .001
	2.5%	-0.005	-0.032	-0.012	-0.081	-0.054	-0.166	-0.667	-1.125	-0.844	-0.026
	97.5%	0.042	0.049	0.096	0.115	0.182	0.249	0.495	1.175	0.696	0.025
Skew		1.527	0.024	1.738	-0.081	1.882	-0.345	0.403	0.338	0.031	0.006
Kurtosis		3.039	-0.89	5.216	-0.764	6.339	0.284	-0.175	-0.019	0.204	-0.322

Condition 10; Scenario 3; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.8	1.065	0.684	0.771	0.316	0.777	-0.086	-0.121	-0.049	-0.074	-0.052	-0.07	-0.003	-0.009
	$\widehat{M}$	0.604	0.962	0.51	0.731	0.197	0.768	-0.092	-0.124	-0.061	-0.057	-0.045	-0.061	-0.001	-0.01
	$\widehat{\sigma}^2$	0.881	4.31	0.629	2.545	0.548	1.776	0.027	0.056	0.036	0.075	0.029	0.052	0.008	0.012
	2.5%	-0.565	-2.72	-0.413	-2.204	-1.159	-1.776	-0.4	-0.557	-0.429	-0.572	-0.421	-0.522	-0.165	-0.217
	97.5%	3.228	4.72	2.8	3.512	2.129	3.129	0.27	0.292	0.386	0.404	0.297	0.319	0.166	0.215
P	$\widehat{\mu}$	0.008	0.011	0.02	0.025	0.021	0.065	-0.172	-0.242	-0.165	-0.245	-0.13	-0.175	-0.001	-0.002
	$\widehat{M}$	0.006	0.01	0.015	0.024	0.013	0.768	-0.185	-0.248	-0.202	-0.188	-0.114	-0.154	< .001	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.002	0.012	0.109	0.223	0.402	0.836	0.182	0.322	< .001	0.001
	2.5%	-0.006	-0.029	-0.012	-0.072	-0.077	-0.149	-0.8	-1.114	-1.431	-1.906	-1.052	-1.306	-0.041	-0.054
	97.5%	0.032	0.05	0.08	0.114	0.142	0.262	0.539	0.584	1.285	1.347	0.742	0.798	0.041	0.054
Skew		0.93	-0.006	1.02	-0.094	0.986	-0.248	-0.232	-0.062	0.268	-0.1	-0.253	-0.232	-0.356	-0.191
Kurtosis		1.488	-0.787	1.95	-0.615	3.582	0.325	2.217	-0.902	0.574	-0.852	1.107	-0.657	2.473	1.455

Condition 10; Scenario 3; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.826	1.17	0.679	0.845	0.316	0.86	-0.06	-0.018	-0.031	-0.002
	$\widehat{M}$	0.596	1.151	0.52	0.836	0.199	0.853	-0.085	-0.037	-0.031	-0.002
	$\widehat{\sigma}^2$	0.962	4.148	0.637	2.62	0.577	1.695	0.026	0.036	0.026	0.003
	2.5%	-0.538	-2.576	-0.515	-2.17	-1.209	-1.651	-0.336	-0.346	-0.349	-0.105
	97.5%	3.348	4.648	2.648	3.688	2.208	3.224	0.29	0.387	0.298	0.099
P	$\widehat{\mu}$	0.008	0.012	0.019	0.027	0.021	0.072	-0.12	-0.061	-0.078	< .001
	$\widehat{M}$	0.006	0.012	0.015	0.027	0.013	0.071	-0.172	-0.123	-0.078	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.003	0.012	0.104	0.398	0.161	< .001
	2.5%	-0.005	-0.027	-0.015	-0.071	-0.081	-0.138	-0.672	-1.152	-0.872	-0.026
	97.5%	0.033	0.049	0.076	0.12	0.147	0.27	0.58	1.289	0.745	0.025
Skew		1.068	-0.041	0.837	-0.071	0.853	0.012	0.441	0.361	0.041	0.001
Kurtosis		1.876	-0.802	1.661	-0.653	2.915	-0.462	-0.403	-0.282	-0.073	-0.317

Condition 10; Scenario 3; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.547	2.12	0.434	1.562	0.222	1.317	0.032	0.068	0.031	–
	$\widehat{M}$	0.504	2.078	0.371	1.516	0.169	1.394	0.028	0.066	0.032	–
	$\widehat{\sigma}^2$	0.305	2.541	0.219	1.592	0.218	0.992	0.005	0.007	0.006	–
	2.5%	-0.405	-0.962	-0.34	-0.912	-0.701	-0.69	-0.098	-0.093	-0.119	–
	97.5%	1.77	4.742	1.52	3.54	1.396	2.923	0.183	0.257	0.183	–
P	$\widehat{\mu}$	0.005	0.022	0.012	0.051	0.015	0.11	0.064	0.226	0.078	–
	$\widehat{M}$	0.005	0.022	0.011	0.049	0.011	0.117	0.057	0.218	0.08	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.001	0.007	0.022	0.08	0.038	–
	2.5%	-0.004	-0.01	-0.01	-0.03	-0.047	-0.058	-0.197	-0.311	-0.297	–
	97.5%	0.018	0.05	0.043	0.115	0.093	0.245	0.365	0.857	0.458	–
Skew		0.242	-0.184	0.327	-0.251	0.75	-0.552	0.319	0.229	0.039	–
Kurtosis		1.286	-0.648	1.828	-0.571	3.158	1.313	0.086	0.036	0.003	–

TABLE C-31

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 11 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 11; Scenario 1; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	0.554	-0.159	0.428	-0.13	0.246	-0.054	-0.153	-0.072	-0.156	-0.062	-0.106	-0.106	0.022	-0.007
	$\widehat{M}$	0.512	-0.194	0.451	-0.137	0.24	-0.064	-0.158	-0.079	-0.171	-0.076	-0.111	-0.111	0.019	-0.002
B	$\widehat{\sigma}^2$	1.533	1.023	0.928	0.851	0.776	0.538	0.055	0.031	0.083	0.045	0.047	0.047	0.066	0.059
	2.5%	-1.871	-2.159	-1.554	-2.101	-1.626	-1.452	-0.598	-0.398	-0.672	-0.44	-0.529	-0.529	-0.466	-0.495
	97.5%	3.204	1.803	2.313	1.569	2.067	1.417	0.299	0.277	0.44	0.397	0.295	0.295	0.522	0.465
	$\widehat{\mu}$	0.006	-0.002	0.012	-0.004	0.016	-0.005	-0.306	-0.144	-0.52	-0.208	-0.266	-0.266	0.003	-0.001
	$\widehat{M}$	0.005	-0.002	0.013	-0.004	0.016	-0.064	-0.316	-0.158	-0.571	-0.253	-0.277	-0.277	0.002	< .001
P	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.003	0.004	0.219	0.125	0.919	0.5	0.295	0.295	0.001	0.001
	2.5%	-0.019	-0.023	-0.044	-0.068	-0.108	-0.122	-1.195	-0.795	-2.24	-1.466	-1.322	-1.322	-0.058	-0.062
	97.5%	0.032	0.019	0.066	0.051	0.138	0.119	0.598	0.555	1.466	1.324	0.737	0.737	0.065	0.058
Skew		0.233	0.078	0.147	-0.258	0.277	0.195	-0.046	0.109	0.124	0.317	-0.064	-0.064	0.088	-0.079
Kurtosis		1.11	0.819	1.141	0.977	2.496	2.464	-0.19	-0.337	-0.232	0.044	-0.195	-0.195	0.295	0.755

Condition 11; Scenario 1; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.569	-0.162	0.436	-0.132	0.25	-0.055	-0.084	-0.076	-0.055	< .001
	$\widehat{M}$	0.516	-0.196	0.45	-0.139	0.243	-0.065	-0.092	-0.094	-0.053	-0.004
B	$\widehat{\sigma}^2$	1.57	1.015	0.943	0.847	0.787	0.537	0.03	0.045	0.029	0.028
	2.5%	-1.876	-2.141	-1.556	-2.101	-1.63	-1.449	-0.38	-0.427	-0.372	-0.326
	97.5%	3.256	1.788	2.325	1.567	2.059	1.423	0.278	0.4	0.27	0.314
	$\widehat{\mu}$	0.006	-0.002	0.012	-0.004	0.017	-0.005	-0.168	-0.253	-0.137	< .001
	$\widehat{M}$	0.005	-0.002	0.013	-0.005	0.016	-0.005	-0.184	-0.313	-0.134	< .001
P	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.003	0.004	0.12	0.504	0.183	< .001
	2.5%	-0.019	-0.023	-0.044	-0.068	-0.109	-0.121	-0.759	-1.425	-0.93	-0.041
	97.5%	0.033	0.019	0.066	0.051	0.137	0.119	0.556	1.335	0.676	0.039
Skew		0.242	0.094	0.159	-0.248	0.289	0.207	0.289	0.462	0.053	-0.047
Kurtosis		1.076	0.84	1.143	0.975	2.478	2.469	-0.425	-0.134	-0.308	-0.16

Condition 11; Scenario 1; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	0.549	-0.227	0.395	-0.156	0.207	-0.083	-0.153	-0.082	-0.155	-0.071	-0.1	-0.049	0.007	-0.007
	$\widehat{M}$	0.496	-0.265	0.416	-0.166	0.219	-0.091	-0.159	-0.09	-0.172	-0.087	-0.106	-0.046	0.009	-0.007
B	$\widehat{\sigma}^2$	1.332	0.908	0.794	0.738	0.614	0.474	0.05	0.031	0.077	0.043	0.044	0.031	0.058	0.059
	2.5%	-1.582	-2.154	-1.445	-1.84	-1.45	-1.426	-0.572	-0.4	-0.657	-0.442	-0.494	-0.388	-0.472	-0.495
	97.5%	3.083	1.653	2.024	1.51	1.824	1.331	0.286	0.268	0.436	0.375	0.291	0.27	0.489	0.486
	$\widehat{\mu}$	0.005	-0.002	0.011	-0.005	0.014	-0.007	-0.306	-0.164	-0.518	-0.237	-0.251	-0.122	0.001	-0.001
	$\widehat{M}$	0.005	-0.003	0.012	-0.005	0.015	-0.091	-0.318	-0.181	-0.573	-0.292	-0.265	-0.115	0.001	-0.001
P	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.003	0.003	0.202	0.122	0.854	0.477	0.273	0.194	0.001	0.001
	2.5%	-0.016	-0.023	-0.041	-0.06	-0.097	-0.119	-1.144	-0.801	-2.19	-1.473	-1.235	-0.969	-0.059	-0.062
	97.5%	0.031	0.017	0.058	0.049	0.122	0.112	0.571	0.536	1.453	1.252	0.728	0.674	0.061	0.061
Skew		0.234	0.053	0.021	-0.008	-0.031	0.288	0.019	0.12	0.194	0.327	-0.038	-0.094	-0.078	-0.048
Kurtosis		0.783	0.629	1.168	0.618	1.161	1.373	-0.209	-0.317	-0.338	0.005	-0.243	-0.1	0.474	0.629

Condition 11; Scenario 1; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.546	-0.236	0.406	-0.166	0.221	-0.094	-0.094	-0.088	-0.061	< .001
	$\widehat{M}$	0.48	-0.285	0.433	-0.175	0.229	-0.103	-0.106	-0.104	-0.058	-0.004
B	$\widehat{\sigma}^2$	1.43	0.924	0.836	0.757	0.652	0.508	0.029	0.044	0.029	0.028
	2.5%	-1.69	-2.123	-1.451	-1.94	-1.47	-1.478	-0.38	-0.438	-0.372	-0.325
	97.5%	3.17	1.618	2.207	1.505	1.908	1.402	0.266	0.397	0.274	0.314
	$\widehat{\mu}$	0.005	-0.003	0.012	-0.005	0.015	-0.008	-0.189	-0.292	-0.153	< .001
	$\widehat{M}$	0.005	-0.003	0.012	-0.006	0.015	-0.009	-0.212	-0.348	-0.143	< .001
P	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.003	0.004	0.116	0.491	0.18	< .001
	2.5%	-0.017	-0.022	-0.041	-0.063	-0.098	-0.124	-0.76	-1.461	-0.93	-0.041
	97.5%	0.032	0.017	0.063	0.049	0.127	0.117	0.532	1.323	0.686	0.039
Skew		0.278	0.117	0.1	-0.078	0.02	0.004	0.335	0.517	0.088	-0.049
Kurtosis		1.279	0.817	1.272	0.747	1.208	2.07	-0.375	0.02	-0.237	-0.16

Condition 11; Scenario 1; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.218	0.285	0.131	0.187	0.055	0.198	0.041	0.075	0.035	–
	$\widehat{M}$	0.239	0.246	0.195	0.147	0.099	0.181	0.039	0.067	0.037	–
B	$\widehat{\sigma}^2$	0.954	0.685	0.619	0.558	0.529	0.372	0.01	0.017	0.012	–
	2.5%	-1.808	-1.285	-1.631	-1.317	-1.577	-0.915	-0.149	-0.161	-0.181	–
	97.5%	2.142	1.961	1.577	1.636	1.442	1.521	0.243	0.362	0.24	–
	$\widehat{\mu}$	0.002	0.003	0.004	0.006	0.004	0.017	0.082	0.249	0.087	–
	$\widehat{M}$	0.002	0.003	0.006	0.005	0.007	0.015	0.079	0.222	0.091	–
P	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.002	0.003	0.041	0.184	0.078	–
	2.5%	-0.018	-0.014	-0.047	-0.043	-0.105	-0.077	-0.299	-0.536	-0.453	–
	97.5%	0.021	0.021	0.045	0.053	0.096	0.127	0.485	1.207	0.6	–
Skew		-0.102	0.281	-0.187	0.11	-0.01	0.617	0.059	0.222	-0.131	–
Kurtosis		0.759	0.697	0.717	0.473	2.179	1.854	-0.17	0.154	0.031	–

TABLE C-32

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 11 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 11; Scenario 2; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.641	0.293	0.519	0.218	0.225	0.379	-0.086	-0.076	-0.062	-0.029	-0.049	-0.049	-0.001	-0.004
	$\widehat{M}$	0.491	0.221	0.398	0.084	0.147	0.353	-0.084	-0.068	-0.061	-0.023	-0.035	-0.035	0.003	-0.011
	$\widehat{\sigma}^2$	1.213	3.187	0.829	2.19	0.729	1.36	0.053	0.057	0.073	0.075	0.047	0.047	0.059	0.094
	2.5%	-1.033	-2.994	-0.918	-2.683	-1.178	-1.793	-0.537	-0.55	-0.613	-0.552	-0.491	-0.491	-0.446	-0.586
	97.5%	3.3	3.688	2.685	2.983	2.374	2.501	0.292	0.325	0.444	0.47	0.331	0.331	0.453	0.631
P	$\widehat{\mu}$	0.006	0.003	0.015	0.007	0.015	0.032	-0.172	-0.152	-0.208	-0.097	-0.124	-0.124	< .001	< .001
	$\widehat{M}$	0.005	0.002	0.011	0.003	0.01	0.353	-0.167	-0.137	-0.203	-0.076	-0.085	-0.085	< .001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.002	0.003	0.01	0.21	0.228	0.809	0.836	0.296	0.296	0.001	0.001
	2.5%	-0.01	-0.032	-0.026	-0.087	-0.078	-0.15	-1.074	-1.1	-2.042	-1.839	-1.227	-1.227	-0.056	-0.073
	97.5%	0.033	0.039	0.077	0.097	0.158	0.209	0.584	0.65	1.478	1.568	0.829	0.829	0.057	0.079
Skew		0.972	-0.086	1.12	0.023	1.167	-0.019	-1.139	-0.457	-0.314	-0.169	-0.61	-0.61	-0.324	-0.009
Kurtosis		2.344	0.146	4.981	-0.283	5.605	0.029	4.665	0.582	0.674	-0.214	1.528	1.528	2.121	1.891

Condition 11; Scenario 2; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.647	0.293	0.522	0.217	0.227	0.379	-0.054	-0.025	-0.022	-0.001
	$\widehat{M}$	0.493	0.228	0.399	0.086	0.145	0.354	-0.061	-0.03	-0.019	-0.005
	$\widehat{\sigma}^2$	1.235	3.16	0.839	2.177	0.735	1.352	0.034	0.053	0.035	0.028
	2.5%	-1.027	-2.995	-0.917	-2.681	-1.173	-1.777	-0.383	-0.445	-0.405	-0.332
	97.5%	3.327	3.683	2.721	2.98	2.393	2.497	0.281	0.449	0.331	0.34
P	$\widehat{\mu}$	0.006	0.003	0.015	0.007	0.015	0.032	-0.107	-0.083	-0.055	< .001
	$\widehat{M}$	0.005	0.002	0.011	0.003	0.01	0.03	-0.122	-0.102	-0.047	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.002	0.003	0.009	0.135	0.592	0.221	< .001
	2.5%	-0.01	-0.032	-0.026	-0.087	-0.078	-0.149	-0.767	-1.482	-1.013	-0.041
	97.5%	0.033	0.039	0.078	0.097	0.159	0.209	0.561	1.498	0.828	0.043
Skew		0.976	-0.082	1.12	0.027	1.192	-0.013	0.042	0.132	-0.074	-0.01
Kurtosis		2.247	0.164	4.895	-0.278	5.695	0.038	-0.692	-0.576	-0.438	-0.016

Condition 11; Scenario 2; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.631	0.245	0.475	0.214	0.182	0.362	-0.081	-0.081	-0.064	-0.039	-0.044	-0.034	-0.005	-0.005
	$\widehat{M}$	0.499	0.205	0.395	0.09	0.147	0.325	-0.088	-0.073	-0.068	-0.03	-0.036	-0.021	-0.009	-0.01
	$\widehat{\sigma}^2$	0.982	2.832	0.605	1.926	0.557	1.317	0.04	0.057	0.066	0.077	0.041	0.056	0.078	0.094
	2.5%	-0.916	-2.993	-0.862	-2.359	-1.26	-1.686	-0.475	-0.55	-0.569	-0.559	-0.47	-0.509	-0.528	-0.586
	97.5%	3.044	3.286	2.365	2.868	1.927	2.58	0.287	0.325	0.444	0.468	0.322	0.37	0.552	0.626
P	$\widehat{\mu}$	0.006	0.003	0.014	0.007	0.012	0.03	-0.162	-0.163	-0.213	-0.129	-0.111	-0.085	-0.001	-0.001
	$\widehat{M}$	0.005	0.002	0.011	0.003	0.01	0.325	-0.176	-0.146	-0.226	-0.101	-0.091	-0.051	-0.001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.002	0.009	0.16	0.228	0.73	0.859	0.259	0.349	0.001	0.001
	2.5%	-0.009	-0.032	-0.025	-0.077	-0.084	-0.141	-0.951	-1.1	-1.897	-1.862	-1.175	-1.272	-0.066	-0.073
	97.5%	0.03	0.035	0.068	0.093	0.128	0.216	0.575	0.65	1.478	1.561	0.804	0.925	0.069	0.078
Skew		0.827	-0.168	0.402	0.123	0.506	0.085	-0.208	-0.406	-0.066	-0.146	-0.285	-0.222	-0.123	0.085
Kurtosis		1.369	0.231	2.066	-0.292	2.527	0.113	0.251	0.427	0.077	-0.233	0.084	-0.526	1.35	2.016

Condition 11; Scenario 2; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.618	0.232	0.461	0.213	0.181	0.355	-0.058	-0.033	-0.026	-0.001
	$\widehat{M}$	0.493	0.167	0.389	0.082	0.138	0.314	-0.067	-0.04	-0.022	-0.004
	$\widehat{\sigma}^2$	1.029	2.806	0.627	1.951	0.582	1.319	0.035	0.054	0.036	0.028
	2.5%	-0.936	-2.902	-0.912	-2.376	-1.214	-1.709	-0.388	-0.446	-0.405	-0.332
	97.5%	3.091	3.393	2.292	2.868	1.999	2.58	0.304	0.452	0.328	0.34
P	$\widehat{\mu}$	0.006	0.002	0.013	0.007	0.012	0.03	-0.116	-0.11	-0.065	< .001
	$\widehat{M}$	0.005	0.002	0.011	0.003	0.009	0.026	-0.134	-0.133	-0.056	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.002	0.003	0.009	0.138	0.605	0.222	< .001
	2.5%	-0.009	-0.031	-0.026	-0.077	-0.081	-0.143	-0.775	-1.486	-1.013	-0.041
	97.5%	0.031	0.036	0.065	0.093	0.133	0.216	0.608	1.507	0.821	0.043
Skew		0.804	-0.045	0.48	0.123	0.738	0.009	0.098	0.211	-0.061	-0.011
Kurtosis		1.59	0.284	2.525	-0.278	4.004	0.045	-0.667	-0.55	-0.408	-0.012

Condition 11; Scenario 2; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.492	1.092	0.361	0.802	0.157	0.784	0.057	0.1	0.051	–
	$\widehat{M}$	0.452	1.038	0.303	0.718	0.117	0.758	0.058	0.102	0.054	–
	$\widehat{\sigma}^2$	0.589	2.026	0.445	1.396	0.447	0.9	0.01	0.017	0.014	–
	2.5%	-0.864	-1.633	-0.827	-1.399	-1.075	-1.008	-0.132	-0.14	-0.178	–
	97.5%	2.112	3.813	1.803	3.014	1.625	2.544	0.242	0.358	0.264	–
P	$\widehat{\mu}$	0.005	0.012	0.01	0.026	0.01	0.066	0.115	0.334	0.128	–
	$\widehat{M}$	0.005	0.011	0.009	0.023	0.008	0.064	0.115	0.338	0.136	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.002	0.006	0.039	0.187	0.085	–
	2.5%	-0.009	-0.017	-0.024	-0.045	-0.072	-0.084	-0.264	-0.468	-0.446	–
	97.5%	0.021	0.04	0.052	0.098	0.108	0.213	0.484	1.195	0.659	–
Skew		0.25	-0.015	0.426	0.094	0.865	0.048	-0.052	0.051	-0.122	–
Kurtosis		0.981	-0.091	3.685	-0.398	5.301	0.047	-0.357	-0.237	-0.179	–

TABLE C-33

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 11 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 11; Scenario 3; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	1.045	1.679	0.834	1.218	0.473	1.112	-0.093	-0.057	-0.052	0.01	-0.049	-0.049	0.004	-0.019
	$\widehat{M}$	0.825	1.965	0.685	1.411	0.262	1.298	-0.085	-0.011	-0.033	0.023	-0.018	-0.018	-0.001	-0.01
	$\widehat{\sigma}^2$	2.032	4.547	1.08	3.124	1.07	1.905	0.063	0.073	0.096	0.084	0.062	0.062	0.069	0.117
	2.5%	-1.38	-2.854	-1.038	-2.731	-1.11	-2.08	-0.61	-0.665	-0.751	-0.606	-0.598	-0.598	-0.463	-0.706
	97.5%	4.497	5.101	3.272	4.001	3.161	3.248	0.335	0.35	0.486	0.479	0.36	0.36	0.524	0.674
P	$\widehat{\mu}$	0.01	0.018	0.024	0.04	0.032	0.093	-0.186	-0.114	-0.172	0.034	-0.122	-0.122	0.001	-0.002
	$\widehat{M}$	0.008	0.021	0.02	0.046	0.017	1.298	-0.17	-0.021	-0.108	0.077	-0.046	-0.046	< .001	-0.001
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.003	0.005	0.013	0.253	0.294	1.064	0.929	0.388	0.388	0.001	0.002
	2.5%	-0.014	-0.03	-0.03	-0.089	-0.074	-0.174	-1.22	-1.329	-2.503	-2.018	-1.495	-1.495	-0.058	-0.088
	97.5%	0.045	0.054	0.093	0.13	0.211	0.272	0.67	0.7	1.619	1.595	0.899	0.899	0.066	0.084
Skew		0.727	-0.549	0.648	-0.436	1.312	-0.783	-0.762	-0.918	-0.683	-0.418	-0.786	-0.786	-0.684	-0.159
Kurtosis		2.273	0.112	1.69	0.012	3.898	1.504	2.13	1.203	1.098	-0.223	1.464	1.464	8.604	3.991

Condition 11; Scenario 3; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	1.055	1.675	0.839	1.216	0.476	1.111	-0.046	0.006	-0.013	< .001
	$\widehat{M}$	0.829	1.944	0.681	1.403	0.259	1.293	-0.04	0.003	0.005	-0.002
	$\widehat{\sigma}^2$	2.044	4.519	1.088	3.111	1.076	1.893	0.039	0.055	0.039	0.029
	2.5%	-1.417	-2.832	-1.044	-2.711	-1.104	-2.078	-0.45	-0.448	-0.383	-0.318
	97.5%	4.525	5.091	3.275	3.985	3.179	3.254	0.313	0.444	0.354	0.348
P	$\widehat{\mu}$	0.011	0.018	0.024	0.04	0.032	0.093	-0.092	0.021	-0.032	< .001
	$\widehat{M}$	0.008	0.021	0.019	0.046	0.017	0.108	-0.079	0.01	0.014	< .001
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.003	0.005	0.013	0.154	0.61	0.242	< .001
	2.5%	-0.014	-0.03	-0.03	-0.088	-0.073	-0.174	-0.9	-1.496	-0.959	-0.04
	97.5%	0.045	0.054	0.094	0.13	0.212	0.273	0.627	1.48	0.884	0.043
Skew		0.718	-0.544	0.648	-0.431	1.324	-0.779	-0.116	-0.026	-0.181	0.011
Kurtosis		2.194	0.115	1.656	0.013	3.941	1.519	-0.453	-0.456	-0.415	0.147

Condition 11; Scenario 3; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	1.009	1.693	0.785	1.281	0.394	1.131	-0.091	-0.052	-0.045	0.011	-0.042	-0.018	-0.014	-0.016
	$\widehat{M}$	0.835	1.945	0.669	1.425	0.259	1.268	-0.086	-0.011	-0.034	0.036	-0.013	0.019	-0.003	-0.004
	$\widehat{\sigma}^2$	1.622	4.062	0.893	2.696	0.783	1.742	0.059	0.07	0.086	0.086	0.054	0.065	0.078	0.104
	2.5%	-1.158	-2.765	-1.001	-2.235	-1.212	-1.946	-0.587	-0.666	-0.677	-0.62	-0.563	-0.607	-0.576	-0.661
	97.5%	3.866	5.101	2.749	4.015	2.327	3.229	0.336	0.348	0.477	0.483	0.36	0.383	0.539	0.672
P	$\widehat{\mu}$	0.01	0.018	0.022	0.042	0.026	0.095	-0.182	-0.105	-0.149	0.037	-0.105	-0.044	-0.002	-0.002
	$\widehat{M}$	0.008	0.021	0.019	0.046	0.017	1.268	-0.173	-0.022	-0.114	0.12	-0.033	0.046	< .001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.003	0.012	0.235	0.281	0.961	0.961	0.34	0.409	0.001	0.002
	2.5%	-0.012	-0.029	-0.029	-0.073	-0.081	-0.163	-1.174	-1.331	-2.255	-2.068	-1.409	-1.519	-0.072	-0.083
	97.5%	0.039	0.054	0.079	0.13	0.155	0.27	0.673	0.697	1.589	1.61	0.899	0.956	0.067	0.084
Skew		0.68	-0.482	0.376	-0.419	0.584	-0.522	-0.536	-0.772	-0.571	-0.453	-0.501	-0.577	0.101	0.221
Kurtosis		2.094	0.05	1.106	-0.168	1.611	0.164	1.436	0.486	0.858	-0.255	0.269	-0.116	0.887	1.382

Condition 11; Scenario 3; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	1.023	1.71	0.8	1.279	0.397	1.158	-0.047	0.005	-0.015	< .001
	$\widehat{M}$	0.843	1.914	0.705	1.402	0.252	1.276	-0.054	-0.001	0.003	0.001
	$\widehat{\sigma}^2$	1.648	3.97	0.896	2.577	0.839	1.713	0.041	0.058	0.041	0.029
	2.5%	-1.102	-2.565	-1.032	-2.024	-1.203	-1.818	-0.45	-0.457	-0.405	-0.318
	97.5%	3.85	5.065	2.761	3.985	2.35	3.287	0.328	0.451	0.358	0.348
P	$\widehat{\mu}$	0.01	0.018	0.023	0.042	0.026	0.097	-0.095	0.015	-0.036	< .001
	$\widehat{M}$	0.008	0.02	0.02	0.046	0.017	0.107	-0.108	-0.001	0.006	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.004	0.012	0.164	0.648	0.255	< .001
	2.5%	-0.011	-0.027	-0.029	-0.066	-0.08	-0.152	-0.9	-1.525	-1.012	-0.04
	97.5%	0.039	0.054	0.079	0.13	0.157	0.275	0.655	1.505	0.894	0.043
Skew		0.649	-0.476	0.294	-0.35	0.93	0.012	-0.041	-0.011	-0.164	0.012
Kurtosis		2.108	0.074	1.179	-0.176	4.016	0.321	-0.564	-0.597	-0.492	0.152

Condition 11; Scenario 3; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.824	2.468	0.621	1.814	0.351	1.52	0.049	0.105	0.046	–
	$\widehat{M}$	0.756	2.736	0.575	1.922	0.274	1.659	0.05	0.101	0.051	–
	$\widehat{\sigma}^2$	0.982	2.902	0.531	1.925	0.566	1.187	0.01	0.015	0.013	–
	2.5%	-1.19	-1.467	-0.912	-1.396	-0.971	-1.05	-0.159	-0.135	-0.18	–
	97.5%	2.834	5.168	2.021	3.983	2.134	3.219	0.252	0.365	0.27	–
P	$\widehat{\mu}$	0.008	0.026	0.018	0.059	0.023	0.127	0.098	0.351	0.116	–
	$\widehat{M}$	0.008	0.029	0.016	0.062	0.018	0.139	0.099	0.337	0.129	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.003	0.008	0.042	0.17	0.082	–
	2.5%	-0.012	-0.016	-0.026	-0.045	-0.065	-0.088	-0.317	-0.449	-0.45	–
	97.5%	0.028	0.055	0.058	0.129	0.142	0.27	0.503	1.216	0.676	–
Skew		0.204	-0.728	0.02	-0.526	0.943	-0.982	-0.046	0.13	-0.104	–
Kurtosis		2.413	0.839	1.113	0.325	4.057	3.19	-0.049	-0.026	-0.12	–

TABLE C-34

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 12 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 12; Scenario 1; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.722	-0.261	0.579	-0.208	0.307	-0.097	-0.238	-0.113	-0.223	-0.104	-0.151	-0.151	0.018	-0.003
	$\widehat{M}$	0.731	-0.266	0.567	-0.179	0.311	-0.108	-0.252	-0.122	-0.245	-0.119	-0.157	-0.157	0.013	-0.002
	$\widehat{\sigma}^2$	0.862	0.566	0.485	0.468	0.396	0.263	0.031	0.014	0.053	0.022	0.029	0.029	0.027	0.021
	2.5%	-1.222	-1.818	-0.78	-1.666	-1.013	-1.193	-0.571	-0.324	-0.629	-0.364	-0.473	-0.473	-0.302	-0.292
	97.5%	2.629	1.251	1.947	1.083	1.571	0.887	0.159	0.151	0.25	0.226	0.206	0.206	0.357	0.282
P	$\widehat{\mu}$	0.007	-0.003	0.017	-0.007	0.02	-0.008	-0.475	-0.227	-0.742	-0.346	-0.377	-0.377	0.002	< .001
	$\widehat{M}$	0.007	-0.003	0.016	-0.006	0.021	-0.108	-0.504	-0.245	-0.817	-0.397	-0.391	-0.391	0.002	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.002	0.002	0.125	0.056	0.585	0.249	0.183	0.183	< .001	< .001
	2.5%	-0.012	-0.019	-0.022	-0.054	-0.068	-0.1	-1.141	-0.648	-2.097	-1.212	-1.183	-1.183	-0.038	-0.036
	97.5%	0.026	0.013	0.056	0.035	0.105	0.074	0.318	0.302	0.834	0.753	0.515	0.515	0.045	0.035
Skew		0.004	-0.169	-0.064	-0.485	-0.02	-0.144	0.269	0.243	0.241	0.588	0.155	0.155	0.102	-0.022
Kurtosis		1.097	1.362	1.69	3.437	1.972	2.191	0.442	1.103	0.473	0.891	0.497	0.497	0.181	0.207

Condition 12; Scenario 1; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.739	-0.265	0.588	-0.21	0.311	-0.098	-0.136	-0.126	-0.094	0.002
	$\widehat{M}$	0.745	-0.273	0.577	-0.183	0.315	-0.107	-0.154	-0.148	-0.102	0.001
	$\widehat{\sigma}^2$	0.893	0.558	0.497	0.465	0.403	0.262	0.013	0.025	0.018	0.011
	2.5%	-1.22	-1.801	-0.788	-1.659	-1.012	-1.185	-0.321	-0.38	-0.327	-0.2
	97.5%	2.653	1.242	1.981	1.08	1.586	0.882	0.142	0.231	0.189	0.207
P	$\widehat{\mu}$	0.007	-0.003	0.017	-0.007	0.021	-0.008	-0.273	-0.421	-0.235	< .001
	$\widehat{M}$	0.007	-0.003	0.016	-0.006	0.021	-0.009	-0.309	-0.496	-0.256	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.002	0.002	0.054	0.274	0.11	< .001
	2.5%	-0.012	-0.019	-0.023	-0.054	-0.067	-0.099	-0.642	-1.267	-0.818	-0.025
	97.5%	0.027	0.013	0.057	0.035	0.106	0.074	0.284	0.771	0.474	0.026
Skew		0.016	-0.149	-0.047	-0.476	-0.009	-0.133	0.794	0.735	0.438	-0.047
Kurtosis		1.053	1.357	1.658	3.449	1.938	2.181	0.73	0.637	0.379	-0.148

Condition 12; Scenario 1; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.696	-0.333	0.529	-0.234	0.252	-0.111	-0.232	-0.119	-0.212	-0.109	-0.132	-0.082	0.011	-0.003
	$\widehat{M}$	0.702	-0.323	0.529	-0.22	0.277	-0.128	-0.248	-0.128	-0.225	-0.124	-0.138	-0.085	0.009	-0.004
	$\widehat{\sigma}^2$	0.67	0.439	0.355	0.353	0.282	0.231	0.027	0.012	0.046	0.022	0.025	0.017	0.025	0.022
	2.5%	-0.927	-1.78	-0.668	-1.512	-0.937	-1.158	-0.507	-0.325	-0.581	-0.362	-0.431	-0.34	-0.284	-0.302
	97.5%	2.407	0.938	1.671	0.929	1.272	0.859	0.13	0.112	0.24	0.216	0.195	0.176	0.351	0.278
P	$\widehat{\mu}$	0.007	-0.004	0.015	-0.008	0.017	-0.009	-0.463	-0.239	-0.705	-0.363	-0.331	-0.206	0.001	< .001
	$\widehat{M}$	0.007	-0.003	0.015	-0.007	0.018	-0.128	-0.497	-0.256	-0.752	-0.414	-0.344	-0.212	0.001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.107	0.05	0.507	0.241	0.159	0.105	< .001	< .001
	2.5%	-0.009	-0.019	-0.019	-0.049	-0.062	-0.097	-1.015	-0.65	-1.939	-1.206	-1.077	-0.851	-0.036	-0.038
	97.5%	0.024	0.01	0.048	0.03	0.085	0.072	0.259	0.223	0.801	0.722	0.487	0.44	0.044	0.035
Skew		0.048	-0.274	-0.019	-0.183	-0.231	-0.014	0.46	0.347	0.376	0.578	0.164	0.08	0.203	-0.059
Kurtosis		0.32	1.182	1.125	0.641	0.72	0.956	0.214	0.598	-0.063	0.649	0.165	0.237	0.685	0.123

Condition 12; Scenario 1; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.726	-0.303	0.564	-0.222	0.285	-0.109	-0.142	-0.13	-0.095	0.002
	$\widehat{M}$	0.737	-0.302	0.563	-0.2	0.292	-0.119	-0.159	-0.154	-0.101	0.001
	$\widehat{\sigma}^2$	0.78	0.488	0.42	0.383	0.336	0.249	0.013	0.024	0.017	0.011
	2.5%	-1.094	-1.797	-0.727	-1.547	-0.937	-1.201	-0.321	-0.38	-0.327	-0.199
	97.5%	2.575	1.047	1.839	0.98	1.433	0.846	0.126	0.229	0.174	0.207
P	$\widehat{\mu}$	0.007	-0.003	0.016	-0.007	0.019	-0.009	-0.283	-0.433	-0.238	< .001
	$\widehat{M}$	0.007	-0.003	0.016	-0.006	0.019	-0.01	-0.318	-0.514	-0.253	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.051	0.267	0.106	< .001
	2.5%	-0.011	-0.019	-0.021	-0.05	-0.062	-0.101	-0.643	-1.267	-0.818	-0.025
	97.5%	0.026	0.011	0.053	0.032	0.096	0.071	0.251	0.762	0.434	0.026
Skew		0.049	-0.225	-0.039	-0.181	-0.147	0.002	0.824	0.737	0.351	-0.049
Kurtosis		0.641	1.052	0.862	0.705	1.467	1.52	0.887	0.675	0.227	-0.147

Condition 12; Scenario 1; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.225	0.304	0.154	0.201	0.04	0.212	0.039	0.071	0.031	–
	$\widehat{M}$	0.296	0.26	0.178	0.177	0.071	0.184	0.035	0.067	0.03	–
	$\widehat{\sigma}^2$	0.465	0.301	0.279	0.241	0.233	0.158	0.004	0.007	0.006	–
	2.5%	-1.365	-0.661	-1.008	-0.693	-1.05	-0.515	-0.079	-0.075	-0.112	–
	97.5%	1.463	1.491	1.051	1.214	0.936	1.038	0.172	0.24	0.184	–
P	$\widehat{\mu}$	0.002	0.003	0.004	0.007	0.003	0.018	0.077	0.238	0.079	–
	$\widehat{M}$	0.003	0.003	0.005	0.006	0.005	0.015	0.069	0.223	0.074	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.001	0.016	0.08	0.037	–
	2.5%	-0.014	-0.007	-0.029	-0.023	-0.07	-0.043	-0.158	-0.248	-0.279	–
	97.5%	0.015	0.016	0.03	0.039	0.062	0.087	0.344	0.8	0.461	–
Skew		-0.571	0.457	-0.54	0.37	-0.459	0.327	0.235	0.337	0.208	–
Kurtosis		0.893	0.695	1.487	1.183	1.292	1.306	0.209	0.382	0.478	–

TABLE C-35

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 12 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 12; Scenario 2; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	0.623	-0.032	0.523	-0.075	0.25	0.119	-0.136	-0.156	-0.117	-0.122	-0.088	-0.088	0.017	0.001
	$\widehat{M}$	0.439	-0.297	0.354	-0.272	0.119	0.016	-0.137	-0.182	-0.109	-0.139	-0.079	-0.079	0.013	-0.005
B	$\widehat{\sigma}^2$	0.962	2.233	0.564	1.537	0.479	0.908	0.03	0.039	0.044	0.054	0.029	0.029	0.021	0.038
	2.5%	-0.674	-2.491	-0.468	-2.113	-0.695	-1.554	-0.485	-0.482	-0.605	-0.512	-0.459	-0.459	-0.26	-0.361
	97.5%	3.189	3.415	2.482	2.53	2.022	2.076	0.204	0.242	0.321	0.361	0.233	0.233	0.289	0.423
	$\widehat{\mu}$	0.006	< .001	0.015	-0.002	0.017	0.01	-0.273	-0.312	-0.39	-0.406	-0.221	-0.221	0.002	< .001
	$\widehat{M}$	0.004	-0.003	0.01	-0.009	0.008	0.016	-0.275	-0.363	-0.363	-0.464	-0.198	-0.198	0.002	-0.001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.002	0.006	0.12	0.156	0.493	0.599	0.179	0.179	< .001	0.001
	2.5%	-0.007	-0.026	-0.013	-0.069	-0.046	-0.13	-0.971	-0.964	-2.018	-1.706	-1.148	-1.148	-0.033	-0.045
	97.5%	0.032	0.036	0.071	0.082	0.135	0.174	0.409	0.484	1.071	1.205	0.582	0.582	0.036	0.053
Skew		1.702	0.541	1.632	0.416	2.071	0.279	-0.995	0.31	-0.495	0.216	-0.741	-0.741	0.154	0.156
Kurtosis		5.061	-0.108	4.467	-0.173	9.454	-0.237	5.423	-0.515	2.648	-0.249	3.478	3.478	0.498	0.472

Condition 12; Scenario 2; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.627	-0.029	0.525	-0.073	0.251	0.12	-0.112	-0.086	-0.071	0.009
	$\widehat{M}$	0.434	-0.293	0.355	-0.269	0.117	0.019	-0.128	-0.104	-0.082	0.006
B	$\widehat{\sigma}^2$	0.991	2.203	0.577	1.524	0.485	0.899	0.021	0.032	0.021	0.013
	2.5%	-0.674	-2.461	-0.473	-2.116	-0.692	-1.541	-0.341	-0.381	-0.333	-0.214
	97.5%	3.265	3.406	2.515	2.526	2.049	2.073	0.229	0.336	0.225	0.227
	$\widehat{\mu}$	0.006	< .001	0.015	-0.002	0.017	0.01	-0.224	-0.287	-0.176	0.001
	$\widehat{M}$	0.004	-0.003	0.01	-0.009	0.008	0.002	-0.257	-0.348	-0.205	0.001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.002	0.006	0.084	0.35	0.133	< .001
	2.5%	-0.007	-0.026	-0.014	-0.069	-0.046	-0.129	-0.682	-1.269	-0.833	-0.027
	97.5%	0.033	0.036	0.072	0.082	0.137	0.174	0.457	1.121	0.562	0.028
Skew		1.695	0.549	1.644	0.42	2.089	0.282	0.661	0.617	0.328	-0.06
Kurtosis		4.909	-0.088	4.502	-0.162	9.449	-0.228	0.127	0.256	-0.095	-0.1

Condition 12; Scenario 2; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	0.559	-0.127	0.447	-0.115	0.181	0.068	-0.13	-0.161	-0.109	-0.133	-0.077	-0.114	0.006	0.001
	$\widehat{M}$	0.423	-0.326	0.339	-0.274	0.118	-0.022	-0.134	-0.188	-0.116	-0.159	-0.078	-0.118	0.001	-0.01
B	$\widehat{\sigma}^2$	0.663	1.85	0.361	1.29	0.252	0.853	0.023	0.038	0.035	0.055	0.022	0.038	0.032	0.038
	2.5%	-0.623	-2.277	-0.467	-2.038	-0.693	-1.534	-0.431	-0.477	-0.477	-0.537	-0.364	-0.464	-0.324	-0.372
	97.5%	2.58	2.992	1.892	2.326	1.386	2.067	0.216	0.252	0.355	0.365	0.23	0.26	0.398	0.422
	$\widehat{\mu}$	0.006	-0.001	0.013	-0.004	0.012	0.006	-0.26	-0.323	-0.362	-0.444	-0.194	-0.284	0.001	< .001
	$\widehat{M}$	0.004	-0.003	0.01	-0.009	0.008	-0.022	-0.268	-0.376	-0.385	-0.528	-0.195	-0.294	< .001	-0.001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.006	0.091	0.15	0.388	0.607	0.138	0.241	< .001	0.001
	2.5%	-0.006	-0.024	-0.013	-0.066	-0.046	-0.129	-0.862	-0.953	-1.592	-1.791	-0.91	-1.159	-0.041	-0.047
	97.5%	0.026	0.032	0.054	0.076	0.092	0.173	0.432	0.504	1.185	1.218	0.574	0.652	0.05	0.053
Skew		1.107	0.512	0.966	0.462	0.657	0.391	0.136	0.388	0.339	0.316	-0.002	0.073	0.189	0.153
Kurtosis		2.267	0.028	1.54	-0.001	2.981	-0.106	1.118	-0.379	1.172	-0.249	0.646	-0.568	0.804	0.423

Condition 12; Scenario 2; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.549	-0.12	0.452	-0.126	0.193	0.06	-0.118	-0.097	-0.077	0.01
	$\widehat{M}$	0.403	-0.337	0.338	-0.28	0.117	-0.021	-0.138	-0.122	-0.087	0.007
B	$\widehat{\sigma}^2$	0.716	1.888	0.393	1.281	0.281	0.834	0.02	0.031	0.021	0.013
	2.5%	-0.64	-2.273	-0.447	-2.015	-0.666	-1.523	-0.342	-0.381	-0.334	-0.214
	97.5%	2.645	3.039	2.023	2.314	1.532	2.036	0.236	0.337	0.221	0.227
	$\widehat{\mu}$	0.005	-0.001	0.013	-0.004	0.013	0.005	-0.237	-0.325	-0.193	0.001
	$\widehat{M}$	0.004	-0.004	0.01	-0.009	0.008	-0.002	-0.275	-0.407	-0.218	0.001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.006	0.082	0.349	0.131	< .001
	2.5%	-0.006	-0.024	-0.013	-0.065	-0.044	-0.128	-0.683	-1.269	-0.834	-0.027
	97.5%	0.026	0.032	0.058	0.075	0.102	0.17	0.473	1.123	0.553	0.028
Skew		1.141	0.604	1.11	0.474	1.04	0.006	0.768	0.754	0.349	-0.07
Kurtosis		2.639	0.195	1.98	-0.022	4.279	-0.07	0.455	0.532	-0.054	-0.095

Condition 12; Scenario 2; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.382	1.112	0.289	0.791	0.117	0.726	0.052	0.093	0.044	–
	$\widehat{M}$	0.353	0.94	0.247	0.7	0.11	0.661	0.05	0.087	0.042	–
B	$\widehat{\sigma}^2$	0.279	1.258	0.181	0.865	0.159	0.505	0.005	0.007	0.006	–
	2.5%	-0.552	-0.715	-0.44	-0.806	-0.649	-0.495	-0.079	-0.057	-0.104	–
	97.5%	1.566	3.58	1.248	2.675	0.942	2.158	0.193	0.264	0.191	–
	$\widehat{\mu}$	0.004	0.012	0.008	0.026	0.008	0.061	0.104	0.309	0.111	–
	$\widehat{M}$	0.004	0.01	0.007	0.023	0.007	0.055	0.099	0.29	0.106	–
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.004	0.019	0.077	0.037	–
	2.5%	-0.006	-0.008	-0.013	-0.026	-0.043	-0.041	-0.159	-0.19	-0.261	–
	97.5%	0.016	0.038	0.036	0.087	0.063	0.181	0.387	0.879	0.478	–
Skew		0.396	0.464	0.673	0.318	0.387	0.256	0.196	0.366	0.154	–
Kurtosis		1.105	-0.269	1.81	-0.356	4.358	-0.329	-0.022	0.333	0.059	–

TABLE C-36

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 12 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 12; Scenario 3; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	1.148	1.144	0.893	0.845	0.474	0.928	-0.139	-0.125	-0.097	-0.044	-0.078	-0.078	-0.011	-0.005
	$\widehat{M}$	0.785	1.15	0.611	0.91	0.193	1.135	-0.116	-0.111	-0.074	-0.013	-0.042	-0.042	-0.011	-0.002
	$\widehat{\sigma}^2$	1.949	4.258	1.172	3.081	0.969	1.679	0.059	0.06	0.086	0.068	0.051	0.051	0.03	0.059
	2.5%	-0.553	-2.716	-0.418	-2.419	-0.754	-1.803	-0.675	-0.614	-0.902	-0.569	-0.658	-0.658	-0.339	-0.501
	97.5%	4.851	4.556	3.71	3.601	3.321	2.977	0.285	0.295	0.419	0.414	0.259	0.259	0.284	0.506
P	$\widehat{\mu}$	0.011	0.012	0.025	0.027	0.032	0.078	-0.277	-0.251	-0.324	-0.145	-0.194	-0.194	-0.001	-0.001
	$\widehat{M}$	0.008	0.012	0.017	0.03	0.013	1.135	-0.231	-0.223	-0.245	-0.041	-0.105	-0.105	-0.001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.004	0.012	0.236	0.24	0.951	0.76	0.318	0.318	< .001	0.001
	2.5%	-0.006	-0.029	-0.012	-0.079	-0.05	-0.151	-1.351	-1.229	-3.008	-1.896	-1.646	-1.646	-0.042	-0.063
	97.5%	0.048	0.048	0.106	0.117	0.221	0.249	0.569	0.59	1.396	1.38	0.649	0.649	0.035	0.063
Skew		1.599	-0.139	1.57	-0.33	1.858	-0.597	-1.529	-0.258	-1.105	-0.327	-1.628	-1.628	-2.719	-0.199
Kurtosis		3.82	-0.651	3.461	-0.326	4.787	0.127	5.928	-0.596	2.861	-0.334	4.979	4.979	35.501	3.57

Condition 12; Scenario 3; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	1.158	1.143	0.898	0.844	0.477	0.928	-0.078	-0.026	-0.028	-0.005
	$\widehat{M}$	0.784	1.14	0.608	0.907	0.195	1.13	-0.085	-0.035	-0.017	-0.008
	$\widehat{\sigma}^2$	1.974	4.218	1.187	3.062	0.977	1.665	0.028	0.036	0.023	0.011
	2.5%	-0.559	-2.705	-0.417	-2.408	-0.749	-1.786	-0.377	-0.371	-0.341	-0.214
	97.5%	4.896	4.555	3.76	3.602	3.339	2.972	0.273	0.387	0.25	0.202
P	$\widehat{\mu}$	0.012	0.012	0.026	0.027	0.032	0.078	-0.157	-0.086	-0.069	-0.001
	$\widehat{M}$	0.008	0.012	0.017	0.029	0.013	0.095	-0.169	-0.115	-0.042	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.004	0.012	0.11	0.404	0.144	< .001
	2.5%	-0.006	-0.029	-0.012	-0.078	-0.05	-0.15	-0.753	-1.237	-0.852	-0.027
	97.5%	0.049	0.048	0.107	0.117	0.223	0.249	0.548	1.29	0.627	0.025
Skew		1.567	-0.133	1.562	-0.328	1.864	-0.593	0.181	0.203	-0.185	-0.155
Kurtosis		3.606	-0.646	3.375	-0.318	4.779	0.136	-0.387	-0.167	-0.074	0.481

Condition 12; Scenario 3; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.995	1.135	0.766	0.879	0.35	0.897	-0.121	-0.122	-0.076	-0.05	-0.057	-0.051	-0.002	-0.005
	$\widehat{M}$	0.754	1.153	0.593	0.919	0.198	1.038	-0.116	-0.113	-0.076	-0.024	-0.036	-0.021	-0.004	-0.004
	$\widehat{\sigma}^2$	1.276	3.793	0.688	2.462	0.593	1.601	0.043	0.058	0.064	0.075	0.036	0.049	0.036	0.053
	2.5%	-0.582	-2.35	-0.403	-2.003	-0.956	-1.756	-0.59	-0.594	-0.681	-0.584	-0.508	-0.509	-0.387	-0.481
	97.5%	3.746	4.663	2.724	3.53	2.336	3.005	0.285	0.294	0.417	0.416	0.287	0.329	0.407	0.473
P	$\widehat{\mu}$	0.01	0.012	0.022	0.029	0.023	0.075	-0.243	-0.244	-0.253	-0.168	-0.142	-0.127	< .001	-0.001
	$\widehat{M}$	0.008	0.012	0.017	0.03	0.013	1.038	-0.232	-0.226	-0.255	-0.08	-0.091	-0.051	-0.001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.003	0.011	0.17	0.234	0.716	0.832	0.228	0.309	0.001	0.001
	2.5%	-0.006	-0.025	-0.012	-0.065	-0.064	-0.147	-1.181	-1.188	-2.27	-1.948	-1.271	-1.273	-0.048	-0.06
	97.5%	0.037	0.049	0.078	0.115	0.156	0.252	0.569	0.588	1.389	1.386	0.717	0.823	0.051	0.059
Skew		1.056	-0.047	0.983	-0.139	1.12	-0.42	-0.64	-0.189	-0.636	-0.251	-0.757	-0.407	0.1	0.146
Kurtosis		2.6	-0.43	1.711	-0.515	3.312	-0.184	2.166	-0.687	2.066	-0.506	2.081	-0.443	1.243	1.988

Condition 12; Scenario 3; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	1.023	1.201	0.803	0.936	0.353	0.945	-0.081	-0.026	-0.03	-0.005
	$\widehat{M}$	0.776	1.2	0.615	0.954	0.197	1.087	-0.094	-0.044	-0.026	-0.007
	$\widehat{\sigma}^2$	1.273	3.608	0.768	2.372	0.654	1.615	0.03	0.042	0.027	0.011
	2.5%	-0.559	-2.261	-0.422	-1.902	-1.012	-1.7	-0.38	-0.378	-0.348	-0.214
	97.5%	3.913	4.556	2.969	3.54	2.516	3.051	0.29	0.401	0.307	0.202
P	$\widehat{\mu}$	0.01	0.013	0.023	0.03	0.024	0.079	-0.163	-0.088	-0.074	-0.001
	$\widehat{M}$	0.008	0.013	0.018	0.031	0.013	0.091	-0.188	-0.147	-0.064	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.003	0.011	0.119	0.469	0.166	< .001
	2.5%	-0.006	-0.024	-0.012	-0.062	-0.067	-0.142	-0.76	-1.26	-0.871	-0.027
	97.5%	0.039	0.048	0.085	0.115	0.168	0.256	0.58	1.337	0.767	0.025
Skew		1.129	-0.077	0.995	-0.15	1.082	0.011	0.272	0.293	-0.034	-0.155
Kurtosis		2.787	-0.506	1.398	-0.62	2.923	-0.203	-0.421	-0.391	-0.216	0.487

Condition 12; Scenario 3; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.726	2.387	0.529	1.778	0.254	1.53	0.049	0.098	0.049	–
	$\widehat{M}$	0.661	2.502	0.462	1.922	0.182	1.72	0.052	0.096	0.051	–
	$\widehat{\sigma}^2$	0.376	2.042	0.26	1.502	0.246	0.811	0.005	0.007	0.005	–
	2.5%	-0.32	-0.531	-0.338	-0.617	-0.603	-0.523	-0.076	-0.051	-0.093	–
	97.5%	1.994	4.626	1.641	3.604	1.501	2.965	0.196	0.272	0.193	–
P	$\widehat{\mu}$	0.007	0.025	0.015	0.058	0.017	0.128	0.099	0.328	0.122	–
	$\widehat{M}$	0.007	0.026	0.013	0.062	0.012	0.144	0.105	0.319	0.128	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.001	0.006	0.019	0.074	0.032	–
	2.5%	-0.003	-0.006	-0.01	-0.02	-0.04	-0.044	-0.152	-0.171	-0.232	–
	97.5%	0.02	0.049	0.047	0.117	0.1	0.248	0.392	0.908	0.483	–
Skew		0.291	-0.406	0.591	-0.601	0.93	-0.782	0.124	0.2	-0.063	–
Kurtosis		1.158	-0.097	1.564	0.424	3.439	0.918	0.167	0.229	0.124	–

TABLE C-37

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 13 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 13; Scenario 1; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.514	-0.208	0.402	-0.142	0.162	-0.03	-0.144	-0.07	-0.133	-0.055	-0.097	-0.097	< .001	-0.003
	$\widehat{M}$	0.486	-0.222	0.385	-0.167	0.178	-0.061	-0.164	-0.083	-0.145	-0.072	-0.1	-0.1	-0.004	< .001
	$\widehat{\sigma}^2$	1.346	0.911	0.879	0.797	0.654	0.541	0.047	0.029	0.074	0.043	0.047	0.047	0.03	0.028
	2.5%	-1.756	-2.052	-1.486	-1.86	-1.505	-1.356	-0.57	-0.395	-0.611	-0.414	-0.538	-0.538	-0.314	-0.331
	97.5%	2.888	1.732	2.238	1.614	1.832	1.48	0.288	0.275	0.408	0.392	0.318	0.318	0.362	0.324
P	$\widehat{\mu}$	0.005	-0.002	0.012	-0.005	0.011	-0.003	-0.289	-0.14	-0.444	-0.182	-0.242	-0.242	< .001	-0.001
	$\widehat{M}$	0.005	-0.002	0.011	-0.005	0.012	-0.061	-0.327	-0.166	-0.485	-0.243	-0.249	-0.249	-0.001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.003	0.004	0.188	0.114	0.817	0.475	0.293	0.293	0.002	0.002
	2.5%	-0.018	-0.022	-0.042	-0.06	-0.1	-0.114	-1.14	-0.789	-2.038	-1.379	-1.345	-1.345	-0.078	-0.083
	97.5%	0.029	0.018	0.064	0.052	0.122	0.124	0.576	0.55	1.361	1.307	0.794	0.794	0.091	0.081
Skew		0.482	0.004	0.422	-0.177	0.182	0.261	0.038	0.172	0.012	0.316	-0.033	-0.033	0.163	-0.009
Kurtosis		2.76	0.903	2.357	1.717	1.844	2.569	0.103	-0.224	0.027	-0.252	-0.193	-0.193	0.284	0.517

Condition 13; Scenario 1; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.526	-0.21	0.409	-0.143	0.166	-0.031	-0.083	-0.07	-0.058	-0.003
	$\widehat{M}$	0.489	-0.22	0.393	-0.164	0.18	-0.062	-0.102	-0.091	-0.061	-0.001
	$\widehat{\sigma}^2$	1.374	0.903	0.891	0.794	0.661	0.54	0.027	0.044	0.03	0.013
	2.5%	-1.772	-2.026	-1.491	-1.852	-1.513	-1.358	-0.358	-0.41	-0.363	-0.23
	97.5%	2.923	1.723	2.27	1.614	1.849	1.478	0.28	0.373	0.298	0.212
P	$\widehat{\mu}$	0.005	-0.002	0.012	-0.005	0.011	-0.003	-0.166	-0.234	-0.145	-0.001
	$\widehat{M}$	0.005	-0.002	0.011	-0.005	0.012	-0.005	-0.205	-0.304	-0.154	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.003	0.004	0.11	0.484	0.188	0.001
	2.5%	-0.018	-0.021	-0.043	-0.06	-0.101	-0.114	-0.716	-1.366	-0.906	-0.057
	97.5%	0.029	0.018	0.065	0.052	0.123	0.124	0.561	1.244	0.746	0.053
Skew		0.495	0.017	0.438	-0.171	0.196	0.268	0.414	0.394	0.211	-0.121
Kurtosis		2.782	0.904	2.393	1.715	1.869	2.568	-0.268	-0.4	-0.451	-0.021

Condition 13; Scenario 1; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.503	-0.267	0.361	-0.16	0.147	-0.067	-0.141	-0.078	-0.135	-0.068	-0.091	-0.057	0.004	-0.004
	$\widehat{M}$	0.489	-0.267	0.366	-0.185	0.154	-0.093	-0.162	-0.092	-0.143	-0.091	-0.09	-0.06	0.004	-0.001
	$\widehat{\sigma}^2$	1.123	0.78	0.684	0.67	0.566	0.456	0.042	0.028	0.068	0.041	0.043	0.032	0.027	0.027
	2.5%	-1.488	-1.978	-1.426	-1.777	-1.346	-1.322	-0.521	-0.395	-0.591	-0.423	-0.495	-0.381	-0.302	-0.313
	97.5%	2.636	1.415	2.024	1.525	1.699	1.384	0.285	0.271	0.391	0.369	0.318	0.296	0.339	0.306
P	$\widehat{\mu}$	0.005	-0.003	0.01	-0.005	0.01	-0.006	-0.283	-0.155	-0.449	-0.228	-0.227	-0.142	0.001	-0.001
	$\widehat{M}$	0.005	-0.003	0.01	-0.006	0.01	-0.093	-0.324	-0.184	-0.478	-0.303	-0.227	-0.151	0.001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.003	0.003	0.17	0.111	0.759	0.46	0.271	0.197	0.002	0.002
	2.5%	-0.015	-0.021	-0.041	-0.058	-0.09	-0.111	-1.043	-0.789	-1.971	-1.409	-1.238	-0.952	-0.075	-0.078
	97.5%	0.026	0.015	0.058	0.05	0.113	0.116	0.57	0.542	1.302	1.23	0.794	0.741	0.085	0.077
Skew		0.533	0.057	0.211	-0.021	0.219	0.223	0.18	0.211	0.089	0.308	-0.045	0.062	0.087	0.037
Kurtosis		3.255	1.023	2.204	0.641	2.452	0.539	-0.009	-0.145	-0.105	-0.201	-0.086	-0.256	0.124	0.217

Condition 13; Scenario 1; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.524	-0.245	0.396	-0.153	0.162	-0.06	-0.091	-0.08	-0.061	-0.003
	$\widehat{M}$	0.491	-0.248	0.378	-0.19	0.166	-0.082	-0.111	-0.103	-0.061	< .001
	$\widehat{\sigma}^2$	1.186	0.795	0.732	0.657	0.581	0.463	0.026	0.042	0.029	0.013
	2.5%	-1.532	-1.977	-1.382	-1.741	-1.372	-1.357	-0.362	-0.41	-0.366	-0.23
	97.5%	2.725	1.411	2.136	1.499	1.747	1.404	0.272	0.349	0.286	0.212
P	$\widehat{\mu}$	0.005	-0.003	0.011	-0.005	0.011	-0.005	-0.181	-0.267	-0.153	-0.001
	$\widehat{M}$	0.005	-0.003	0.011	-0.006	0.011	-0.007	-0.222	-0.343	-0.154	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.003	0.003	0.105	0.463	0.18	0.001
	2.5%	-0.015	-0.021	-0.039	-0.057	-0.091	-0.114	-0.724	-1.366	-0.916	-0.057
	97.5%	0.027	0.015	0.061	0.049	0.116	0.118	0.545	1.162	0.715	0.053
Skew		0.505	0.052	0.299	0.021	0.206	0.003	0.453	0.433	0.196	-0.121
Kurtosis		3.055	0.984	1.909	0.42	2.214	0.521	-0.139	-0.322	-0.352	-0.022

Condition 13; Scenario 1; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.188	0.219	0.117	0.172	-0.006	0.202	0.035	0.065	0.027	–
	$\widehat{M}$	0.204	0.16	0.163	0.127	0.015	0.154	0.032	0.061	0.026	–
	$\widehat{\sigma}^2$	0.861	0.632	0.576	0.525	0.467	0.388	0.01	0.017	0.013	–
	2.5%	-1.702	-1.193	-1.484	-1.112	-1.534	-0.918	-0.159	-0.167	-0.18	–
	97.5%	1.944	1.941	1.518	1.661	1.286	1.562	0.241	0.33	0.254	–
P	$\widehat{\mu}$	0.002	0.002	0.003	0.006	< .001	0.017	0.071	0.218	0.067	–
	$\widehat{M}$	0.002	0.002	0.005	0.004	0.001	0.013	0.062	0.202	0.065	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.002	0.003	0.041	0.186	0.08	–
	2.5%	-0.017	-0.013	-0.042	-0.036	-0.102	-0.077	-0.317	-0.556	-0.45	–
	97.5%	0.019	0.021	0.043	0.054	0.086	0.131	0.482	1.099	0.636	–
Skew		0.082	0.286	-0.046	0.232	-0.141	0.548	0.053	0.166	0.136	–
Kurtosis		1.726	0.45	1.236	0.876	1.12	1.569	0.177	-0.058	-0.183	–

TABLE C-38

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 13 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 13; Scenario 2; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.515	0.165	0.455	0.114	0.177	0.224	-0.083	-0.08	-0.049	-0.046	-0.052	-0.052	0.012	-0.003
	$\widehat{M}$	0.383	0.094	0.359	0.071	0.092	0.167	-0.088	-0.083	-0.06	-0.045	-0.042	-0.042	0.007	-0.007
	$\widehat{\sigma}^2$	1.037	3.018	0.833	2.115	0.643	1.483	0.038	0.05	0.054	0.076	0.039	0.039	0.024	0.05
	2.5%	-1.154	-3.05	-0.965	-2.565	-1.179	-2.081	-0.481	-0.515	-0.481	-0.551	-0.452	-0.452	-0.267	-0.398
	97.5%	2.845	3.405	2.637	2.897	2.072	2.547	0.321	0.337	0.433	0.465	0.333	0.333	0.32	0.444
P	$\widehat{\mu}$	0.005	0.002	0.013	0.004	0.012	0.019	-0.166	-0.16	-0.162	-0.153	-0.129	-0.129	0.003	-0.001
	$\widehat{M}$	0.004	0.001	0.01	0.002	0.006	0.167	-0.177	-0.166	-0.203	-0.152	-0.106	-0.106	0.002	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.002	0.003	0.01	0.154	0.202	0.601	0.849	0.246	0.246	0.002	0.003
	2.5%	-0.012	-0.032	-0.028	-0.083	-0.079	-0.174	-0.962	-1.031	-1.605	-1.837	-1.129	-1.129	-0.067	-0.1
	97.5%	0.028	0.036	0.075	0.094	0.138	0.213	0.642	0.673	1.442	1.551	0.833	0.833	0.08	0.111
Skew		0.99	-0.083	1.586	-0.027	2.035	-0.246	-0.305	-0.101	-0.011	-0.119	-0.436	-0.436	0.278	0.287
Kurtosis		2.979	0.304	7.787	0.943	12.844	1.435	1.425	-0.506	0.204	-0.386	1.803	1.803	0.473	1.808

Condition 13; Scenario 2; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.52	0.165	0.457	0.114	0.178	0.224	-0.06	-0.026	-0.033	0.004
	$\widehat{M}$	0.385	0.09	0.36	0.073	0.09	0.164	-0.077	-0.045	-0.034	0.005
	$\widehat{\sigma}^2$	1.057	2.995	0.843	2.105	0.647	1.474	0.03	0.046	0.032	0.013
	2.5%	-1.149	-3.046	-0.969	-2.559	-1.181	-2.065	-0.374	-0.393	-0.379	-0.22
	97.5%	2.86	3.391	2.682	2.894	2.095	2.545	0.296	0.414	0.308	0.23
P	$\widehat{\mu}$	0.005	0.002	0.013	0.004	0.012	0.019	-0.119	-0.085	-0.083	0.001
	$\widehat{M}$	0.004	0.001	0.01	0.002	0.006	0.014	-0.154	-0.149	-0.085	0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.002	0.003	0.01	0.12	0.511	0.201	0.001
	2.5%	-0.011	-0.032	-0.028	-0.083	-0.079	-0.173	-0.747	-1.31	-0.948	-0.055
	97.5%	0.029	0.036	0.077	0.094	0.14	0.213	0.591	1.381	0.772	0.058
Skew		1.009	-0.078	1.597	-0.025	2.045	-0.243	0.215	0.282	-0.004	-0.028
Kurtosis		2.976	0.306	7.717	0.96	12.793	1.452	-0.292	-0.518	-0.434	-0.295

Condition 13; Scenario 2; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.504	0.161	0.4	0.167	0.144	0.23	-0.079	-0.082	-0.051	-0.054	-0.047	-0.047	-0.006	-0.005
	$\widehat{M}$	0.392	0.087	0.346	0.097	0.107	0.154	-0.085	-0.085	-0.069	-0.064	-0.043	-0.039	-0.006	-0.006
	$\widehat{\sigma}^2$	0.881	2.819	0.622	1.963	0.467	1.368	0.035	0.051	0.052	0.079	0.036	0.054	0.04	0.047
	2.5%	-1.131	-2.958	-0.977	-2.318	-1.194	-1.907	-0.428	-0.513	-0.469	-0.562	-0.421	-0.498	-0.394	-0.396
	97.5%	2.624	3.405	2.304	2.959	1.665	2.547	0.314	0.328	0.433	0.467	0.348	0.349	0.373	0.419
P	$\widehat{\mu}$	0.005	0.002	0.011	0.005	0.01	0.019	-0.157	-0.165	-0.171	-0.179	-0.117	-0.116	-0.002	-0.001
	$\widehat{M}$	0.004	0.001	0.01	0.003	0.007	0.154	-0.171	-0.171	-0.229	-0.212	-0.108	-0.098	-0.002	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.002	0.002	0.01	0.139	0.204	0.583	0.875	0.222	0.338	0.003	0.003
	2.5%	-0.011	-0.031	-0.028	-0.075	-0.08	-0.16	-0.857	-1.027	-1.564	-1.873	-1.053	-1.246	-0.098	-0.099
	97.5%	0.026	0.036	0.066	0.096	0.111	0.213	0.627	0.656	1.442	1.556	0.869	0.873	0.093	0.105
Skew		0.806	0.01	0.72	0.187	0.498	0.081	-0.078	-0.093	0.162	-0.053	-0.034	-0.165	0.049	0.124
Kurtosis		2.348	0.352	3.143	1.032	2.494	0.761	0.506	-0.552	-0.161	-0.53	-0.012	-0.615	1.335	0.96

Condition 13; Scenario 2; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.525	0.175	0.414	0.176	0.156	0.229	-0.062	-0.032	-0.034	0.004
	$\widehat{M}$	0.408	0.129	0.358	0.122	0.099	0.16	-0.085	-0.055	-0.035	0.005
	$\widehat{\sigma}^2$	0.898	2.773	0.651	1.96	0.496	1.371	0.031	0.047	0.034	0.013
	2.5%	-1.06	-2.826	-0.981	-2.31	-1.181	-1.939	-0.374	-0.399	-0.382	-0.22
	97.5%	2.717	3.365	2.329	2.964	1.809	2.545	0.299	0.419	0.331	0.23
P	$\widehat{\mu}$	0.005	0.002	0.012	0.006	0.01	0.019	-0.124	-0.108	-0.086	0.001
	$\widehat{M}$	0.004	0.001	0.01	0.004	0.007	0.013	-0.169	-0.184	-0.088	0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.002	0.002	0.01	0.123	0.528	0.212	0.001
	2.5%	-0.011	-0.03	-0.028	-0.075	-0.079	-0.162	-0.747	-1.33	-0.955	-0.055
	97.5%	0.027	0.036	0.067	0.096	0.121	0.213	0.598	1.398	0.827	0.058
Skew		0.763	0.027	0.756	0.132	0.726	0.01	0.256	0.339	0.033	-0.032
Kurtosis		1.894	0.035	3.105	0.816	2.497	0.712	-0.31	-0.523	-0.469	-0.295

Condition 13; Scenario 2; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.381	0.979	0.3	0.726	0.105	0.659	0.048	0.092	0.043	–
	$\widehat{M}$	0.33	0.895	0.284	0.637	0.077	0.606	0.044	0.088	0.047	–
	$\widehat{\sigma}^2$	0.549	1.945	0.456	1.376	0.397	0.968	0.01	0.016	0.013	–
	2.5%	-1.035	-1.623	-0.891	-1.426	-1.112	-1.105	-0.142	-0.134	-0.178	–
	97.5%	2.041	3.534	1.641	2.896	1.512	2.57	0.257	0.345	0.258	–
P	$\widehat{\mu}$	0.004	0.01	0.009	0.024	0.007	0.055	0.095	0.307	0.108	–
	$\widehat{M}$	0.003	0.009	0.008	0.021	0.005	0.051	0.09	0.294	0.117	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.002	0.007	0.04	0.176	0.079	–
	2.5%	-0.01	-0.017	-0.025	-0.046	-0.074	-0.092	-0.284	-0.445	-0.445	–
	97.5%	0.02	0.037	0.047	0.094	0.101	0.215	0.514	1.15	0.644	–
Skew		0.229	-0.092	1.39	-0.008	1.687	-0.208	0.084	0.151	-0.048	–
Kurtosis		1.153	0.4	15.177	1.142	14.314	1.681	0.092	-0.242	-0.157	–

TABLE C-39

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 13 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 13; Scenario 3; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.993	1.452	0.785	1.021	0.417	1.005	-0.084	-0.067	-0.052	-0.011	-0.045	-0.045	-0.007	-0.001
	$\widehat{M}$	0.736	1.619	0.59	1.289	0.203	1.108	-0.073	-0.033	-0.048	0.013	-0.025	-0.025	-0.011	-0.008
	$\widehat{\sigma}^2$	1.759	4.758	1.23	3.585	1.023	1.928	0.056	0.071	0.078	0.088	0.053	0.053	0.027	0.057
	2.5%	-0.955	-3.082	-0.78	-3.093	-1.239	-1.897	-0.569	-0.654	-0.631	-0.621	-0.565	-0.565	-0.322	-0.428
	97.5%	3.96	4.981	3.418	3.908	2.751	3.34	0.33	0.35	0.462	0.485	0.355	0.355	0.32	0.511
P	$\widehat{\mu}$	0.01	0.015	0.022	0.033	0.028	0.084	-0.168	-0.135	-0.174	-0.038	-0.112	-0.112	-0.002	< .001
	$\widehat{M}$	0.007	0.017	0.017	0.042	0.013	1.108	-0.147	-0.066	-0.161	0.045	-0.061	-0.061	-0.003	-0.002
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.004	0.005	0.014	0.225	0.284	0.864	0.979	0.334	0.334	0.002	0.004
	2.5%	-0.01	-0.033	-0.022	-0.101	-0.083	-0.159	-1.139	-1.308	-2.104	-2.069	-1.413	-1.413	-0.081	-0.107
	97.5%	0.04	0.053	0.098	0.127	0.183	0.28	0.66	0.7	1.54	1.617	0.886	0.886	0.08	0.128
Skew		1.097	-0.395	1.738	-0.645	1.232	-0.389	-0.82	-0.643	-0.524	-0.396	-1.084	-1.084	-0.183	0.601
Kurtosis		2.917	-0.264	7.49	0.389	5.427	0.202	2.653	0.179	1.194	-0.146	3.812	3.812	1.975	3.185

Condition 13; Scenario 3; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	1.001	1.45	0.789	1.019	0.42	1.004	-0.046	-0.006	-0.014	-0.006
	$\widehat{M}$	0.733	1.615	0.585	1.278	0.204	1.103	-0.045	-0.016	-0.001	-0.008
	$\widehat{\sigma}^2$	1.781	4.731	1.243	3.568	1.029	1.915	0.036	0.051	0.035	0.013
	2.5%	-0.956	-3.069	-0.781	-3.079	-1.232	-1.89	-0.397	-0.41	-0.403	-0.23
	97.5%	3.991	4.977	3.388	3.91	2.77	3.339	0.31	0.428	0.329	0.232
P	$\widehat{\mu}$	0.01	0.015	0.023	0.033	0.028	0.084	-0.092	-0.022	-0.035	-0.002
	$\widehat{M}$	0.007	0.017	0.017	0.042	0.014	0.092	-0.09	-0.053	-0.003	-0.002
	$\widehat{\sigma}^2$	< .001	0.001	0.001	0.004	0.005	0.013	0.143	0.563	0.217	0.001
	2.5%	-0.01	-0.032	-0.022	-0.1	-0.082	-0.158	-0.795	-1.368	-1.007	-0.057
	97.5%	0.04	0.053	0.097	0.127	0.185	0.28	0.619	1.427	0.823	0.058
Skew		1.094	-0.39	1.751	-0.643	1.234	-0.386	-0.074	0.053	-0.298	0.058
Kurtosis		2.86	-0.261	7.554	0.394	5.345	0.206	-0.526	-0.446	-0.164	0.141

Condition 13; Scenario 3; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.973	1.564	0.753	1.156	0.351	1.08	-0.081	-0.061	-0.045	-0.006	-0.033	-0.022	-0.003	-0.003
	$\widehat{M}$	0.767	1.775	0.611	1.351	0.22	1.129	-0.078	-0.026	-0.052	0.017	-0.021	0.028	-0.008	-0.005
	$\widehat{\sigma}^2$	1.337	4.143	0.899	2.911	0.791	1.778	0.051	0.071	0.069	0.087	0.047	0.069	0.043	0.05
	2.5%	-0.936	-2.841	-0.774	-2.504	-1.386	-1.612	-0.537	-0.651	-0.599	-0.622	-0.509	-0.572	-0.379	-0.413
	97.5%	3.58	4.897	3.019	3.92	2.376	3.438	0.328	0.35	0.459	0.481	0.362	0.386	0.4	0.456
P	$\widehat{\mu}$	0.01	0.017	0.022	0.038	0.023	0.09	-0.163	-0.123	-0.151	-0.02	-0.083	-0.054	-0.001	-0.001
	$\widehat{M}$	0.008	0.019	0.017	0.044	0.015	1.129	-0.155	-0.051	-0.173	0.055	-0.053	0.07	-0.002	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.004	0.012	0.205	0.282	0.772	0.961	0.292	0.434	0.003	0.003
	2.5%	-0.009	-0.03	-0.022	-0.081	-0.092	-0.135	-1.074	-1.302	-1.997	-2.075	-1.272	-1.431	-0.095	-0.103
	97.5%	0.036	0.052	0.086	0.127	0.158	0.288	0.657	0.7	1.529	1.603	0.907	0.964	0.1	0.114
Skew		0.696	-0.394	1.103	-0.533	0.458	-0.283	-0.566	-0.608	-0.245	-0.341	-0.658	-0.6	0.379	0.358
Kurtosis		1.782	-0.056	4.937	0.372	1.883	-0.003	1.974	0.005	0.405	-0.453	1.909	-0.085	3.698	2.854

Condition 13; Scenario 3; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.996	1.63	0.735	1.255	0.36	1.123	-0.041	-0.003	-0.009	-0.006
	$\widehat{M}$	0.782	1.852	0.613	1.442	0.233	1.185	-0.046	-0.017	0.006	-0.008
	$\widehat{\sigma}^2$	1.316	3.942	0.953	2.91	0.801	1.751	0.039	0.054	0.038	0.013
	2.5%	-0.891	-2.528	-1.176	-2.456	-1.38	-1.555	-0.397	-0.407	-0.41	-0.231
	97.5%	3.51	4.892	3.107	4.137	2.392	3.51	0.328	0.437	0.351	0.232
P	$\widehat{\mu}$	0.01	0.017	0.021	0.041	0.024	0.094	-0.081	-0.009	-0.023	-0.002
	$\widehat{M}$	0.008	0.02	0.018	0.047	0.015	0.1	-0.093	-0.056	0.016	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.004	0.012	0.157	0.605	0.239	0.001
	2.5%	-0.009	-0.027	-0.034	-0.08	-0.092	-0.13	-0.795	-1.357	-1.025	-0.058
	97.5%	0.035	0.052	0.089	0.134	0.159	0.294	0.655	1.455	0.877	0.058
Skew		0.715	-0.439	0.615	-0.421	0.233	0.012	-0.041	0.041	-0.262	0.054
Kurtosis		1.55	-0.039	1.901	0.191	1.609	0.09	-0.679	-0.62	-0.291	0.14

Condition 13; Scenario 3; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.788	2.245	0.597	1.619	0.32	1.397	0.041	0.085	0.039	–
	$\widehat{M}$	0.69	2.368	0.516	1.817	0.215	1.497	0.046	0.082	0.043	–
	$\widehat{\sigma}^2$	0.818	2.998	0.661	2.279	0.599	1.254	0.011	0.017	0.013	–
	2.5%	-0.827	-1.661	-0.641	-1.603	-1.03	-0.913	-0.17	-0.159	-0.206	–
	97.5%	2.616	4.956	2.313	3.912	1.92	3.326	0.252	0.347	0.256	–
P	$\widehat{\mu}$	0.008	0.024	0.017	0.053	0.021	0.117	0.083	0.282	0.098	–
	$\widehat{M}$	0.007	0.025	0.015	0.059	0.014	0.125	0.093	0.274	0.109	–
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.002	0.003	0.009	0.045	0.188	0.082	–
	2.5%	-0.008	-0.018	-0.018	-0.052	-0.069	-0.076	-0.341	-0.531	-0.514	–
	97.5%	0.026	0.052	0.066	0.127	0.128	0.278	0.504	1.158	0.64	–
Skew		0.686	-0.567	1.987	-0.823	1.491	-0.469	-0.043	0.085	-0.274	–
Kurtosis		4.375	0.371	14.182	1.249	11.019	0.649	-0.048	-0.146	0.144	–

TABLE C-40

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 14 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 14; Scenario 1; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	0.639	-0.267	0.53	-0.225	0.297	-0.128	-0.214	-0.116	-0.207	-0.106	-0.151	-0.151	0.014	-0.007
	$\widehat{M}$	0.621	-0.269	0.52	-0.202	0.316	-0.108	-0.225	-0.128	-0.221	-0.12	-0.158	-0.158	0.018	-0.004
B	$\widehat{\sigma}^2$	0.771	0.492	0.434	0.407	0.281	0.249	0.025	0.014	0.039	0.019	0.024	0.024	0.013	0.011
	2.5%	-1.19	-1.725	-0.705	-1.52	-0.711	-1.173	-0.478	-0.326	-0.561	-0.33	-0.433	-0.433	-0.219	-0.205
	97.5%	2.409	1.284	1.811	1.061	1.32	0.8	0.133	0.155	0.215	0.199	0.152	0.152	0.24	0.195
	$\widehat{\mu}$	0.006	-0.003	0.015	-0.007	0.02	-0.011	-0.427	-0.232	-0.689	-0.352	-0.377	-0.377	0.003	-0.002
	$\widehat{M}$	0.006	-0.003	0.015	-0.007	0.021	-0.108	-0.45	-0.255	-0.737	-0.399	-0.394	-0.394	0.005	-0.001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.098	0.056	0.429	0.215	0.151	0.151	0.001	0.001
	2.5%	-0.012	-0.018	-0.02	-0.049	-0.047	-0.098	-0.957	-0.654	-1.869	-1.101	-1.083	-1.083	-0.055	-0.051
	97.5%	0.024	0.014	0.052	0.034	0.088	0.067	0.266	0.311	0.716	0.664	0.381	0.381	0.06	0.049
Skew		0.301	0.126	0.27	-0.367	0.154	-0.296	0.371	0.471	0.32	0.584	0.162	0.162	-0.012	-0.029
Kurtosis		3.596	1.57	4.12	3.314	2.093	2.326	0.506	0.556	0.459	0.966	0.343	0.343	0.037	0.483

Condition 14; Scenario 1; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.653	-0.27	0.537	-0.227	0.302	-0.129	-0.138	-0.129	-0.103	-0.001
	$\widehat{M}$	0.636	-0.275	0.528	-0.204	0.315	-0.11	-0.158	-0.148	-0.112	< .001
B	$\widehat{\sigma}^2$	0.792	0.487	0.442	0.404	0.286	0.248	0.013	0.021	0.016	0.006
	2.5%	-1.204	-1.718	-0.71	-1.521	-0.713	-1.16	-0.317	-0.351	-0.329	-0.149
	97.5%	2.444	1.278	1.842	1.059	1.336	0.799	0.138	0.191	0.148	0.143
	$\widehat{\mu}$	0.007	-0.003	0.015	-0.007	0.02	-0.011	-0.275	-0.432	-0.257	< .001
	$\widehat{M}$	0.006	-0.003	0.015	-0.007	0.021	-0.009	-0.317	-0.493	-0.28	< .001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.054	0.23	0.099	< .001
	2.5%	-0.012	-0.018	-0.02	-0.049	-0.048	-0.097	-0.635	-1.17	-0.822	-0.037
	97.5%	0.024	0.014	0.053	0.034	0.089	0.067	0.276	0.636	0.371	0.036
Skew		0.297	0.143	0.276	-0.357	0.163	-0.286	0.842	0.776	0.406	-0.089
Kurtosis		3.426	1.586	4.003	3.309	2.087	2.321	0.763	0.832	0.37	-0.021

Condition 14; Scenario 1; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	0.605	-0.332	0.462	-0.238	0.252	-0.144	-0.206	-0.12	-0.2	-0.112	-0.135	-0.089	0.003	-0.007
	$\widehat{M}$	0.597	-0.331	0.486	-0.228	0.282	-0.126	-0.217	-0.133	-0.207	-0.126	-0.14	-0.093	0.007	-0.003
B	$\widehat{\sigma}^2$	0.602	0.392	0.311	0.329	0.222	0.219	0.022	0.014	0.036	0.019	0.022	0.016	0.012	0.011
	2.5%	-0.903	-1.59	-0.642	-1.382	-0.659	-1.13	-0.465	-0.326	-0.529	-0.345	-0.416	-0.324	-0.209	-0.209
	97.5%	2.119	0.832	1.472	0.973	1.096	0.706	0.134	0.157	0.22	0.193	0.147	0.156	0.219	0.197
	$\widehat{\mu}$	0.006	-0.004	0.013	-0.008	0.017	-0.012	-0.412	-0.241	-0.667	-0.373	-0.338	-0.223	0.001	-0.002
	$\widehat{M}$	0.006	-0.004	0.014	-0.007	0.019	-0.126	-0.435	-0.266	-0.691	-0.417	-0.35	-0.233	0.002	-0.001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.089	0.055	0.395	0.217	0.136	0.099	0.001	0.001
	2.5%	-0.009	-0.017	-0.018	-0.045	-0.044	-0.095	-0.93	-0.653	-1.762	-1.149	-1.041	-0.81	-0.052	-0.052
	97.5%	0.021	0.009	0.042	0.032	0.073	0.059	0.268	0.315	0.732	0.644	0.367	0.389	0.055	0.049
Skew		0.169	0.254	-0.54	0.244	0.165	-0.084	0.568	0.582	0.437	0.606	0.167	0.213	-0.06	-0.082
Kurtosis		2.916	2.139	2.497	1.603	2.763	1.352	0.567	0.718	0.421	0.905	0.276	0.282	0.094	0.464

Condition 14; Scenario 1; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.634	-0.309	0.501	-0.234	0.279	-0.142	-0.143	-0.136	-0.104	-0.001
	$\widehat{M}$	0.621	-0.316	0.508	-0.217	0.303	-0.119	-0.161	-0.152	-0.11	< .001
B	$\widehat{\sigma}^2$	0.645	0.411	0.343	0.33	0.248	0.228	0.013	0.02	0.016	0.006
	2.5%	-0.954	-1.578	-0.646	-1.404	-0.662	-1.138	-0.32	-0.351	-0.329	-0.149
	97.5%	2.267	0.92	1.655	0.957	1.181	0.701	0.133	0.196	0.134	0.144
	$\widehat{\mu}$	0.006	-0.003	0.014	-0.008	0.019	-0.012	-0.286	-0.453	-0.259	< .001
	$\widehat{M}$	0.006	-0.003	0.014	-0.007	0.02	-0.01	-0.323	-0.507	-0.275	< .001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.002	0.051	0.225	0.098	< .001
	2.5%	-0.01	-0.017	-0.018	-0.046	-0.044	-0.095	-0.64	-1.17	-0.822	-0.037
	97.5%	0.023	0.01	0.047	0.031	0.079	0.059	0.267	0.655	0.335	0.036
Skew		-0.038	0.136	-0.315	0.148	0.186	0.002	0.864	0.84	0.419	-0.085
Kurtosis		1.26	1.613	1.675	1.239	2.644	1.402	0.905	1.045	0.559	-0.018

Condition 14; Scenario 1; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.158	0.264	0.125	0.168	0.038	0.168	0.031	0.06	0.023	–
	$\widehat{M}$	0.221	0.252	0.159	0.153	0.073	0.152	0.03	0.056	0.021	–
B	$\widehat{\sigma}^2$	0.411	0.295	0.25	0.223	0.17	0.151	0.004	0.007	0.006	–
	2.5%	-1.29	-0.729	-0.907	-0.687	-0.78	-0.644	-0.091	-0.09	-0.118	–
	97.5%	1.202	1.57	1.059	1.231	0.814	0.93	0.158	0.224	0.175	–
	$\widehat{\mu}$	0.002	0.003	0.004	0.005	0.003	0.014	0.062	0.201	0.058	–
	$\widehat{M}$	0.002	0.003	0.005	0.005	0.005	0.013	0.059	0.186	0.052	–
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.001	0.016	0.075	0.035	–
	2.5%	-0.013	-0.008	-0.026	-0.022	-0.052	-0.054	-0.181	-0.302	-0.296	–
	97.5%	0.012	0.017	0.03	0.04	0.054	0.078	0.316	0.745	0.438	–
Skew		-0.617	0.488	-0.547	0.436	-0.298	0.229	0.174	0.254	0.172	–
Kurtosis		1.51	1.273	1.852	1.306	1.121	1.236	0.245	0.583	0.366	–

TABLE C-41

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 14 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 14; Scenario 2; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\hat{\mu}$	0.591	-0.118	0.492	-0.152	0.182	0.117	-0.127	-0.166	-0.087	-0.108	-0.067	-0.067	0.006	-0.007
	$\hat{M}$	0.373	-0.423	0.325	-0.386	0.1	-0.005	-0.122	-0.186	-0.094	-0.13	-0.069	-0.069	0.007	-0.005
	$\hat{\sigma}^2$	0.981	2.251	0.571	1.507	0.353	1.006	0.027	0.038	0.034	0.055	0.023	0.023	0.009	0.019
	2.5%	-0.679	-2.478	-0.486	-2.164	-0.757	-1.508	-0.45	-0.501	-0.459	-0.51	-0.369	-0.369	-0.183	-0.286
	97.5%	3.518	3.253	2.591	2.506	1.713	2.166	0.206	0.22	0.301	0.364	0.239	0.239	0.189	0.265
P	$\hat{\mu}$	0.006	-0.001	0.014	-0.005	0.012	0.01	-0.254	-0.332	-0.291	-0.36	-0.167	-0.167	0.002	-0.002
	$\hat{M}$	0.004	-0.005	0.009	-0.013	0.007	-0.005	-0.244	-0.372	-0.312	-0.432	-0.17	-0.17	0.002	-0.001
	$\hat{\sigma}^2$	< .001	< .001	< .001	0.002	0.002	0.007	0.106	0.15	0.381	0.609	0.143	0.143	0.001	0.001
	2.5%	-0.007	-0.026	-0.014	-0.07	-0.05	-0.126	-0.899	-1.002	-1.529	-1.699	-0.922	-0.922	-0.046	-0.071
	97.5%	0.035	0.034	0.074	0.081	0.114	0.181	0.414	0.441	1.005	1.214	0.597	0.597	0.047	0.066
Skew		1.653	0.641	1.682	0.533	1.753	0.284	-1.062	0.289	-0.055	0.298	-0.277	-0.277	-0.051	-0.027
Kurtosis		4.44	-0.032	4.386	-0.198	9.925	0.274	7.252	-0.43	1.624	-0.588	2.015	2.015	0.552	0.124

Condition 14; Scenario 2; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\hat{\mu}$	0.594	-0.115	0.493	-0.15	0.183	0.118	-0.11	-0.068	-0.057	0.003
	$\hat{M}$	0.366	-0.416	0.318	-0.386	0.099	-0.004	-0.13	-0.098	-0.072	0.004
	$\hat{\sigma}^2$	1.009	2.226	0.582	1.495	0.357	0.998	0.018	0.031	0.02	0.005
	2.5%	-0.685	-2.463	-0.49	-2.155	-0.748	-1.497	-0.315	-0.347	-0.316	-0.132
	97.5%	3.549	3.246	2.624	2.501	1.723	2.163	0.211	0.331	0.253	0.142
P	$\hat{\mu}$	0.006	-0.001	0.014	-0.005	0.012	0.01	-0.219	-0.228	-0.143	0.001
	$\hat{M}$	0.004	-0.004	0.009	-0.013	0.007	< .001	-0.26	-0.327	-0.179	0.001
	$\hat{\sigma}^2$	< .001	< .001	< .001	0.002	0.002	0.007	0.071	0.341	0.126	< .001
	2.5%	-0.007	-0.026	-0.014	-0.07	-0.05	-0.125	-0.63	-1.158	-0.789	-0.033
	97.5%	0.035	0.034	0.075	0.081	0.115	0.181	0.422	1.102	0.633	0.035
Skew		1.655	0.647	1.69	0.536	1.792	0.286	0.72	0.572	0.389	-0.106
Kurtosis		4.357	-0.015	4.368	-0.188	10.127	0.29	0.301	-0.01	0.085	0.116

Condition 14; Scenario 2; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\hat{\mu}$	0.501	-0.2	0.416	-0.187	0.143	0.068	-0.118	-0.17	-0.084	-0.119	-0.061	-0.102	-0.005	-0.009
	$\hat{M}$	0.356	-0.473	0.311	-0.39	0.091	-0.049	-0.121	-0.191	-0.1	-0.15	-0.07	-0.113	-0.003	-0.005
	$\hat{\sigma}^2$	0.62	1.881	0.36	1.236	0.27	0.944	0.018	0.036	0.031	0.057	0.021	0.037	0.014	0.019
	2.5%	-0.609	-2.345	-0.48	-1.926	-0.771	-1.499	-0.36	-0.498	-0.398	-0.512	-0.336	-0.429	-0.236	-0.278
	97.5%	2.502	3.049	1.92	2.232	1.407	2.171	0.188	0.221	0.333	0.385	0.243	0.297	0.229	0.259
P	$\hat{\mu}$	0.005	-0.002	0.012	-0.006	0.01	0.006	-0.235	-0.34	-0.28	-0.395	-0.153	-0.255	-0.001	-0.002
	$\hat{M}$	0.004	-0.005	0.009	-0.013	0.006	-0.049	-0.242	-0.383	-0.334	-0.499	-0.176	-0.283	-0.001	-0.001
	$\hat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.007	0.07	0.143	0.343	0.631	0.129	0.234	0.001	0.001
	2.5%	-0.006	-0.025	-0.014	-0.063	-0.051	-0.126	-0.721	-0.997	-1.326	-1.705	-0.839	-1.073	-0.059	-0.07
	97.5%	0.025	0.032	0.055	0.073	0.094	0.182	0.377	0.442	1.11	1.285	0.607	0.743	0.057	0.065
Skew		0.944	0.733	1.148	0.614	1.692	0.416	0.389	0.4	0.321	0.38	-0.019	0.241	0.03	-0.015
Kurtosis		2.425	0.447	2.873	-0.053	12.447	0.392	0.982	-0.34	1.097	-0.531	2.17	-0.341	0.161	0.161

Condition 14; Scenario 2; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\hat{\mu}$	0.524	-0.158	0.438	-0.165	0.16	0.07	-0.114	-0.078	-0.061	0.003
	$\hat{M}$	0.344	-0.424	0.323	-0.369	0.099	-0.046	-0.135	-0.112	-0.079	0.005
	$\hat{\sigma}^2$	0.732	1.943	0.417	1.28	0.304	0.957	0.018	0.031	0.02	0.005
	2.5%	-0.648	-2.313	-0.486	-1.998	-0.733	-1.538	-0.313	-0.357	-0.31	-0.131
	97.5%	2.752	3.113	2.182	2.309	1.454	2.152	0.191	0.336	0.274	0.143
P	$\hat{\mu}$	0.005	-0.002	0.013	-0.005	0.011	0.006	-0.227	-0.261	-0.153	0.001
	$\hat{M}$	0.003	-0.004	0.009	-0.012	0.007	-0.004	-0.27	-0.372	-0.197	0.001
	$\hat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.007	0.07	0.349	0.125	< .001
	2.5%	-0.006	-0.024	-0.014	-0.065	-0.049	-0.129	-0.627	-1.19	-0.775	-0.033
	97.5%	0.028	0.033	0.062	0.075	0.097	0.18	0.382	1.12	0.684	0.036
Skew		1.284	0.733	1.315	0.571	1.872	0.007	0.777	0.708	0.495	-0.105
Kurtosis		3.476	0.261	3.421	-0.115	13.259	0.573	0.481	0.23	0.357	0.116

Condition 14; Scenario 2; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\hat{\mu}$	0.337	0.994	0.262	0.695	0.071	0.696	0.043	0.087	0.044	–
	$\hat{M}$	0.31	0.726	0.227	0.526	0.065	0.595	0.037	0.083	0.041	–
	$\hat{\sigma}^2$	0.255	1.316	0.178	0.885	0.14	0.573	0.004	0.008	0.006	–
	2.5%	-0.591	-0.809	-0.483	-0.834	-0.684	-0.576	-0.075	-0.076	-0.093	–
	97.5%	1.419	3.427	1.197	2.647	0.866	2.257	0.186	0.267	0.203	–
P	$\hat{\mu}$	0.003	0.011	0.007	0.023	0.005	0.058	0.086	0.289	0.111	–
	$\hat{M}$	0.003	0.008	0.007	0.017	0.004	0.05	0.074	0.276	0.103	–
	$\hat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.004	0.018	0.088	0.038	–
	2.5%	-0.006	-0.009	-0.014	-0.027	-0.046	-0.048	-0.15	-0.253	-0.232	–
	97.5%	0.014	0.036	0.034	0.086	0.058	0.189	0.372	0.89	0.506	–
Skew		0.062	0.573	0.552	0.448	0.19	0.296	0.317	0.247	0.258	–
Kurtosis		2.318	-0.236	2.175	-0.291	4.144	0.105	0.05	0.067	0.196	–

TABLE C-42

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 14 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 14; Scenario 3; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	1.006	1.074	0.829	0.717	0.419	0.811	-0.126	-0.137	-0.085	-0.068	-0.075	-0.075	0.007	-0.003
	$\widehat{M}$	0.663	1.044	0.581	0.732	0.193	0.935	-0.117	-0.127	-0.068	-0.057	-0.059	-0.059	0.009	-0.006
	$\widehat{\sigma}^2$	1.558	4.327	1.05	3.063	0.744	1.704	0.044	0.058	0.061	0.065	0.038	0.038	0.013	0.027
	2.5%	-0.5	-3.094	-0.392	-2.477	-0.771	-1.774	-0.608	-0.596	-0.694	-0.556	-0.534	-0.534	-0.205	-0.332
	97.5%	4.153	4.492	3.426	3.569	2.696	2.881	0.253	0.278	0.392	0.395	0.303	0.303	0.226	0.324
P	$\widehat{\mu}$	0.01	0.011	0.024	0.023	0.028	0.068	-0.252	-0.274	-0.283	-0.228	-0.188	-0.188	0.002	-0.001
	$\widehat{M}$	0.007	0.011	0.017	0.024	0.013	0.935	-0.235	-0.254	-0.225	-0.191	-0.148	-0.148	0.002	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.003	0.012	0.178	0.231	0.682	0.727	0.239	0.239	0.001	0.002
	2.5%	-0.005	-0.033	-0.011	-0.081	-0.051	-0.149	-1.218	-1.192	-2.315	-1.855	-1.336	-1.336	-0.051	-0.083
	97.5%	0.042	0.048	0.098	0.116	0.18	0.241	0.507	0.557	1.308	1.315	0.757	0.757	0.057	0.081
Skew		1.514	-0.208	1.543	-0.197	1.77	-0.404	-1.167	-0.151	-0.843	-0.212	-0.817	-0.817	0.355	0.468
Kurtosis		3.503	-0.564	4.169	-0.552	5.308	-0.208	5.238	-0.76	2.904	-0.396	2.471	2.471	5.243	5.329

Condition 14; Scenario 3; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	1.013	1.075	0.833	0.717	0.422	0.811	-0.082	-0.037	-0.041	0.002
	$\widehat{M}$	0.668	1.042	0.578	0.73	0.191	0.937	-0.095	-0.05	-0.042	0.002
	$\widehat{\sigma}^2$	1.58	4.291	1.064	3.045	0.75	1.69	0.025	0.033	0.023	0.005
	2.5%	-0.503	-3.084	-0.393	-2.457	-0.762	-1.773	-0.36	-0.383	-0.352	-0.136
	97.5%	4.124	4.483	3.45	3.565	2.723	2.877	0.251	0.357	0.268	0.141
P	$\widehat{\mu}$	0.01	0.011	0.024	0.023	0.028	0.068	-0.163	-0.124	-0.104	< .001
	$\widehat{M}$	0.007	0.011	0.017	0.024	0.013	0.079	-0.19	-0.164	-0.105	0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.003	0.012	0.098	0.371	0.146	< .001
	2.5%	-0.005	-0.033	-0.011	-0.08	-0.051	-0.149	-0.719	-1.278	-0.881	-0.034
	97.5%	0.041	0.047	0.098	0.116	0.181	0.241	0.502	1.19	0.669	0.035
Skew		1.495	-0.204	1.544	-0.195	1.781	-0.402	0.298	0.212	0.01	0.038
Kurtosis		3.362	-0.559	4.132	-0.547	5.332	-0.201	-0.328	-0.112	-0.082	-0.41

Condition 14; Scenario 3; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.924	1.08	0.738	0.763	0.339	0.811	-0.109	-0.128	-0.073	-0.069	-0.061	-0.075	-0.005	-0.003
	$\widehat{M}$	0.683	1.107	0.591	0.738	0.204	0.888	-0.114	-0.121	-0.075	-0.052	-0.054	-0.048	-0.005	-0.004
	$\widehat{\sigma}^2$	1.068	3.841	0.694	2.658	0.515	1.627	0.034	0.057	0.05	0.07	0.032	0.051	0.018	0.025
	2.5%	-0.474	-2.905	-0.392	-2.292	-0.841	-1.68	-0.487	-0.58	-0.545	-0.562	-0.474	-0.524	-0.284	-0.315
	97.5%	3.298	4.427	2.816	3.53	2.102	2.967	0.251	0.281	0.392	0.403	0.303	0.29	0.259	0.322
P	$\widehat{\mu}$	0.009	0.011	0.021	0.025	0.023	0.068	-0.218	-0.257	-0.242	-0.231	-0.154	-0.187	-0.001	-0.001
	$\widehat{M}$	0.007	0.012	0.017	0.024	0.014	0.888	-0.228	-0.243	-0.252	-0.172	-0.137	-0.12	-0.001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.002	0.011	0.136	0.228	0.553	0.777	0.198	0.319	0.001	0.002
	2.5%	-0.005	-0.031	-0.011	-0.075	-0.056	-0.141	-0.974	-1.16	-1.817	-1.874	-1.186	-1.309	-0.071	-0.079
	97.5%	0.033	0.047	0.08	0.115	0.14	0.249	0.502	0.562	1.308	1.345	0.758	0.724	0.065	0.08
Skew		1.08	-0.221	0.944	-0.135	1.149	-0.293	-0.372	-0.102	-0.31	-0.122	-0.422	-0.365	0.382	0.34
Kurtosis		1.965	-0.427	2.257	-0.47	3.661	-0.344	1.925	-0.825	1.74	-0.724	1.92	-0.521	5.494	4.118

Condition 14; Scenario 3; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.931	1.189	0.745	0.852	0.35	0.885	-0.078	-0.031	-0.037	0.002
	$\widehat{M}$	0.712	1.25	0.607	0.867	0.198	0.944	-0.098	-0.049	-0.041	0.001
	$\widehat{\sigma}^2$	1.087	3.838	0.694	2.62	0.551	1.56	0.028	0.038	0.026	0.005
	2.5%	-0.525	-2.772	-0.515	-2.113	-0.947	-1.619	-0.362	-0.396	-0.355	-0.136
	97.5%	3.355	4.498	2.707	3.661	2.246	3.036	0.267	0.389	0.291	0.142
P	$\widehat{\mu}$	0.009	0.013	0.021	0.028	0.023	0.074	-0.155	-0.103	-0.091	0.001
	$\widehat{M}$	0.007	0.013	0.017	0.028	0.013	0.079	-0.196	-0.16	-0.101	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.002	0.011	0.112	0.427	0.163	< .001
	2.5%	-0.005	-0.029	-0.015	-0.069	-0.063	-0.136	-0.725	-1.322	-0.887	-0.034
	97.5%	0.034	0.048	0.077	0.119	0.15	0.254	0.534	1.296	0.728	0.036
Skew		0.963	-0.238	0.693	-0.148	1.242	0.011	0.348	0.241	0.071	0.044
Kurtosis		1.331	-0.444	1.38	-0.474	3.678	-0.264	-0.478	-0.277	-0.167	-0.398

Condition 14; Scenario 3; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.65	2.285	0.504	1.661	0.25	1.423	0.041	0.082	0.037	–
	$\widehat{M}$	0.567	2.393	0.451	1.761	0.19	1.532	0.042	0.077	0.034	–
	$\widehat{\sigma}^2$	0.366	2.222	0.263	1.586	0.229	0.857	0.005	0.007	0.006	–
	2.5%	-0.37	-0.725	-0.343	-0.793	-0.597	-0.566	-0.092	-0.066	-0.112	–
	97.5%	1.991	4.57	1.635	3.621	1.426	2.915	0.182	0.251	0.183	–
P	$\widehat{\mu}$	0.006	0.024	0.014	0.054	0.017	0.119	0.082	0.272	0.092	–
	$\widehat{M}$	0.006	0.025	0.013	0.057	0.013	0.129	0.083	0.257	0.085	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.001	0.006	0.019	0.073	0.035	–
	2.5%	-0.004	-0.008	-0.01	-0.026	-0.04	-0.047	-0.184	-0.222	-0.279	–
	97.5%	0.02	0.048	0.047	0.118	0.095	0.244	0.364	0.837	0.458	–
Skew		0.54	-0.458	0.421	-0.481	0.808	-0.598	0.133	0.196	0.058	–
Kurtosis		1.026	-0.163	2.937	0.234	2.46	0.59	-0.009	0.123	0.335	–

TABLE C-43

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 15 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 15; Scenario 1; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.738	0.007	0.63	0.001	0.346	0.073	-0.198	-0.046	-0.195	-0.032	-0.114	-0.114	0.007	< .001
	$\widehat{M}$	0.746	-0.037	0.605	-0.024	0.313	0.094	-0.178	-0.05	-0.176	-0.042	-0.112	-0.112	-0.007	-0.009
	$\widehat{\sigma}^2$	2.376	1.22	1.427	0.851	1.081	0.489	0.103	0.035	0.151	0.045	0.077	0.077	0.138	0.099
	2.5%	-2.312	-2.25	-1.813	-1.979	-1.564	-1.325	-0.833	-0.395	-0.97	-0.417	-0.665	-0.665	-0.702	-0.597
	97.5%	3.893	2.264	2.974	1.747	2.635	1.41	0.36	0.313	0.472	0.402	0.39	0.39	0.754	0.626
P	$\widehat{\mu}$	0.007	< .001	0.018	< .001	0.023	0.006	-0.395	-0.092	-0.651	-0.107	-0.286	-0.286	0.001	< .001
	$\widehat{M}$	0.007	< .001	0.017	-0.001	0.021	0.094	-0.356	-0.099	-0.588	-0.137	-0.28	-0.28	-0.001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.005	0.003	0.41	0.14	1.678	0.502	0.484	0.484	0.002	0.002
	2.5%	-0.023	-0.024	-0.052	-0.064	-0.104	-0.111	-1.665	-0.79	-3.235	-1.39	-1.663	-1.663	-0.088	-0.075
	97.5%	0.039	0.024	0.085	0.057	0.176	0.118	0.72	0.626	1.573	1.338	0.974	0.974	0.094	0.078
Skew		0.554	-0.096	0.662	-0.191	0.69	0.105	-0.353	0.02	-0.224	0.113	-0.334	-0.334	0.223	0.137
Kurtosis		3.061	0.718	4.266	0.485	3.319	1.086	0.164	-0.461	-0.526	-0.278	0.148	0.148	1.276	0.408

Condition 15; Scenario 1; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.752	0.003	0.638	-0.001	0.347	0.072	-0.068	-0.055	-0.037	-0.009
	$\widehat{M}$	0.776	-0.045	0.607	-0.028	0.309	0.091	-0.075	-0.063	-0.035	-0.013
	$\widehat{\sigma}^2$	2.399	1.208	1.443	0.846	1.091	0.489	0.037	0.053	0.036	0.053
	2.5%	-2.322	-2.213	-1.829	-1.969	-1.576	-1.327	-0.396	-0.46	-0.394	-0.445
	97.5%	3.9	2.254	2.959	1.752	2.642	1.407	0.309	0.394	0.332	0.446
P	$\widehat{\mu}$	0.008	< .001	0.018	< .001	0.023	0.006	-0.136	-0.183	-0.093	-0.001
	$\widehat{M}$	0.008	< .001	0.017	-0.001	0.021	0.008	-0.15	-0.211	-0.088	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.005	0.003	0.148	0.585	0.222	0.001
	2.5%	-0.023	-0.023	-0.052	-0.064	-0.105	-0.111	-0.793	-1.533	-0.986	-0.056
	97.5%	0.039	0.024	0.085	0.057	0.176	0.118	0.618	1.313	0.831	0.056
Skew		0.541	-0.074	0.643	-0.178	0.69	0.109	0.161	0.169	0.022	0.107
Kurtosis		3.005	0.707	4.086	0.485	3.296	1.073	-0.666	-0.588	-0.364	0.11

Condition 15; Scenario 1; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.698	-0.062	0.574	-0.036	0.301	0.037	-0.197	-0.053	-0.196	-0.04	-0.11	-0.028	0.005	0.001
	$\widehat{M}$	0.721	-0.084	0.567	-0.044	0.28	0.075	-0.178	-0.055	-0.182	-0.051	-0.108	-0.019	-0.004	-0.008
	$\widehat{\sigma}^2$	2.167	1.177	1.222	0.798	0.937	0.497	0.1	0.035	0.148	0.046	0.074	0.034	0.101	0.097
	2.5%	-2.198	-2.32	-1.748	-1.945	-1.53	-1.401	-0.828	-0.395	-0.953	-0.427	-0.664	-0.398	-0.605	-0.595
	97.5%	3.631	2.149	2.742	1.692	2.428	1.343	0.362	0.313	0.466	0.402	0.378	0.33	0.631	0.626
P	$\widehat{\mu}$	0.007	-0.001	0.016	-0.001	0.02	0.003	-0.394	-0.105	-0.653	-0.133	-0.274	-0.07	0.001	< .001
	$\widehat{M}$	0.007	-0.001	0.016	-0.001	0.019	0.075	-0.355	-0.11	-0.609	-0.168	-0.269	-0.049	< .001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.004	0.003	0.399	0.139	1.644	0.509	0.464	0.211	0.002	0.002
	2.5%	-0.022	-0.025	-0.05	-0.063	-0.102	-0.117	-1.656	-0.79	-3.178	-1.422	-1.66	-0.996	-0.076	-0.074
	97.5%	0.036	0.023	0.078	0.055	0.162	0.112	0.725	0.626	1.554	1.338	0.946	0.826	0.079	0.078
Skew		0.469	-0.134	0.22	-0.104	0.657	-0.061	-0.369	0.046	-0.219	0.161	-0.383	-0.128	0.06	0.114
Kurtosis		3.08	1.029	2.121	0.206	3.21	0.817	0.264	-0.422	-0.506	-0.237	0.253	0.127	0.294	0.339

Condition 15; Scenario 1; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.687	-0.099	0.564	-0.064	0.296	0.01	-0.076	-0.066	-0.044	-0.008
	$\widehat{M}$	0.718	-0.104	0.562	-0.073	0.279	0.05	-0.091	-0.077	-0.04	-0.013
	$\widehat{\sigma}^2$	2.128	1.148	1.206	0.806	0.92	0.496	0.036	0.052	0.035	0.053
	2.5%	-2.19	-2.303	-1.71	-1.982	-1.536	-1.412	-0.396	-0.461	-0.394	-0.444
	97.5%	3.633	1.985	2.776	1.689	2.396	1.312	0.309	0.41	0.315	0.446
P	$\widehat{\mu}$	0.007	-0.001	0.016	-0.002	0.02	0.001	-0.152	-0.219	-0.111	-0.001
	$\widehat{M}$	0.007	-0.001	0.016	-0.002	0.019	0.004	-0.182	-0.257	-0.1	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.004	0.003	0.144	0.58	0.217	0.001
	2.5%	-0.022	-0.024	-0.049	-0.064	-0.102	-0.118	-0.793	-1.538	-0.986	-0.056
	97.5%	0.036	0.021	0.079	0.055	0.16	0.11	0.618	1.368	0.789	0.056
Skew		0.49	-0.119	0.301	-0.205	0.634	0.003	0.227	0.253	0.019	0.105
Kurtosis		3.152	0.947	1.74	0.447	3.285	0.842	-0.568	-0.469	-0.369	0.114

Condition 15; Scenario 1; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.384	0.471	0.293	0.315	0.151	0.323	0.059	0.098	0.045	–
	$\widehat{M}$	0.424	0.405	0.314	0.298	0.16	0.323	0.059	0.098	0.044	–
	$\widehat{\sigma}^2$	1.502	0.812	0.919	0.564	0.733	0.364	0.011	0.016	0.013	–
	2.5%	-2.096	-1.23	-1.681	-1.11	-1.49	-0.838	-0.135	-0.14	-0.183	–
	97.5%	2.759	2.39	2.09	1.805	1.83	1.524	0.263	0.344	0.274	–
P	$\widehat{\mu}$	0.004	0.005	0.008	0.01	0.01	0.027	0.118	0.328	0.112	–
	$\widehat{M}$	0.004	0.004	0.009	0.01	0.011	0.027	0.119	0.326	0.111	–
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.003	0.003	0.042	0.181	0.082	–
	2.5%	-0.021	-0.013	-0.048	-0.036	-0.099	-0.07	-0.27	-0.467	-0.458	–
	97.5%	0.028	0.025	0.06	0.059	0.122	0.128	0.526	1.146	0.684	–
Skew		0.572	0.201	0.503	0.129	0.656	0.221	0.073	0.036	-0.057	–
Kurtosis		5.465	0.737	5.135	0.166	4.806	1.113	-0.237	-0.177	0.094	–

TABLE C-44

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 15 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 15; Scenario 2; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	0.888	0.581	0.687	0.466	0.317	0.567	-0.117	-0.059	-0.082	0.002	-0.051	-0.051	-0.019	0.005
	$\widehat{M}$	0.718	0.575	0.541	0.476	0.16	0.669	-0.103	-0.032	-0.044	0.03	-0.029	-0.029	-0.005	-0.015
B	$\widehat{\sigma}^2$	2.248	3.295	1.334	2.329	1.044	1.423	0.081	0.068	0.122	0.083	0.074	0.074	0.141	0.185
	2.5%	-1.259	-2.996	-1.087	-2.607	-1.202	-1.829	-0.722	-0.604	-0.906	-0.557	-0.661	-0.661	-0.72	-0.784
	97.5%	4.271	3.808	3.165	3.239	2.654	2.656	0.365	0.359	0.488	0.503	0.387	0.387	0.641	0.901
	$\widehat{\mu}$	0.009	0.006	0.02	0.015	0.021	0.047	-0.235	-0.118	-0.272	0.005	-0.128	-0.128	-0.002	0.001
	$\widehat{M}$	0.007	0.006	0.015	0.015	0.011	0.669	-0.206	-0.064	-0.147	0.102	-0.072	-0.072	-0.001	-0.002
P	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.002	0.005	0.01	0.323	0.274	1.361	0.924	0.461	0.461	0.002	0.003
	2.5%	-0.013	-0.032	-0.031	-0.085	-0.08	-0.153	-1.444	-1.208	-3.023	-1.857	-1.654	-1.654	-0.09	-0.098
	97.5%	0.043	0.04	0.09	0.105	0.177	0.223	0.729	0.719	1.627	1.678	0.968	0.968	0.08	0.113
Skew		1.656	-0.258	1.938	-0.191	1.571	-0.251	-0.607	-0.6	-0.717	-0.233	-1.042	-1.042	-1.349	0.122
Kurtosis		8.475	0.701	11.86	0.052	5.238	0.333	1.373	0.568	0.714	-0.581	2.688	2.688	11.31	1.409

Condition 15; Scenario 2; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.896	0.577	0.691	0.464	0.318	0.565	-0.053	-0.006	-0.004	-0.008
	$\widehat{M}$	0.715	0.565	0.544	0.468	0.164	0.661	-0.048	0.002	0.007	-0.005
B	$\widehat{\sigma}^2$	2.267	3.267	1.348	2.314	1.049	1.413	0.046	0.063	0.041	0.056
	2.5%	-1.264	-2.992	-1.064	-2.595	-1.197	-1.83	-0.443	-0.48	-0.404	-0.462
	97.5%	4.288	3.802	3.225	3.228	2.655	2.652	0.339	0.453	0.362	0.454
	$\widehat{\mu}$	0.009	0.006	0.02	0.015	0.021	0.047	-0.106	-0.02	-0.009	-0.001
	$\widehat{M}$	0.007	0.006	0.016	0.015	0.011	0.055	-0.097	0.005	0.018	-0.001
P	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.002	0.005	0.01	0.183	0.695	0.255	0.001
	2.5%	-0.013	-0.032	-0.03	-0.084	-0.08	-0.153	-0.886	-1.6	-1.01	-0.058
	97.5%	0.043	0.04	0.092	0.105	0.177	0.222	0.679	1.511	0.906	0.057
Skew		1.639	-0.251	1.926	-0.184	1.58	-0.243	-0.033	-0.089	-0.099	0.053
Kurtosis		8.266	0.711	11.602	0.056	5.279	0.341	-0.748	-0.651	-0.597	-0.183

Condition 15; Scenario 2; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	0.817	0.532	0.609	0.463	0.239	0.552	-0.111	-0.064	-0.077	-0.005	-0.04	-0.008	0.006	0.001
	$\widehat{M}$	0.657	0.53	0.515	0.452	0.148	0.615	-0.102	-0.042	-0.046	0.016	-0.024	0.007	-0.011	-0.017
B	$\widehat{\sigma}^2$	1.65	2.902	0.881	2.12	0.724	1.349	0.075	0.067	0.113	0.086	0.06	0.059	0.158	0.18
	2.5%	-1.138	-2.832	-1.083	-2.412	-1.278	-1.752	-0.647	-0.604	-0.847	-0.56	-0.553	-0.501	-0.759	-0.794
	97.5%	3.603	3.497	2.624	3.256	2.282	2.704	0.369	0.359	0.478	0.51	0.38	0.407	0.851	0.893
	$\widehat{\mu}$	0.008	0.006	0.017	0.015	0.016	0.046	-0.223	-0.128	-0.255	-0.017	-0.1	-0.02	0.001	< .001
	$\widehat{M}$	0.007	0.006	0.015	0.015	0.01	0.615	-0.203	-0.083	-0.155	0.053	-0.058	0.019	-0.002	-0.002
P	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.002	0.003	0.009	0.3	0.267	1.254	0.953	0.374	0.371	0.002	0.003
	2.5%	-0.011	-0.03	-0.031	-0.078	-0.085	-0.147	-1.294	-1.208	-2.821	-1.867	-1.383	-1.253	-0.095	-0.099
	97.5%	0.036	0.037	0.075	0.106	0.152	0.227	0.739	0.719	1.595	1.699	0.949	1.017	0.106	0.112
Skew		1.257	-0.156	0.564	-0.045	0.654	-0.043	-0.476	-0.369	-0.668	-0.276	-0.568	-0.323	0.305	0.149
Kurtosis		6.441	0.035	1.906	-0.027	1.607	0.097	0.984	-0.34	0.679	-0.333	0.775	-0.318	0.654	1

Condition 15; Scenario 2; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.833	0.556	0.635	0.467	0.246	0.558	-0.058	-0.008	-0.005	-0.007
	$\widehat{M}$	0.669	0.555	0.53	0.454	0.148	0.631	-0.068	-0.009	0.005	-0.004
B	$\widehat{\sigma}^2$	1.71	2.969	0.955	2.106	0.804	1.375	0.046	0.064	0.043	0.056
	2.5%	-1.144	-2.796	-1.046	-2.402	-1.302	-1.72	-0.443	-0.48	-0.407	-0.462
	97.5%	3.657	3.538	2.746	3.228	2.382	2.705	0.343	0.462	0.367	0.458
	$\widehat{\mu}$	0.008	0.006	0.018	0.015	0.016	0.047	-0.117	-0.028	-0.013	-0.001
	$\widehat{M}$	0.007	0.006	0.015	0.015	0.01	0.053	-0.137	-0.032	0.01	-0.001
P	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.002	0.004	0.01	0.184	0.71	0.266	0.001
	2.5%	-0.011	-0.03	-0.03	-0.078	-0.087	-0.144	-0.886	-1.6	-1.017	-0.058
	97.5%	0.037	0.037	0.078	0.105	0.159	0.227	0.685	1.541	0.919	0.057
Skew		1.253	-0.154	0.673	-0.024	0.802	0.01	0.036	-0.037	-0.059	0.05
Kurtosis		6.106	0.077	2.001	-0.086	2.971	0.27	-0.721	-0.68	-0.632	-0.197

Condition 15; Scenario 2; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.671	1.426	0.47	1.07	0.215	0.991	0.075	0.134	0.07	–
	$\widehat{M}$	0.617	1.453	0.395	1.075	0.133	1.079	0.075	0.137	0.073	–
B	$\widehat{\sigma}^2$	1.171	2.078	0.723	1.443	0.619	0.907	0.01	0.015	0.013	–
	2.5%	-1.064	-1.331	-0.983	-1.25	-1.093	-0.912	-0.126	-0.108	-0.153	–
	97.5%	2.658	4.012	2.009	3.288	1.887	2.693	0.268	0.371	0.296	–
	$\widehat{\mu}$	0.007	0.015	0.013	0.035	0.014	0.083	0.149	0.448	0.174	–
	$\widehat{M}$	0.006	0.015	0.011	0.035	0.009	0.09	0.151	0.457	0.182	–
P	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.002	0.003	0.006	0.041	0.168	0.081	–
	2.5%	-0.011	-0.014	-0.028	-0.041	-0.073	-0.076	-0.252	-0.359	-0.383	–
	97.5%	0.027	0.042	0.057	0.107	0.126	0.225	0.535	1.236	0.74	–
Skew		1.976	-0.382	2.143	-0.166	1.575	-0.204	-0.095	-0.083	-0.05	–
Kurtosis		17.066	1.73	18.665	0.125	7.588	0.692	-0.246	-0.143	-0.151	–

TABLE C-45

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 15 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 15; Scenario 3; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\hat{\mu}$	1.388	1.754	1.022	1.416	0.53	1.247	-0.124	-0.032	-0.083	0.032	-0.051	-0.051	-0.035	-0.022
	$\hat{M}$	1.004	2.177	0.754	1.696	0.286	1.433	-0.069	0.005	-0.025	0.062	-0.001	-0.001	-0.025	-0.034
	$\hat{\sigma}^2$	3.204	4.476	2.05	3.072	1.395	1.796	0.119	0.076	0.155	0.088	0.091	0.091	0.139	0.159
	2.5%	-1.142	-2.72	-1.129	-2.46	-1.311	-1.854	-1.052	-0.679	-1.191	-0.607	-0.866	-0.866	-0.778	-0.827
	97.5%	5.621	4.99	4.415	4.096	3.508	3.57	0.382	0.389	0.51	0.536	0.398	0.398	0.614	0.778
P	$\hat{\mu}$	0.014	0.019	0.029	0.046	0.035	0.104	-0.248	-0.063	-0.277	0.107	-0.126	-0.126	-0.004	-0.003
	$\hat{M}$	0.01	0.023	0.021	0.055	0.019	1.433	-0.138	0.01	-0.083	0.207	-0.002	-0.002	-0.003	-0.004
	$\hat{\sigma}^2$	< .001	0.001	0.002	0.003	0.006	0.013	0.478	0.305	1.724	0.976	0.567	0.567	0.002	0.002
	2.5%	-0.011	-0.029	-0.032	-0.08	-0.087	-0.155	-2.105	-1.359	-3.97	-2.023	-2.165	-2.165	-0.097	-0.103
	97.5%	0.056	0.053	0.126	0.133	0.234	0.299	0.764	0.778	1.701	1.788	0.995	0.995	0.077	0.097
Skew		2.114	-0.726	1.705	-0.73	1.481	-0.645	-1.116	-0.959	-0.947	-0.562	-1.372	-1.372	-0.732	0.299
Kurtosis		10.228	0.592	7.316	0.764	4.095	1.231	1.698	1.841	0.98	0.256	3.205	3.205	3.907	2.67

Condition 15; Scenario 3; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\hat{\mu}$	1.399	1.745	1.028	1.411	0.531	1.243	-0.032	0.014	0.011	-0.022
	$\hat{M}$	1.008	2.189	0.754	1.68	0.281	1.425	-0.019	0.033	0.03	-0.02
	$\hat{\sigma}^2$	3.197	4.46	2.061	3.062	1.399	1.784	0.047	0.06	0.04	0.05
	2.5%	-1.127	-2.698	-1.131	-2.441	-1.307	-1.821	-0.46	-0.505	-0.422	-0.442
	97.5%	5.591	5	4.464	4.037	3.504	3.561	0.355	0.463	0.362	0.416
P	$\hat{\mu}$	0.014	0.018	0.029	0.046	0.035	0.104	-0.064	0.047	0.027	-0.003
	$\hat{M}$	0.01	0.023	0.022	0.055	0.019	0.119	-0.039	0.107	0.076	-0.002
	$\hat{\sigma}^2$	< .001	< .001	0.002	0.003	0.006	0.013	0.187	0.67	0.248	0.001
	2.5%	-0.011	-0.029	-0.032	-0.079	-0.087	-0.153	-0.92	-1.684	-1.054	-0.055
	97.5%	0.056	0.053	0.127	0.131	0.234	0.298	0.711	1.545	0.904	0.052
Skew		2.025	-0.723	1.654	-0.725	1.494	-0.636	-0.262	-0.236	-0.404	0.122
Kurtosis		9.138	0.586	6.708	0.76	4.166	1.223	-0.23	-0.291	-0.144	-0.088

Condition 15; Scenario 3; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\hat{\mu}$	1.23	1.81	0.944	1.435	0.433	1.244	-0.11	-0.031	-0.073	0.027	-0.041	0.019	-0.019	-0.008
	$\hat{M}$	0.978	2.171	0.773	1.679	0.269	1.391	-0.065	0.001	-0.023	0.064	-0.001	0.07	-0.034	-0.03
	$\hat{\sigma}^2$	2.282	4.035	1.49	2.765	1.087	1.713	0.11	0.077	0.146	0.092	0.078	0.065	0.133	0.165
	2.5%	-1.364	-2.351	-1.129	-2.074	-1.367	-1.585	-0.993	-0.68	-1.177	-0.656	-0.76	-0.571	-0.697	-0.713
	97.5%	4.724	5.108	3.752	4.096	2.87	3.57	0.379	0.398	0.507	0.527	0.399	0.407	0.731	0.759
P	$\hat{\mu}$	0.012	0.019	0.027	0.047	0.029	0.104	-0.219	-0.062	-0.244	0.09	-0.104	0.048	-0.002	-0.001
	$\hat{M}$	0.01	0.023	0.022	0.054	0.018	1.391	-0.131	0.003	-0.076	0.213	-0.002	0.175	-0.004	-0.004
	$\hat{\sigma}^2$	< .001	< .001	0.001	0.003	0.005	0.012	0.44	0.308	1.623	1.018	0.484	0.408	0.002	0.003
	2.5%	-0.014	-0.025	-0.032	-0.067	-0.091	-0.133	-1.986	-1.36	-3.923	-2.186	-1.901	-1.427	-0.087	-0.089
	97.5%	0.047	0.054	0.107	0.133	0.191	0.299	0.758	0.795	1.69	1.756	0.998	1.017	0.091	0.095
Skew		0.921	-0.471	0.998	-0.541	0.954	-0.438	-1.072	-0.714	-0.93	-0.6	-1.045	-0.87	0.298	0.959
Kurtosis		1.847	-0.118	3.397	0.546	3.247	0.56	1.71	0.419	1.065	0.17	1.885	0.799	2.449	5.832

Condition 15; Scenario 3; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\hat{\mu}$	1.283	1.803	0.959	1.468	0.443	1.265	-0.03	0.015	0.01	-0.021
	$\hat{M}$	0.996	2.194	0.754	1.694	0.264	1.405	-0.026	0.026	0.029	-0.02
	$\hat{\sigma}^2$	2.474	4.082	1.602	2.752	1.167	1.693	0.049	0.064	0.042	0.05
	2.5%	-1.304	-2.375	-1.132	-2.079	-1.412	-1.572	-0.46	-0.51	-0.43	-0.442
	97.5%	5.059	5.102	3.837	4.146	2.895	3.614	0.364	0.466	0.377	0.416
P	$\hat{\mu}$	0.013	0.019	0.027	0.048	0.03	0.106	-0.06	0.049	0.024	-0.003
	$\hat{M}$	0.01	0.023	0.022	0.055	0.018	0.118	-0.052	0.085	0.072	-0.002
	$\hat{\sigma}^2$	< .001	< .001	0.001	0.003	0.005	0.012	0.198	0.707	0.262	0.001
	2.5%	-0.013	-0.025	-0.032	-0.068	-0.094	-0.132	-0.92	-1.701	-1.074	-0.055
	97.5%	0.051	0.054	0.11	0.135	0.193	0.303	0.727	1.554	0.943	0.052
Skew		1.067	-0.509	1.093	-0.562	1.231	0.012	-0.206	-0.212	-0.345	0.111
Kurtosis		2.624	0.064	3.729	0.454	4.477	0.476	-0.353	-0.41	-0.246	-0.095

Condition 15; Scenario 3; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\hat{\mu}$	1.124	2.595	0.764	2.003	0.397	1.657	0.069	0.128	0.063	–
	$\hat{M}$	0.928	2.87	0.641	2.211	0.297	1.752	0.07	0.127	0.073	–
	$\hat{\sigma}^2$	1.753	2.626	1.152	1.848	0.753	1.087	0.01	0.014	0.011	–
	2.5%	-0.814	-0.896	-0.917	-1.093	-1.066	-0.701	-0.13	-0.118	-0.158	–
	97.5%	3.567	5.119	2.931	4.094	2.166	3.499	0.265	0.354	0.26	–
P	$\hat{\mu}$	0.011	0.027	0.022	0.065	0.026	0.139	0.137	0.428	0.158	–
	$\hat{M}$	0.009	0.03	0.018	0.072	0.02	0.147	0.14	0.42	0.183	–
	$\hat{\sigma}^2$	< .001	< .001	0.001	0.002	0.003	0.008	0.04	0.161	0.071	–
	2.5%	-0.008	-0.009	-0.026	-0.036	-0.071	-0.059	-0.26	-0.392	-0.395	–
	97.5%	0.036	0.054	0.084	0.133	0.144	0.293	0.531	1.181	0.65	–
Skew		3.628	-0.991	2.714	-0.899	1.662	-0.654	-0.124	-0.089	-0.202	–
Kurtosis		31.632	2.368	21.562	1.831	7.334	2.01	0.175	0.096	-0.015	–

TABLE C-46

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 16 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 16; Scenario 1; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	1.004	-0.15	0.775	-0.093	0.44	-0.012	-0.336	-0.1	-0.299	-0.086	-0.183	-0.183	0.005	-0.012
	$\widehat{M}$	0.982	-0.161	0.764	-0.088	0.417	0.009	-0.337	-0.107	-0.297	-0.097	-0.176	-0.176	0.006	-0.022
	$\widehat{\sigma}^2$	1.193	0.6	0.636	0.495	0.553	0.259	0.064	0.017	0.098	0.023	0.043	0.043	0.053	0.04
	2.5%	-1.41	-1.656	-0.907	-1.633	-1.009	-1.089	-0.866	-0.346	-0.927	-0.36	-0.598	-0.598	-0.477	-0.389
	97.5%	3.397	1.414	2.451	1.225	2.077	0.969	0.175	0.155	0.281	0.265	0.217	0.217	0.456	0.392
P	$\widehat{\mu}$	0.01	-0.002	0.022	-0.003	0.029	-0.001	-0.672	-0.2	-0.998	-0.287	-0.458	-0.458	0.001	-0.002
	$\widehat{M}$	0.01	-0.002	0.022	-0.003	0.028	0.009	-0.673	-0.214	-0.99	-0.325	-0.439	-0.439	0.001	-0.003
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.002	0.002	0.257	0.068	1.094	0.258	0.27	0.27	0.001	0.001
	2.5%	-0.014	-0.018	-0.026	-0.053	-0.067	-0.091	-1.733	-0.691	-3.091	-1.201	-1.497	-1.497	-0.06	-0.049
	97.5%	0.034	0.015	0.07	0.04	0.138	0.081	0.351	0.309	0.937	0.884	0.543	0.543	0.057	0.049
Skew		0.043	-0.241	-0.008	-0.276	0.421	-0.267	-0.164	0.324	-0.135	0.573	-0.207	-0.207	-0.112	0.191
Kurtosis		1.035	1.253	1.418	1.244	1.794	1.6	1.101	0.725	-0.065	1.675	0.578	0.578	0.308	0.116

Condition 16; Scenario 1; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	1.024	-0.157	0.786	-0.097	0.442	-0.014	-0.14	-0.118	-0.086	-0.018
	$\widehat{M}$	1.006	-0.173	0.772	-0.094	0.419	0.008	-0.154	-0.134	-0.09	-0.013
	$\widehat{\sigma}^2$	1.225	0.591	0.651	0.49	0.56	0.258	0.019	0.027	0.018	0.021
	2.5%	-1.445	-1.637	-0.922	-1.615	-1.005	-1.084	-0.37	-0.39	-0.335	-0.291
	97.5%	3.457	1.403	2.457	1.221	2.064	0.973	0.142	0.25	0.198	0.259
P	$\widehat{\mu}$	0.01	-0.002	0.022	-0.003	0.029	-0.001	-0.28	-0.394	-0.216	-0.002
	$\widehat{M}$	0.01	-0.002	0.022	-0.003	0.028	0.001	-0.309	-0.446	-0.226	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.001	0.002	0.002	0.075	0.302	0.113	< .001
	2.5%	-0.014	-0.017	-0.026	-0.052	-0.067	-0.091	-0.74	-1.301	-0.839	-0.036
	97.5%	0.035	0.015	0.07	0.04	0.138	0.082	0.284	0.834	0.494	0.032
Skew		0.035	-0.218	-0.002	-0.255	0.423	-0.25	0.648	0.613	0.215	-0.026
Kurtosis		1.024	1.254	1.398	1.235	1.801	1.587	0.666	0.521	0.078	-0.441

Condition 16; Scenario 1; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.986	-0.192	0.739	-0.109	0.396	-0.021	-0.331	-0.105	-0.29	-0.088	-0.166	-0.066	0.003	-0.013
	$\widehat{M}$	0.966	-0.19	0.744	-0.11	0.388	0.002	-0.336	-0.108	-0.281	-0.099	-0.163	-0.056	-0.005	-0.023
	$\widehat{\sigma}^2$	1.084	0.543	0.55	0.445	0.461	0.251	0.059	0.017	0.093	0.023	0.037	0.017	0.046	0.04
	2.5%	-1.081	-1.656	-0.753	-1.578	-0.968	-1.116	-0.813	-0.349	-0.861	-0.361	-0.538	-0.328	-0.413	-0.389
	97.5%	3.22	1.196	2.283	1.158	1.84	0.941	0.16	0.149	0.281	0.265	0.209	0.195	0.453	0.395
P	$\widehat{\mu}$	0.01	-0.002	0.021	-0.004	0.026	-0.002	-0.662	-0.211	-0.966	-0.294	-0.416	-0.166	< .001	-0.002
	$\widehat{M}$	0.01	-0.002	0.021	-0.004	0.026	0.002	-0.673	-0.216	-0.935	-0.33	-0.407	-0.14	-0.001	-0.003
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.002	0.002	0.234	0.066	1.034	0.252	0.233	0.109	0.001	0.001
	2.5%	-0.011	-0.018	-0.022	-0.051	-0.065	-0.093	-1.627	-0.698	-2.87	-1.204	-1.345	-0.82	-0.052	-0.049
	97.5%	0.032	0.013	0.065	0.038	0.123	0.079	0.319	0.298	0.937	0.884	0.523	0.487	0.057	0.049
Skew		0.032	-0.281	-0.189	-0.087	0.159	-0.232	0.103	0.303	-0.088	0.463	-0.103	-0.138	0.161	0.193
Kurtosis		0.683	0.998	1.301	0.769	1.071	1.379	0.634	0.851	-0.172	1.226	0.21	0.684	0.371	0.07

Condition 16; Scenario 1; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.979	-0.168	0.75	-0.108	0.416	-0.031	-0.141	-0.119	-0.086	-0.018
	$\widehat{M}$	0.986	-0.191	0.76	-0.099	0.406	-0.001	-0.158	-0.135	-0.089	-0.013
	$\widehat{\sigma}^2$	1.192	0.576	0.626	0.472	0.519	0.265	0.019	0.028	0.018	0.021
	2.5%	-1.435	-1.641	-0.979	-1.615	-1.005	-1.146	-0.37	-0.39	-0.335	-0.291
	97.5%	3.149	1.284	2.399	1.164	1.948	0.935	0.183	0.276	0.199	0.259
P	$\widehat{\mu}$	0.01	-0.002	0.021	-0.004	0.028	-0.003	-0.281	-0.396	-0.216	-0.002
	$\widehat{M}$	0.01	-0.002	0.022	-0.003	0.027	< .001	-0.315	-0.45	-0.223	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	0.001	< .001	0.002	0.002	0.077	0.308	0.113	< .001
	2.5%	-0.014	-0.017	-0.028	-0.052	-0.067	-0.096	-0.74	-1.301	-0.839	-0.036
	97.5%	0.031	0.014	0.069	0.038	0.13	0.078	0.366	0.92	0.497	0.032
Skew		-0.23	-0.168	-0.267	-0.228	0.28	0.002	0.749	0.668	0.212	-0.028
Kurtosis		0.9	1.436	1.319	1.202	1.253	2.099	0.893	0.609	0.076	-0.439

Condition 16; Scenario 1; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.421	0.442	0.271	0.322	0.124	0.313	0.049	0.095	0.04	–
	$\widehat{M}$	0.461	0.425	0.3	0.296	0.136	0.321	0.049	0.095	0.041	–
	$\widehat{\sigma}^2$	0.597	0.305	0.327	0.253	0.291	0.148	0.004	0.007	0.005	–
	2.5%	-1.461	-0.578	-1.068	-0.585	-0.991	-0.449	-0.068	-0.06	-0.119	–
	97.5%	1.787	1.625	1.293	1.351	1.194	1.079	0.17	0.272	0.188	–
P	$\widehat{\mu}$	0.004	0.005	0.008	0.01	0.008	0.026	0.099	0.318	0.1	–
	$\widehat{M}$	0.005	0.005	0.009	0.01	0.009	0.027	0.097	0.315	0.103	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.001	0.001	0.016	0.073	0.034	–
	2.5%	-0.015	-0.006	-0.031	-0.019	-0.066	-0.038	-0.136	-0.201	-0.297	–
	97.5%	0.018	0.017	0.037	0.044	0.08	0.09	0.34	0.906	0.469	–
Skew		-0.547	0.258	-0.6	0.247	-0.055	0.182	0.217	0.203	-0.126	–
Kurtosis		0.851	0.278	1.326	0.963	0.933	0.782	0.698	0.523	0.418	–

TABLE C-47

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 16 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 16; Scenario 2; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.968	0.261	0.743	0.183	0.364	0.394	-0.191	-0.139	-0.137	-0.061	-0.094	-0.094	-0.008	-0.02
	$\widehat{M}$	0.702	0.188	0.589	0.127	0.163	0.445	-0.168	-0.14	-0.101	-0.054	-0.071	-0.071	-0.002	-0.031
	$\widehat{\sigma}^2$	1.636	2.474	0.895	1.909	0.737	0.963	0.067	0.051	0.097	0.057	0.046	0.046	0.044	0.078
	2.5%	-0.774	-2.635	-0.549	-2.377	-0.712	-1.488	-0.801	-0.563	-1.003	-0.527	-0.578	-0.578	-0.422	-0.539
	97.5%	4.491	3.335	3.029	2.772	2.9	2.007	0.27	0.266	0.399	0.372	0.256	0.256	0.401	0.548
P	$\widehat{\mu}$	0.01	0.003	0.021	0.006	0.024	0.033	-0.383	-0.278	-0.456	-0.204	-0.234	-0.234	-0.001	-0.002
	$\widehat{M}$	0.007	0.002	0.017	0.004	0.011	0.445	-0.335	-0.28	-0.334	-0.182	-0.177	-0.177	< .001	-0.004
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.002	0.003	0.007	0.267	0.204	1.082	0.635	0.286	0.286	0.001	0.001
	2.5%	-0.008	-0.028	-0.016	-0.077	-0.047	-0.125	-1.601	-1.127	-3.344	-1.758	-1.446	-1.446	-0.053	-0.067
	97.5%	0.045	0.035	0.086	0.09	0.193	0.168	0.54	0.532	1.331	1.239	0.64	0.64	0.05	0.068
Skew		1.376	0.035	1.605	-0.119	1.709	-0.256	-1.254	-0.149	-1.075	-0.249	-1.185	-1.185	-0.393	1.04
Kurtosis		2.821	-0.19	5.011	0.432	4.256	0.114	4.517	-0.322	2.183	-0.055	3.907	3.907	2.613	10.956

Condition 16; Scenario 2; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.977	0.26	0.748	0.182	0.365	0.394	-0.113	-0.05	-0.044	-0.011
	$\widehat{M}$	0.702	0.194	0.596	0.121	0.162	0.44	-0.136	-0.053	-0.042	-0.009
	$\widehat{\sigma}^2$	1.676	2.436	0.913	1.89	0.743	0.952	0.029	0.039	0.026	0.019
	2.5%	-0.783	-2.617	-0.55	-2.351	-0.714	-1.47	-0.397	-0.41	-0.351	-0.279
	97.5%	4.56	3.311	3.068	2.769	2.906	2.001	0.251	0.368	0.246	0.273
P	$\widehat{\mu}$	0.01	0.003	0.021	0.006	0.024	0.033	-0.225	-0.167	-0.11	-0.001
	$\widehat{M}$	0.007	0.002	0.017	0.004	0.011	0.037	-0.272	-0.176	-0.107	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.002	0.003	0.007	0.116	0.433	0.16	< .001
	2.5%	-0.008	-0.028	-0.016	-0.076	-0.048	-0.123	-0.793	-1.366	-0.878	-0.035
	97.5%	0.046	0.035	0.088	0.09	0.194	0.168	0.502	1.226	0.615	0.034
Skew		1.369	0.044	1.606	-0.113	1.723	-0.249	0.385	0.154	0.018	0.065
Kurtosis		2.785	-0.17	4.975	0.451	4.319	0.121	-0.382	-0.338	-0.386	0.153

Condition 16; Scenario 2; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	0.863	0.169	0.644	0.126	0.298	0.331	-0.175	-0.145	-0.125	-0.073	-0.08	-0.062	-0.01	-0.021
	$\widehat{M}$	0.667	0.11	0.552	0.041	0.149	0.36	-0.158	-0.149	-0.105	-0.073	-0.07	-0.048	-0.013	-0.029
	$\widehat{\sigma}^2$	1.198	2.138	0.549	1.612	0.526	0.937	0.051	0.048	0.079	0.058	0.034	0.041	0.061	0.068
	2.5%	-0.72	-2.503	-0.478	-2.092	-0.697	-1.511	-0.677	-0.563	-0.872	-0.542	-0.473	-0.496	-0.48	-0.544
	97.5%	3.451	2.949	2.303	2.689	2.472	2.007	0.279	0.266	0.392	0.372	0.262	0.292	0.497	0.515
P	$\widehat{\mu}$	0.009	0.002	0.018	0.004	0.02	0.028	-0.351	-0.291	-0.415	-0.242	-0.199	-0.154	-0.001	-0.003
	$\widehat{M}$	0.007	0.001	0.016	0.001	0.01	0.36	-0.316	-0.298	-0.349	-0.243	-0.174	-0.119	-0.002	-0.004
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.002	0.007	0.204	0.192	0.875	0.644	0.212	0.258	0.001	0.001
	2.5%	-0.007	-0.026	-0.014	-0.068	-0.046	-0.127	-1.355	-1.127	-2.906	-1.807	-1.183	-1.239	-0.06	-0.068
	97.5%	0.035	0.031	0.066	0.087	0.165	0.168	0.559	0.532	1.307	1.239	0.655	0.729	0.062	0.064
Skew		0.993	0.1	0.661	0.183	1.563	-0.138	-0.543	0.014	-0.878	-0.17	-0.488	-0.295	-0.094	-0.031
Kurtosis		1.196	-0.421	1.078	-0.391	4.112	-0.249	2.059	-0.585	1.992	-0.255	1.136	-0.198	0.688	0.298

Condition 16; Scenario 2; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.868	0.187	0.659	0.142	0.28	0.358	-0.118	-0.055	-0.046	-0.01
	$\widehat{M}$	0.68	0.115	0.56	0.069	0.145	0.373	-0.145	-0.062	-0.048	-0.009
	$\widehat{\sigma}^2$	1.282	2.208	0.607	1.643	0.503	0.939	0.029	0.04	0.026	0.019
	2.5%	-0.773	-2.546	-0.494	-2.189	-0.727	-1.489	-0.397	-0.414	-0.351	-0.279
	97.5%	3.476	2.977	2.585	2.642	2.173	2.039	0.261	0.372	0.249	0.273
P	$\widehat{\mu}$	0.009	0.002	0.019	0.005	0.019	0.03	-0.237	-0.185	-0.116	-0.001
	$\widehat{M}$	0.007	0.001	0.016	0.002	0.01	0.031	-0.29	-0.207	-0.121	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.002	0.002	0.007	0.117	0.442	0.161	< .001
	2.5%	-0.008	-0.027	-0.014	-0.071	-0.048	-0.125	-0.793	-1.379	-0.879	-0.035
	97.5%	0.035	0.032	0.074	0.086	0.145	0.171	0.523	1.242	0.622	0.034
Skew		0.747	0.154	0.801	0.132	1.282	0.007	0.549	0.221	0.08	0.061
Kurtosis		1.199	-0.229	1.388	-0.446	2.976	-0.117	-0.04	-0.339	-0.328	0.147

Condition 16; Scenario 2; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.562	1.493	0.383	1.104	0.165	1.014	0.075	0.133	0.067	–
	$\widehat{M}$	0.52	1.424	0.359	1.05	0.115	1.032	0.078	0.135	0.069	–
	$\widehat{\sigma}^2$	0.392	1.109	0.23	0.863	0.237	0.451	0.004	0.006	0.005	–
	2.5%	-0.539	-0.524	-0.454	-0.502	-0.65	-0.314	-0.049	-0.02	-0.085	–
	97.5%	1.856	3.504	1.374	2.842	1.384	2.159	0.193	0.279	0.202	–
P	$\widehat{\mu}$	0.006	0.016	0.011	0.036	0.011	0.085	0.15	0.442	0.168	–
	$\widehat{M}$	0.005	0.015	0.01	0.034	0.008	0.086	0.155	0.45	0.172	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.001	0.003	0.016	0.063	0.033	–
	2.5%	-0.005	-0.006	-0.013	-0.016	-0.043	-0.026	-0.099	-0.068	-0.213	–
	97.5%	0.019	0.037	0.039	0.092	0.092	0.181	0.387	0.93	0.506	–
Skew		0.353	0.014	0.099	-0.113	0.866	-0.147	-0.051	0.061	-0.131	–
Kurtosis		0.344	-0.138	0.917	0.784	2.626	-0.111	0.035	0.233	0.251	–

TABLE C-48

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 16 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 16; Scenario 3; Method 1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	1.516	1.482	1.087	1.182	0.578	1.216	-0.171	-0.096	-0.118	-0.01	-0.076	-0.076	-0.023	-0.012
	$\widehat{M}$	1.169	1.724	0.804	1.422	0.302	1.385	-0.135	-0.056	-0.066	0.015	-0.032	-0.032	-0.018	-0.021
	$\widehat{\sigma}^2$	2.631	3.804	1.285	2.734	1.124	1.245	0.101	0.064	0.132	0.065	0.068	0.068	0.056	0.076
	2.5%	-0.592	-2.547	-0.392	-2.297	-0.89	-1.284	-0.95	-0.63	-1.116	-0.565	-0.787	-0.787	-0.486	-0.525
	97.5%	5.62	4.504	3.91	3.582	3.242	2.96	0.363	0.313	0.454	0.462	0.341	0.341	0.431	0.572
P	$\widehat{\mu}$	0.015	0.016	0.031	0.038	0.039	0.102	-0.341	-0.193	-0.393	-0.034	-0.19	-0.19	-0.003	-0.001
	$\widehat{M}$	0.012	0.018	0.023	0.046	0.02	1.385	-0.271	-0.113	-0.219	0.051	-0.079	-0.079	-0.002	-0.003
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.005	0.009	0.405	0.256	1.471	0.725	0.427	0.427	0.001	0.001
	2.5%	-0.006	-0.027	-0.011	-0.075	-0.059	-0.108	-1.899	-1.261	-3.719	-1.884	-1.967	-1.967	-0.061	-0.066
	97.5%	0.056	0.048	0.112	0.116	0.216	0.248	0.726	0.626	1.513	1.539	0.853	0.853	0.054	0.071
	Skew	1.222	-0.423	1.441	-0.546	1.347	-0.622	-1.068	-0.469	-1.007	-0.395	-1.502	-1.502	-0.765	0.256
	Kurtosis	2.108	-0.423	3.73	-0.194	2.521	0.668	2.321	-0.3	1.299	0.184	3.909	3.909	5.307	2.58

Condition 16; Scenario 3; Method 2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	1.524	1.476	1.092	1.179	0.579	1.214	-0.079	-0.018	-0.015	-0.015
	$\widehat{M}$	1.186	1.704	0.811	1.414	0.304	1.378	-0.079	-0.011	-0.001	-0.012
	$\widehat{\sigma}^2$	2.637	3.773	1.292	2.717	1.125	1.234	0.038	0.045	0.028	0.022
	2.5%	-0.575	-2.522	-0.383	-2.282	-0.892	-1.247	-0.435	-0.453	-0.379	-0.307
	97.5%	5.709	4.49	3.927	3.586	3.247	2.959	0.292	0.402	0.29	0.272
P	$\widehat{\mu}$	0.015	0.016	0.031	0.038	0.039	0.102	-0.158	-0.058	-0.037	-0.002
	$\widehat{M}$	0.012	0.018	0.023	0.046	0.02	0.115	-0.156	-0.036	-0.001	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.005	0.009	0.151	0.502	0.174	< .001
	2.5%	-0.006	-0.027	-0.011	-0.074	-0.059	-0.104	-0.871	-1.51	-0.947	-0.038
	97.5%	0.057	0.048	0.112	0.117	0.216	0.248	0.585	1.339	0.724	0.034
	Skew	1.205	-0.414	1.441	-0.539	1.36	-0.611	-0.005	-0.145	-0.365	0.098
	Kurtosis	2.061	-0.435	3.751	-0.201	2.576	0.654	-0.564	-0.074	0.137	0.084

Condition 16; Scenario 3; Method 3.1

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	1.374	1.451	0.995	1.165	0.469	1.205	-0.159	-0.098	-0.098	-0.008	-0.059	-0.013	-0.009	-0.013
	$\widehat{M}$	1.131	1.692	0.786	1.369	0.272	1.364	-0.135	-0.069	-0.059	0.019	-0.026	0.024	-0.011	-0.021
	$\widehat{\sigma}^2$	1.908	3.528	0.938	2.485	0.84	1.236	0.092	0.064	0.118	0.07	0.055	0.047	0.06	0.078
	2.5%	-0.626	-2.434	-0.431	-2.137	-0.993	-1.24	-0.868	-0.606	-1.008	-0.584	-0.648	-0.513	-0.462	-0.551
	97.5%	4.751	4.475	3.365	3.605	2.807	2.993	0.362	0.318	0.454	0.462	0.343	0.346	0.501	0.581
P	$\widehat{\mu}$	0.014	0.015	0.028	0.038	0.031	0.101	-0.319	-0.197	-0.325	-0.027	-0.148	-0.033	-0.001	-0.002
	$\widehat{M}$	0.011	0.018	0.022	0.044	0.018	1.364	-0.27	-0.138	-0.198	0.063	-0.067	0.06	-0.002	-0.003
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.004	0.009	0.37	0.254	1.311	0.783	0.346	0.297	0.001	0.001
	2.5%	-0.006	-0.026	-0.012	-0.069	-0.066	-0.104	-1.736	-1.212	-3.362	-1.948	-1.619	-1.283	-0.058	-0.069
	97.5%	0.047	0.047	0.096	0.117	0.187	0.251	0.723	0.636	1.513	1.541	0.857	0.866	0.063	0.073
	Skew	0.883	-0.423	0.862	-0.408	1.034	-0.521	-1.014	-0.354	-0.981	-0.455	-1.096	-0.7	0.154	0.244
	Kurtosis	0.757	-0.353	0.967	-0.303	2.031	0.398	2.528	-0.469	1.434	0.145	2.498	0.548	2.304	2.465

Condition 16; Scenario 3; Method 3.2

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	1.375	1.436	0.99	1.168	0.467	1.202	-0.081	-0.016	-0.015	-0.015
	$\widehat{M}$	1.141	1.699	0.788	1.357	0.271	1.346	-0.087	-0.013	-0.002	-0.01
	$\widehat{\sigma}^2$	1.958	3.561	0.932	2.441	0.874	1.253	0.039	0.048	0.03	0.022
	2.5%	-0.619	-2.403	-0.439	-2.098	-1.042	-1.28	-0.435	-0.453	-0.383	-0.305
	97.5%	4.842	4.498	3.363	3.605	2.837	3.142	0.298	0.411	0.297	0.272
P	$\widehat{\mu}$	0.014	0.015	0.028	0.038	0.031	0.101	-0.162	-0.053	-0.036	-0.002
	$\widehat{M}$	0.011	0.018	0.022	0.044	0.018	0.113	-0.174	-0.043	-0.004	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	0.003	0.004	0.009	0.156	0.536	0.185	< .001
	2.5%	-0.006	-0.025	-0.013	-0.068	-0.069	-0.107	-0.871	-1.51	-0.957	-0.038
	97.5%	0.048	0.048	0.096	0.117	0.189	0.263	0.596	1.368	0.743	0.034
	Skew	0.929	-0.413	0.852	-0.413	1.04	0.009	0.068	-0.093	-0.298	0.106
	Kurtosis	1.072	-0.386	0.834	-0.257	2.016	0.352	-0.591	-0.246	-0.033	0.116

Condition 16; Scenario 3; Method 4

D	Stat.	α_1	α_2	β_1	β_2	γ_1	γ_2	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.972	2.745	0.651	2.105	0.329	1.788	0.071	0.125	0.062	–
	$\widehat{M}$	0.907	2.929	0.581	2.252	0.241	1.856	0.069	0.124	0.064	–
	$\widehat{\sigma}^2$	0.577	1.495	0.317	1.076	0.344	0.498	0.004	0.006	0.005	–
	2.5%	-0.333	0.043	-0.295	-0.247	-0.671	0.171	-0.056	-0.021	-0.076	–
	97.5%	2.692	4.674	1.945	3.679	1.781	2.989	0.212	0.288	0.198	–
P	$\widehat{\mu}$	0.01	0.029	0.019	0.068	0.022	0.15	0.142	0.417	0.155	–
	$\widehat{M}$	0.009	0.031	0.017	0.073	0.016	0.155	0.139	0.413	0.16	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0.001	0.002	0.003	0.016	0.064	0.03	–
	2.5%	-0.003	< .001	-0.008	-0.008	-0.045	0.014	-0.111	-0.069	-0.19	–
	97.5%	0.027	0.049	0.056	0.12	0.119	0.25	0.424	0.959	0.495	–
	Skew	0.53	-0.673	0.511	-0.811	0.944	-0.49	0.157	0.094	-0.167	–
	Kurtosis	1.018	0.555	0.799	1.012	2.468	0.966	0.424	0.684	0.781	–

TABLE C-49

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 17 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 17; Scenario 1; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.071	-0.06	0.02	-0.037	0.005	0.005	-0.014	-0.013	0.005	0.005	0.007	< .001
	$\widehat{M}$	-0.068	-0.1	0.03	-0.032	0.015	0.019	-0.008	-0.005	0.011	0.011	0.005	-0.01
	$\widehat{\sigma}^2$	0.062	0.214	0.185	0.033	0.011	0.013	0.011	0.012	0.012	0.012	0.015	0.014
	2.5%	-0.577	-0.734	-0.673	-0.383	-0.206	-0.234	-0.224	-0.226	-0.219	-0.219	-0.225	-0.224
	97.5%	0.409	0.644	0.723	0.305	0.178	0.183	0.169	0.178	0.194	0.194	0.265	0.235
P	$\widehat{\mu}$	-0.001	-0.002	< .001	-0.002	0.009	0.011	-0.045	-0.042	0.013	0.013	0.002	< .001
	$\widehat{M}$	-0.001	-0.003	0.001	-0.002	0.029	0.038	-0.026	-0.017	0.029	0.029	0.001	-0.003
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.045	0.054	0.123	0.132	0.077	0.077	0.001	0.001
	2.5%	-0.006	-0.021	-0.015	-0.026	-0.411	-0.468	-0.747	-0.753	-0.547	-0.547	-0.056	-0.056
	97.5%	0.004	0.018	0.017	0.02	0.356	0.365	0.562	0.595	0.486	0.486	0.066	0.059
Skew		-0.106	4.053	-3.52	-0.057	-1.365	-2.429	-0.888	-0.647	-0.853	-0.853	0.165	0.191
Kurtosis		0.307	37.639	32.05	-0.184	6.942	18.404	4.567	2.193	3.221	3.221	0.443	0.168

Condition 17; Scenario 1; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.072	-0.099	0.05	-0.038	0.008	-0.009	0.006	0.003
	$\widehat{M}$	-0.071	-0.103	0.057	-0.032	0.019	-0.008	0.013	0.001
	$\widehat{\sigma}^2$	0.061	0.124	0.119	0.033	0.009	0.005	0.009	0.007
	2.5%	-0.577	-0.74	-0.589	-0.386	-0.162	-0.155	-0.17	-0.16
	97.5%	0.41	0.552	0.638	0.304	0.143	0.124	0.159	0.175
P	$\widehat{\mu}$	-0.001	-0.003	0.001	-0.003	0.016	-0.031	0.016	0.001
	$\widehat{M}$	-0.001	-0.003	0.001	-0.002	0.038	-0.028	0.032	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.036	0.057	0.056	< .001
	2.5%	-0.006	-0.021	-0.014	-0.026	-0.325	-0.515	-0.424	-0.04
	97.5%	0.004	0.016	0.015	0.02	0.287	0.415	0.398	0.044
Skew		-0.144	0.915	-0.702	-0.07	-2.41	-0.139	-1.767	0.117
Kurtosis		0.242	9.391	8.025	-0.209	15.683	0.06	12.412	0.097

Condition 17; Scenario 1; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.071	-0.07	0.028	-0.037	0.007	0.007	-0.013	-0.012	0.007	0.004	< .001	< .001
	$\widehat{M}$	-0.068	-0.1	0.029	-0.032	0.017	0.02	-0.007	-0.005	0.011	0.01	-0.011	-0.009
	$\widehat{\sigma}^2$	0.062	0.183	0.155	0.033	0.011	0.013	0.011	0.012	0.011	0.012	0.014	0.014
	2.5%	-0.577	-0.734	-0.654	-0.383	-0.198	-0.229	-0.224	-0.225	-0.212	-0.222	-0.226	-0.224
	97.5%	0.409	0.598	0.723	0.305	0.179	0.183	0.169	0.178	0.193	0.183	0.235	0.235
P	$\widehat{\mu}$	-0.001	-0.002	0.001	-0.002	0.013	0.014	-0.042	-0.04	0.017	0.01	< .001	< .001
	$\widehat{M}$	-0.001	-0.003	0.001	-0.002	0.033	0.039	-0.024	-0.017	0.029	0.025	-0.003	-0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.042	0.051	0.122	0.129	0.071	0.073	0.001	0.001
	2.5%	-0.006	-0.021	-0.015	-0.026	-0.396	-0.458	-0.745	-0.75	-0.529	-0.556	-0.056	-0.056
	97.5%	0.004	0.017	0.017	0.02	0.358	0.366	0.564	0.593	0.482	0.458	0.059	0.059
Skew		-0.106	3.987	-2.893	-0.058	-1.203	-2.408	-0.868	-0.63	-0.455	-0.763	0.17	0.187
Kurtosis		0.299	44.303	30.672	-0.187	6.455	19.426	4.557	2.199	0.557	2.258	0.076	0.166

Condition 17; Scenario 1; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.072	-0.102	0.058	-0.038	0.011	-0.009	0.01	0.003
	$\widehat{M}$	-0.071	-0.102	0.059	-0.032	0.019	-0.008	0.013	0.001
	$\widehat{\sigma}^2$	0.061	0.097	0.093	0.033	0.007	0.005	0.007	0.007
	2.5%	-0.577	-0.709	-0.58	-0.386	-0.154	-0.155	-0.157	-0.16
	97.5%	0.41	0.538	0.634	0.304	0.143	0.124	0.157	0.175
P	$\widehat{\mu}$	-0.001	-0.003	0.001	-0.003	0.023	-0.031	0.024	0.001
	$\widehat{M}$	-0.001	-0.003	0.001	-0.002	0.038	-0.028	0.033	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.027	0.057	0.042	< .001
	2.5%	-0.006	-0.02	-0.013	-0.026	-0.308	-0.515	-0.393	-0.04
	97.5%	0.004	0.015	0.015	0.02	0.287	0.415	0.392	0.044
Skew		-0.141	0.098	0.013	-0.071	-0.999	-0.138	-0.285	0.118
Kurtosis		0.241	0.186	0.175	-0.21	4.935	0.059	-0.035	0.098

Condition 17; Scenario 1; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.03	0.048	0.005	0.007	0.015	0.029	0.005	–
	$\widehat{M}$	0.033	0.011	0.017	0.011	0.024	0.03	0.011	–
	$\widehat{\sigma}^2$	0.062	0.212	0.184	0.033	0.006	0.005	0.006	–
	2.5%	-0.477	-0.615	-0.724	-0.343	-0.135	-0.103	-0.144	–
	97.5%	0.512	0.793	0.692	0.351	0.135	0.152	0.141	–
P	$\widehat{\mu}$	< .001	0.001	< .001	< .001	0.031	0.095	0.013	–
	$\widehat{M}$	< .001	< .001	< .001	0.001	0.048	0.1	0.028	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.023	0.051	0.04	–
	2.5%	-0.005	-0.018	-0.017	-0.023	-0.269	-0.344	-0.361	–
	97.5%	0.005	0.023	0.016	0.023	0.269	0.506	0.353	–
Skew		-0.138	3.879	-3.516	-0.066	-1.648	-0.488	-1.076	–
Kurtosis		0.228	33.823	30.915	-0.206	9.092	2.165	4.878	–

TABLE C-50

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 17 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 17; Scenario 2; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.074	-0.036	-0.037	-0.034	-0.012	-0.004	-0.019	-0.022	-0.009	-0.009	0.002	0.002
	$\widehat{M}$	-0.064	-0.049	0.122	-0.033	0.003	0.019	-0.008	-0.007	0.005	0.005	0.001	-0.002
	$\widehat{\sigma}^2$	0.058	0.207	1.118	0.033	0.012	0.027	0.014	0.027	0.012	0.012	0.01	0.027
	2.5%	-0.547	-0.596	-3.602	-0.39	-0.254	-0.381	-0.218	-0.361	-0.22	-0.22	-0.194	-0.303
	97.5%	0.392	0.551	1.058	0.308	0.132	0.225	0.142	0.247	0.147	0.147	0.2	0.315
P	$\widehat{\mu}$	-0.001	-0.001	-0.001	-0.002	-0.023	-0.009	-0.065	-0.075	-0.022	-0.022	0.001	< .001
	$\widehat{M}$	-0.001	-0.001	0.003	-0.002	0.005	0.037	-0.026	-0.025	0.013	0.013	< .001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	< .001	0.047	0.107	0.151	0.298	0.077	0.077	0.001	0.002
	2.5%	-0.005	-0.017	-0.083	-0.026	-0.509	-0.763	-0.727	-1.204	-0.551	-0.551	-0.048	-0.076
	97.5%	0.004	0.016	0.024	0.021	0.265	0.451	0.473	0.824	0.367	0.367	0.05	0.079
Skew		-0.045	0.302	-4.178	-0.051	-2.456	-1.875	-2.9	-0.887	-2.221	-2.221	-0.105	0.06
Kurtosis		0.182	53.541	21.824	-0.141	11.059	7.122	20.469	2.881	11.071	11.071	0.505	-0.123

Condition 17; Scenario 2; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.075	-0.078	0.008	-0.034	-0.007	-0.008	-0.001	0.002
	$\widehat{M}$	-0.066	-0.058	0.147	-0.033	0.008	-0.007	0.01	0.001
	$\widehat{\sigma}^2$	0.057	0.164	0.864	0.033	0.013	0.005	0.012	0.008
	2.5%	-0.548	-0.659	-2.704	-0.392	-0.225	-0.15	-0.215	-0.165
	97.5%	0.382	0.467	0.949	0.311	0.132	0.119	0.163	0.176
P	$\widehat{\mu}$	-0.001	-0.002	< .001	-0.002	-0.015	-0.028	-0.001	< .001
	$\widehat{M}$	-0.001	-0.002	0.003	-0.002	0.016	-0.021	0.025	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.054	0.053	0.074	< .001
	2.5%	-0.005	-0.019	-0.062	-0.026	-0.45	-0.499	-0.537	-0.041
	97.5%	0.004	0.013	0.022	0.021	0.263	0.396	0.408	0.044
Skew		-0.026	-2.591	-4.226	-0.06	-3.519	-0.139	-2.674	0.025
Kurtosis		0.146	38.759	22.452	-0.183	19.47	-0.185	14.949	-0.09

Condition 17; Scenario 2; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.076	-0.04	0.012	-0.034	-0.007	< .001	-0.018	-0.019	-0.005	0.018	0.001	0.002
	$\widehat{M}$	-0.064	-0.05	0.125	-0.033	0.004	0.019	-0.008	-0.007	0.005	0.032	-0.003	-0.001
	$\widehat{\sigma}^2$	0.059	0.146	0.845	0.033	0.01	0.025	0.012	0.024	0.011	0.022	0.027	0.027
	2.5%	-0.564	-0.57	-1.349	-0.392	-0.229	-0.353	-0.191	-0.34	-0.198	-0.307	-0.303	-0.318
	97.5%	0.39	0.503	1.058	0.307	0.132	0.225	0.142	0.247	0.147	0.261	0.315	0.318
P	$\widehat{\mu}$	-0.001	-0.001	< .001	-0.002	-0.015	< .001	-0.059	-0.064	-0.013	0.045	< .001	< .001
	$\widehat{M}$	-0.001	-0.001	0.003	-0.002	0.007	0.038	-0.026	-0.024	0.014	0.079	-0.001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.039	0.1	0.132	0.264	0.067	0.137	0.002	0.002
	2.5%	-0.006	-0.016	-0.031	-0.026	-0.458	-0.706	-0.636	-1.133	-0.494	-0.768	-0.076	-0.08
	97.5%	0.004	0.014	0.024	0.02	0.264	0.451	0.473	0.823	0.367	0.652	0.079	0.079
Skew		-0.155	3.897	-4.613	-0.096	-2.306	-1.836	-3.264	-0.546	-2.035	-1.149	0.073	0.098
Kurtosis		0.641	39.56	29.712	-0.096	11.556	7.519	24.933	1.008	11.428	3.958	-0.091	-0.018

Condition 17; Scenario 2; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.074	-0.075	0.123	-0.034	0.003	-0.008	0.007	0.002
	$\widehat{M}$	-0.066	-0.058	0.158	-0.033	0.011	-0.007	0.012	0.002
	$\widehat{\sigma}^2$	0.056	0.083	0.276	0.033	0.008	0.005	0.008	0.008
	2.5%	-0.548	-0.577	-0.829	-0.392	-0.175	-0.15	-0.168	-0.165
	97.5%	0.382	0.417	0.97	0.311	0.132	0.119	0.161	0.176
P	$\widehat{\mu}$	-0.001	-0.002	0.003	-0.002	0.006	-0.028	0.017	< .001
	$\widehat{M}$	-0.001	-0.002	0.004	-0.002	0.022	-0.021	0.029	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.032	0.052	0.048	< .001
	2.5%	-0.005	-0.016	-0.019	-0.026	-0.35	-0.499	-0.42	-0.041
	97.5%	0.004	0.012	0.022	0.021	0.263	0.396	0.402	0.044
Skew		-0.025	-1.49	-2.608	-0.06	-2.976	-0.137	-1.794	0.025
Kurtosis		0.147	10.588	19.134	-0.183	23.386	-0.188	12.851	-0.092

Condition 17; Scenario 2; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.021	0.029	-0.15	0.011	0.003	0.024	< .001	–
	$\widehat{M}$	0.03	0.007	0.011	0.013	0.014	0.028	0.009	–
	$\widehat{\sigma}^2$	0.057	0.159	1.143	0.033	0.008	0.005	0.008	–
	2.5%	-0.453	-0.505	-3.962	-0.351	-0.21	-0.13	-0.2	–
	97.5%	0.483	0.616	0.977	0.355	0.122	0.148	0.142	–
P	$\widehat{\mu}$	< .001	0.001	-0.003	0.001	0.006	0.079	0.001	–
	$\widehat{M}$	< .001	< .001	< .001	0.001	0.027	0.093	0.023	–
	$\widehat{\sigma}^2$	< .001	< .001	0.001	< .001	0.03	0.061	0.048	–
	2.5%	-0.005	-0.014	-0.091	-0.023	-0.42	-0.435	-0.501	–
	97.5%	0.005	0.018	0.022	0.024	0.243	0.492	0.355	–
Skew		-0.025	2.775	-4.006	-0.06	-2.269	-0.899	-1.5	–
Kurtosis		0.141	29.905	19.783	-0.182	9.594	3.53	5.792	–

TABLE C-51

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 17 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 17; Scenario 3; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.065	-0.158	-1.662	-0.045	-0.088	-0.107	-0.066	-0.055	-0.072	-0.072	< .001	-0.007
	$\widehat{M}$	-0.067	-0.098	-0.162	-0.048	-0.016	-0.018	-0.03	-0.017	-0.03	-0.03	0.001	-0.011
	$\widehat{\sigma}^2$	0.062	0.66	9.283	0.035	0.041	0.101	0.042	0.065	0.03	0.03	0.011	0.048
	2.5%	-0.563	-2.142	-8.056	-0.429	-0.617	-0.909	-0.596	-0.638	-0.531	-0.531	-0.199	-0.432
	97.5%	0.402	1.665	1.583	0.324	0.139	0.279	0.332	0.352	0.149	0.149	0.192	0.444
P	$\widehat{\mu}$	-0.001	-0.005	-0.038	-0.003	-0.175	-0.213	-0.221	-0.185	-0.181	-0.181	< .001	-0.002
	$\widehat{M}$	-0.001	-0.003	-0.004	-0.003	-0.032	-0.037	-0.101	-0.055	-0.076	-0.076	< .001	-0.003
	$\widehat{\sigma}^2$	< .001	0.001	0.005	< .001	0.163	0.403	0.47	0.726	0.186	0.186	0.001	0.003
	2.5%	-0.006	-0.061	-0.185	-0.029	-1.233	-1.819	-1.987	-2.128	-1.328	-1.328	-0.05	-0.108
	97.5%	0.004	0.048	0.036	0.022	0.278	0.558	1.108	1.175	0.373	0.373	0.048	0.111
Skew		-0.098	0.225	-0.939	-0.066	-1.673	-1.16	-1.144	-0.632	-1.593	-1.593	0.081	0.258
Kurtosis		0.449	5.731	-0.642	0.107	3.181	0.968	2.957	0.123	3.378	3.378	0.273	0.665

Condition 17; Scenario 3; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.067	-0.379	-1.455	-0.041	-0.095	-0.01	-0.082	-0.001
	$\widehat{M}$	-0.071	-0.15	-0.089	-0.048	-0.008	-0.01	-0.026	0.001
	$\widehat{\sigma}^2$	0.052	0.697	7.506	0.031	0.058	0.005	0.04	0.008
	2.5%	-0.52	-2.639	-7.131	-0.344	-0.756	-0.142	-0.632	-0.166
	97.5%	0.38	0.694	1.302	0.308	0.184	0.123	0.16	0.171
P	$\widehat{\mu}$	-0.001	-0.011	-0.033	-0.003	-0.189	-0.032	-0.204	< .001
	$\widehat{M}$	-0.001	-0.004	-0.002	-0.003	-0.017	-0.033	-0.066	< .001
	$\widehat{\sigma}^2$	< .001	0.001	0.004	< .001	0.231	0.053	0.248	< .001
	2.5%	-0.005	-0.075	-0.164	-0.023	-1.513	-0.475	-1.581	-0.042
	97.5%	0.004	0.02	0.03	0.021	0.369	0.409	0.4	0.043
Skew		0.013	-1.244	-0.965	0.112	-1.542	-0.083	-1.521	0.11
Kurtosis		-0.179	2.654	-0.603	-0.189	1.66	-0.181	2.102	0.022

Condition 17; Scenario 3; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.123	-0.213	-2.286	-0.08	-0.114	-0.155	-0.08	-0.069	-0.093	-0.122	-0.008	-0.008
	$\widehat{M}$	-0.087	-0.141	-0.43	-0.061	-0.029	-0.06	-0.035	-0.033	-0.044	-0.079	-0.014	-0.011
	$\widehat{\sigma}^2$	0.38	0.749	10.43	0.171	0.046	0.117	0.049	0.07	0.034	0.081	0.041	0.043
	2.5%	-1.911	-2.167	-8.24	-1.208	-0.632	-0.971	-0.611	-0.652	-0.576	-0.776	-0.405	-0.411
	97.5%	1.002	1.811	1.516	0.593	0.152	0.273	0.35	0.355	0.151	0.309	0.423	0.436
P	$\widehat{\mu}$	-0.001	-0.006	-0.053	-0.005	-0.227	-0.311	-0.266	-0.229	-0.233	-0.306	-0.002	-0.002
	$\widehat{M}$	-0.001	-0.004	-0.01	-0.004	-0.058	-0.12	-0.117	-0.108	-0.11	-0.196	-0.004	-0.003
	$\widehat{\sigma}^2$	< .001	0.001	0.006	0.001	0.185	0.469	0.547	0.783	0.216	0.509	0.003	0.003
	2.5%	-0.019	-0.062	-0.189	-0.081	-1.265	-1.941	-2.037	-2.173	-1.439	-1.939	-0.101	-0.103
	97.5%	0.01	0.052	0.035	0.039	0.304	0.547	1.166	1.185	0.377	0.772	0.106	0.109
Skew		-0.286	0.008	-0.511	-0.679	-1.12	-0.916	-0.786	-0.524	-1.255	-0.682	0.374	0.313
Kurtosis		9.96	2	-1.311	9.646	0.403	0.144	1.192	-0.246	1.366	0.008	1.156	1.097

Condition 17; Scenario 3; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.067	-0.327	-0.61	-0.041	-0.071	-0.01	-0.059	-0.001
	$\widehat{M}$	-0.071	-0.117	0.116	-0.048	0.001	-0.01	-0.013	0.001
	$\widehat{\sigma}^2$	0.052	0.528	4.224	0.031	0.052	0.005	0.034	0.008
	2.5%	-0.523	-2.518	-5.881	-0.347	-0.764	-0.143	-0.627	-0.166
	97.5%	0.378	0.398	1.4	0.309	0.164	0.123	0.153	0.171
P	$\widehat{\mu}$	-0.001	-0.009	-0.014	-0.003	-0.142	-0.032	-0.146	< .001
	$\widehat{M}$	-0.001	-0.003	0.003	-0.003	0.002	-0.033	-0.031	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.002	< .001	0.21	0.053	0.215	< .001
	2.5%	-0.005	-0.072	-0.135	-0.023	-1.528	-0.477	-1.568	-0.042
	97.5%	0.004	0.011	0.032	0.021	0.327	0.409	0.383	0.043
Skew		0.017	-2.027	-1.645	0.113	-2.014	-0.083	-1.935	0.111
Kurtosis		-0.186	4.11	1.543	-0.189	3.414	-0.181	3.791	0.023

Condition 17; Scenario 3; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.026	-0.022	-1.883	0.004	-0.055	0.005	-0.056	–
	$\widehat{M}$	0.022	-0.041	-0.403	-0.002	-0.002	0.014	-0.024	–
	$\widehat{\sigma}^2$	0.052	0.379	9.15	0.031	0.021	0.011	0.018	–
	2.5%	-0.431	-1.225	-7.963	-0.306	-0.444	-0.256	-0.374	–
	97.5%	0.469	1.636	1.346	0.358	0.121	0.198	0.13	–
P	$\widehat{\mu}$	< .001	-0.001	-0.043	< .001	-0.11	0.017	-0.14	–
	$\widehat{M}$	< .001	-0.001	-0.009	< .001	-0.005	0.05	-0.06	–
	$\widehat{\sigma}^2$	< .001	< .001	0.005	< .001	0.086	0.123	0.11	–
	2.5%	-0.004	-0.035	-0.183	-0.02	-0.887	-0.854	-0.935	–
	97.5%	0.005	0.047	0.031	0.024	0.243	0.662	0.326	–
Skew		0.02	1.141	-0.862	0.115	-1.241	-0.552	-1.079	–
Kurtosis		-0.17	4.918	-0.864	-0.186	1.001	1.376	1.06	–

TABLE C-52

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 18 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 18; Scenario 1; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.079	-0.107	0.022	-0.037	0.013	0.014	-0.007	-0.009	0.009	0.009	0.001	0.001
	$\widehat{M}$	-0.078	-0.112	0.021	-0.037	0.014	0.017	-0.009	-0.007	0.014	0.014	0.001	0.001
	$\widehat{\sigma}^2$	0.023	0.042	0.048	0.012	0.003	0.003	0.004	0.004	0.004	0.004	0.006	0.006
	2.5%	-0.382	-0.508	-0.435	-0.246	-0.097	-0.104	-0.135	-0.135	-0.118	-0.118	-0.15	-0.146
	97.5%	0.21	0.277	0.463	0.171	0.118	0.118	0.115	0.11	0.123	0.123	0.146	0.155
P	$\widehat{\mu}$	-0.001	-0.003	< .001	-0.002	0.027	0.028	-0.025	-0.031	0.023	0.023	< .001	< .001
	$\widehat{M}$	-0.001	-0.003	< .001	-0.003	0.028	0.034	-0.03	-0.026	0.034	0.034	< .001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.013	0.014	0.044	0.043	0.025	0.025	< .001	< .001
	2.5%	-0.004	-0.015	-0.01	-0.016	-0.194	-0.208	-0.45	-0.45	-0.296	-0.296	-0.038	-0.037
	97.5%	0.002	0.008	0.011	0.011	0.236	0.236	0.384	0.366	0.306	0.306	0.037	0.039
Skew		-0.057	0.003	-0.059	-0.076	-0.121	-0.274	-0.051	-0.128	-0.236	-0.236	0.003	0.098
Kurtosis		0.066	-0.016	0.168	0.118	0.004	-0.051	-0.077	0.153	0.193	0.193	0.032	0.161

Condition 18; Scenario 1; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.079	-0.118	0.033	-0.037	0.015	-0.008	0.009	0.001
	$\widehat{M}$	-0.079	-0.123	0.033	-0.037	0.016	-0.007	0.01	-0.001
	$\widehat{\sigma}^2$	0.023	0.037	0.04	0.012	0.002	0.002	0.003	0.003
	2.5%	-0.384	-0.482	-0.371	-0.247	-0.082	-0.096	-0.098	-0.098
	97.5%	0.209	0.255	0.408	0.172	0.1	0.084	0.1	0.104
P	$\widehat{\mu}$	-0.001	-0.003	0.001	-0.002	0.029	-0.026	0.021	< .001
	$\widehat{M}$	-0.001	-0.004	0.001	-0.003	0.032	-0.022	0.026	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.009	0.022	0.017	< .001
	2.5%	-0.004	-0.014	-0.009	-0.016	-0.164	-0.321	-0.245	-0.024
	97.5%	0.002	0.007	0.009	0.011	0.2	0.281	0.25	0.026
Skew		-0.058	0.003	-0.114	-0.075	-0.197	-0.01	-0.217	0.065
Kurtosis		0.067	-0.009	0.089	0.118	-0.033	0.072	-0.085	0.17

Condition 18; Scenario 1; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.079	-0.107	0.021	-0.037	0.013	0.014	-0.007	-0.009	0.009	0.006	0.001	0.001
	$\widehat{M}$	-0.078	-0.112	0.02	-0.037	0.014	0.017	-0.009	-0.007	0.014	0.009	0.002	0.001
	$\widehat{\sigma}^2$	0.023	0.042	0.048	0.012	0.003	0.004	0.004	0.004	0.004	0.004	0.006	0.006
	2.5%	-0.382	-0.508	-0.435	-0.246	-0.097	-0.104	-0.135	-0.135	-0.118	-0.13	-0.146	-0.146
	97.5%	0.21	0.277	0.463	0.171	0.118	0.118	0.115	0.11	0.123	0.121	0.155	0.155
P	$\widehat{\mu}$	-0.001	-0.003	< .001	-0.002	0.027	0.028	-0.025	-0.031	0.024	0.015	< .001	< .001
	$\widehat{M}$	-0.001	-0.003	< .001	-0.003	0.028	0.034	-0.029	-0.026	0.034	0.024	< .001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.013	0.014	0.045	0.043	0.025	0.026	< .001	< .001
	2.5%	-0.004	-0.015	-0.01	-0.016	-0.194	-0.208	-0.45	-0.45	-0.296	-0.325	-0.037	-0.037
	97.5%	0.002	0.008	0.011	0.011	0.236	0.236	0.384	0.366	0.306	0.303	0.039	0.039
Skew		-0.057	0.003	-0.059	-0.076	-0.12	-0.272	-0.052	-0.125	-0.236	-0.177	0.081	0.099
Kurtosis		0.066	-0.016	0.164	0.117	0.004	-0.041	-0.079	0.152	0.19	-0.163	0.159	0.161

Condition 18; Scenario 1; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.079	-0.118	0.033	-0.037	0.015	-0.008	0.009	0.001
	$\widehat{M}$	-0.079	-0.123	0.033	-0.037	0.016	-0.007	0.01	-0.001
	$\widehat{\sigma}^2$	0.023	0.037	0.04	0.012	0.002	0.002	0.003	0.003
	2.5%	-0.384	-0.482	-0.371	-0.247	-0.082	-0.096	-0.098	-0.098
	97.5%	0.21	0.255	0.408	0.172	0.1	0.084	0.1	0.104
P	$\widehat{\mu}$	-0.001	-0.003	0.001	-0.002	0.029	-0.026	0.021	< .001
	$\widehat{M}$	-0.001	-0.004	0.001	-0.003	0.032	-0.022	0.026	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.009	0.022	0.017	< .001
	2.5%	-0.004	-0.014	-0.009	-0.016	-0.164	-0.321	-0.245	-0.024
	97.5%	0.002	0.007	0.009	0.011	0.2	0.28	0.25	0.026
Skew		-0.058	0.004	-0.114	-0.075	-0.195	-0.01	-0.215	0.065
Kurtosis		0.066	-0.008	0.089	0.118	-0.036	0.072	-0.092	0.17

Condition 18; Scenario 1; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.023	0.001	0.009	0.008	0.02	0.031	0.007	–
	$\widehat{M}$	0.024	-0.003	0.013	0.009	0.02	0.032	0.008	–
	$\widehat{\sigma}^2$	0.024	0.042	0.048	0.012	0.002	0.002	0.002	–
	2.5%	-0.283	-0.396	-0.446	-0.204	-0.059	-0.049	-0.086	–
	97.5%	0.313	0.383	0.433	0.217	0.098	0.114	0.091	–
P	$\widehat{\mu}$	< .001	< .001	< .001	0.001	0.039	0.105	0.016	–
	$\widehat{M}$	< .001	< .001	< .001	0.001	0.04	0.107	0.02	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.006	0.018	0.013	–
	2.5%	-0.003	-0.011	-0.01	-0.014	-0.119	-0.162	-0.214	–
	97.5%	0.003	0.011	0.01	0.014	0.196	0.381	0.228	–
Skew		-0.061	0.027	-0.146	-0.076	-0.113	0.009	-0.145	–
Kurtosis		0.064	-0.034	0.167	0.116	-0.047	0.17	-0.077	–

TABLE C-53

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 18 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 18; Scenario 2; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	-0.057	-0.04	0.116	-0.038	< .001	0.019	-0.013	-0.016	-0.002	-0.002	-0.001	-0.003
	$\widehat{M}$	-0.058	-0.049	0.148	-0.035	0.006	0.029	-0.011	-0.016	0.001	0.001	< .001	-0.002
B	$\widehat{\sigma}^2$	0.023	0.059	0.294	0.013	0.004	0.009	0.004	0.009	0.003	0.003	0.004	0.012
	2.5%	-0.343	-0.372	-0.513	-0.264	-0.098	-0.152	-0.116	-0.208	-0.112	-0.112	-0.123	-0.213
	97.5%	0.241	0.298	0.831	0.176	0.092	0.164	0.084	0.152	0.093	0.093	0.126	0.213
	$\widehat{\mu}$	-0.001	-0.001	0.003	-0.003	< .001	0.038	-0.042	-0.052	-0.005	-0.005	< .001	-0.001
	$\widehat{M}$	-0.001	-0.001	0.003	-0.002	0.011	0.058	-0.036	-0.052	0.001	0.001	< .001	< .001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.016	0.034	0.041	0.097	0.02	0.02	< .001	0.001
	2.5%	-0.003	-0.011	-0.012	-0.018	-0.195	-0.304	-0.388	-0.692	-0.28	-0.28	-0.031	-0.053
	97.5%	0.002	0.009	0.019	0.012	0.185	0.328	0.28	0.506	0.234	0.234	0.031	0.053
Skew		0.113	5.665	-6.732	-0.052	-4.611	-2.651	-3.123	-0.891	-1.134	-1.134	0.038	0.011
Kurtosis		0.418	98.569	73.243	0.419	47.182	20.319	33.683	6.467	5.597	5.597	-0.065	-0.155

Condition 18; Scenario 2; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	-0.057	-0.05	0.127	-0.038	0.006	-0.011	0.004	-0.001
	$\widehat{M}$	-0.057	-0.05	0.169	-0.035	0.011	-0.01	0.007	-0.002
B	$\widehat{\sigma}^2$	0.023	0.05	0.25	0.012	0.004	0.002	0.003	0.003
	2.5%	-0.343	-0.36	-0.433	-0.266	-0.087	-0.096	-0.099	-0.107
	97.5%	0.245	0.282	0.684	0.174	0.095	0.07	0.096	0.106
	$\widehat{\mu}$	-0.001	-0.001	0.003	-0.003	0.011	-0.035	0.009	< .001
	$\widehat{M}$	-0.001	-0.001	0.004	-0.002	0.022	-0.034	0.018	-0.001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.017	0.02	0.019	< .001
	2.5%	-0.003	-0.01	-0.01	-0.018	-0.174	-0.32	-0.246	-0.027
	97.5%	0.002	0.008	0.016	0.012	0.19	0.232	0.241	0.027
Skew		0.17	-0.875	-7.562	-0.035	-6.185	-0.062	-1.701	0.07
Kurtosis		0.248	42.736	85.499	0.369	73.254	0.05	12.067	-0.172

Condition 18; Scenario 2; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	-0.057	-0.042	0.129	-0.038	0.001	0.021	-0.012	-0.015	-0.001	0.021	-0.003	-0.003
	$\widehat{M}$	-0.058	-0.049	0.146	-0.036	0.006	0.029	-0.011	-0.015	0.002	0.024	-0.002	-0.002
B	$\widehat{\sigma}^2$	0.023	0.051	0.213	0.013	0.003	0.008	0.003	0.009	0.003	0.007	0.012	0.012
	2.5%	-0.343	-0.368	-0.504	-0.264	-0.097	-0.144	-0.113	-0.208	-0.11	-0.157	-0.211	-0.213
	97.5%	0.241	0.291	0.831	0.174	0.092	0.164	0.084	0.151	0.094	0.173	0.212	0.214
	$\widehat{\mu}$	-0.001	-0.001	0.003	-0.003	0.003	0.041	-0.039	-0.049	-0.003	0.052	-0.001	-0.001
	$\widehat{M}$	-0.001	-0.001	0.003	-0.002	0.012	0.058	-0.036	-0.05	0.004	0.06	-0.001	-0.001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.013	0.031	0.037	0.095	0.019	0.045	0.001	0.001
	2.5%	-0.003	-0.011	-0.012	-0.018	-0.193	-0.288	-0.376	-0.692	-0.276	-0.393	-0.053	-0.053
	97.5%	0.002	0.008	0.019	0.012	0.185	0.328	0.281	0.502	0.235	0.433	0.053	0.054
Skew		0.168	5.898	-6.412	-0.047	-3.371	-2.25	-2.673	-0.848	-0.801	-0.502	0.003	0.012
Kurtosis		0.249	115.762	86.304	0.405	35.784	18.93	30.305	6.083	3.431	1.191	-0.169	-0.164

Condition 18; Scenario 2; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	-0.057	-0.05	0.155	-0.038	0.008	-0.011	0.005	-0.001
	$\widehat{M}$	-0.057	-0.051	0.169	-0.035	0.011	-0.01	0.008	-0.002
B	$\widehat{\sigma}^2$	0.023	0.03	0.087	0.012	0.003	0.002	0.003	0.003
	2.5%	-0.343	-0.357	-0.41	-0.266	-0.083	-0.096	-0.095	-0.107
	97.5%	0.245	0.275	0.684	0.174	0.095	0.07	0.096	0.106
	$\widehat{\mu}$	-0.001	-0.001	0.004	-0.003	0.016	-0.035	0.013	< .001
	$\widehat{M}$	-0.001	-0.001	0.004	-0.002	0.022	-0.034	0.02	-0.001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.011	0.02	0.017	< .001
	2.5%	-0.003	-0.01	-0.009	-0.018	-0.166	-0.32	-0.238	-0.027
	97.5%	0.002	0.008	0.016	0.012	0.19	0.232	0.241	0.027
Skew		0.173	-1.533	-1.417	-0.034	-4.628	-0.062	-1.228	0.07
Kurtosis		0.245	18.216	15.044	0.368	74.625	0.049	10.391	-0.172

Condition 18; Scenario 2; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.038	0.015	-0.001	0.006	0.014	0.026	0.004	–
	$\widehat{M}$	0.037	0.005	0.039	0.01	0.018	0.026	0.006	–
B	$\widehat{\sigma}^2$	0.023	0.049	0.305	0.013	0.002	0.002	0.002	–
	2.5%	-0.249	-0.308	-0.648	-0.22	-0.071	-0.052	-0.093	–
	97.5%	0.345	0.356	0.707	0.218	0.09	0.099	0.086	–
	$\widehat{\mu}$	< .001	< .001	< .001	< .001	0.028	0.087	0.009	–
	$\widehat{M}$	< .001	< .001	0.001	0.001	0.035	0.088	0.016	–
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.01	0.018	0.014	–
	2.5%	-0.002	-0.009	-0.015	-0.015	-0.142	-0.174	-0.232	–
	97.5%	0.003	0.01	0.016	0.015	0.18	0.331	0.215	–
Skew		0.179	5.214	-6.432	-0.034	-3.097	-0.307	-0.935	–
Kurtosis		0.251	72.439	65.853	0.367	27.113	1.23	3.665	–

TABLE C-54

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 18 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 18; Scenario 3; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.065	-0.158	-1.186	-0.038	-0.08	-0.077	-0.072	-0.068	-0.059	-0.059	0.001	0.001
	$\widehat{M}$	-0.065	-0.056	0.084	-0.033	-0.011	0.017	-0.017	-0.033	-0.015	-0.015	< .001	-0.002
	$\widehat{\sigma}^2$	0.033	0.503	7.455	0.015	0.034	0.074	0.035	0.041	0.02	0.02	0.004	0.02
	2.5%	-0.41	-1.731	-7.837	-0.272	-0.583	-0.825	-0.602	-0.549	-0.489	-0.489	-0.124	-0.27
	97.5%	0.262	1.543	1.023	0.186	0.083	0.218	0.116	0.24	0.082	0.082	0.131	0.285
P	$\widehat{\mu}$	-0.001	-0.004	-0.027	-0.003	-0.161	-0.153	-0.238	-0.228	-0.147	-0.147	< .001	< .001
	$\widehat{M}$	-0.001	-0.002	0.002	-0.002	-0.023	0.033	-0.058	-0.109	-0.038	-0.038	< .001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.004	< .001	0.138	0.295	0.386	0.452	0.128	0.128	< .001	0.001
	2.5%	-0.004	-0.049	-0.18	-0.018	-1.166	-1.649	-2.006	-1.832	-1.224	-1.224	-0.031	-0.068
	97.5%	0.003	0.044	0.024	0.012	0.166	0.436	0.388	0.801	0.205	0.205	0.033	0.071
Skew		-1.245	-0.343	-1.385	-0.104	-2.018	-1.529	-2.1	-0.913	-1.932	-1.932	-0.638	0.193
Kurtosis		13.911	9.005	0.314	1.066	4.199	1.682	5.59	0.985	3.466	3.466	5.731	0.493

Condition 18; Scenario 3; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.061	-0.287	-1.067	-0.036	-0.082	-0.007	-0.059	0.002
	$\widehat{M}$	-0.061	-0.071	0.139	-0.039	-0.003	-0.005	-0.008	0.001
	$\widehat{\sigma}^2$	0.021	0.443	6.349	0.012	0.047	0.002	0.028	0.003
	2.5%	-0.356	-2.154	-6.984	-0.248	-0.707	-0.089	-0.57	-0.104
	97.5%	0.229	0.35	0.95	0.173	0.098	0.076	0.091	0.112
P	$\widehat{\mu}$	-0.001	-0.008	-0.025	-0.002	-0.165	-0.022	-0.147	0.001
	$\widehat{M}$	-0.001	-0.002	0.003	-0.003	-0.006	-0.018	-0.021	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.003	< .001	0.188	0.02	0.172	< .001
	2.5%	-0.004	-0.062	-0.161	-0.017	-1.415	-0.297	-1.424	-0.026
	97.5%	0.002	0.01	0.022	0.012	0.196	0.252	0.228	0.028
Skew		-0.011	-1.605	-1.361	0.007	-1.866	-0.215	-2.127	0.107
Kurtosis		0.114	2.679	0.165	-0.185	2.391	0.194	4.18	0.088

Condition 18; Scenario 3; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.199	-0.253	-1.821	-0.127	-0.116	-0.129	-0.099	-0.09	-0.091	-0.09	-0.001	0.001
	$\widehat{M}$	-0.075	-0.092	-0.039	-0.049	-0.021	-0.015	-0.026	-0.047	-0.027	-0.026	-0.004	0.001
	$\widehat{\sigma}^2$	0.438	0.608	9.169	0.18	0.044	0.091	0.045	0.049	0.029	0.058	0.018	0.018
	2.5%	-2.424	-1.887	-7.942	-1.565	-0.614	-0.856	-0.625	-0.585	-0.526	-0.664	-0.267	-0.267
	97.5%	0.402	1.66	0.996	0.29	0.083	0.214	0.209	0.252	0.085	0.221	0.275	0.275
P	$\widehat{\mu}$	-0.002	-0.007	-0.042	-0.008	-0.233	-0.259	-0.331	-0.301	-0.227	-0.225	< .001	< .001
	$\widehat{M}$	-0.001	-0.003	-0.001	-0.003	-0.042	-0.03	-0.085	-0.158	-0.067	-0.065	-0.001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.005	0.001	0.176	0.365	0.501	0.539	0.181	0.365	0.001	0.001
	2.5%	-0.024	-0.054	-0.183	-0.104	-1.228	-1.712	-2.083	-1.949	-1.315	-1.661	-0.067	-0.067
	97.5%	0.004	0.047	0.023	0.019	0.166	0.428	0.697	0.841	0.213	0.552	0.069	0.069
Skew		-1.955	0.17	-0.812	-2.133	-1.279	-1.084	-1.251	-0.66	-1.445	-1.031	0.169	0.148
Kurtosis		8.29	2.634	-1.061	8.649	0.32	0.129	1.348	0.053	1.132	0.456	0.877	0.8

Condition 18; Scenario 3; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.06	-0.199	-0.323	-0.036	-0.05	-0.007	-0.035	0.002
	$\widehat{M}$	-0.062	-0.049	0.208	-0.039	0.002	-0.005	0.001	0.001
	$\widehat{\sigma}^2$	0.021	0.291	2.964	0.012	0.036	0.002	0.02	0.003
	2.5%	-0.356	-2.02	-5.557	-0.248	-0.71	-0.09	-0.542	-0.104
	97.5%	0.229	0.26	0.979	0.173	0.089	0.076	0.085	0.112
P	$\widehat{\mu}$	-0.001	-0.006	-0.007	-0.002	-0.1	-0.022	-0.088	0.001
	$\widehat{M}$	-0.001	-0.001	0.005	-0.003	0.003	-0.018	0.002	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.002	< .001	0.143	0.02	0.125	< .001
	2.5%	-0.004	-0.058	-0.128	-0.017	-1.421	-0.302	-1.355	-0.026
	97.5%	0.002	0.007	0.023	0.012	0.178	0.252	0.213	0.028
Skew		-0.016	-2.613	-2.408	0.007	-2.701	-0.21	-2.846	0.107
Kurtosis		0.126	7.054	4.482	-0.179	6.426	0.199	8.067	0.089

Condition 18; Scenario 3; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.033	-0.006	-1.45	0.009	-0.037	0.009	-0.035	–
	$\widehat{M}$	0.032	-0.015	-0.145	0.007	0.006	0.021	-0.007	–
	$\widehat{\sigma}^2$	0.022	0.214	7.546	0.012	0.013	0.005	0.009	–
	2.5%	-0.265	-0.851	-7.747	-0.203	-0.331	-0.171	-0.301	–
	97.5%	0.328	1.456	0.819	0.219	0.085	0.109	0.078	–
P	$\widehat{\mu}$	< .001	< .001	-0.033	0.001	-0.073	0.031	-0.088	–
	$\widehat{M}$	< .001	< .001	-0.003	< .001	0.012	0.069	-0.018	–
	$\widehat{\sigma}^2$	< .001	< .001	0.004	< .001	0.054	0.053	0.058	–
	2.5%	-0.003	-0.024	-0.178	-0.014	-0.662	-0.57	-0.753	–
	97.5%	0.003	0.042	0.019	0.015	0.169	0.362	0.195	–
Skew		-0.006	2.044	-1.298	0.008	-1.463	-1.157	-1.53	–
Kurtosis		0.128	9.699	-0.055	-0.168	1.23	2.757	2.195	–

TABLE C-55

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 19 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 19; Scenario 1; Method 1

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha_1.\beta_1}$	$\rho_{\alpha_2.\beta_2}$	$\rho_{\alpha_1.\gamma_1}$	$\rho_{\alpha_2.\gamma_2}$	$\rho_{\beta_1.\gamma_1}$	$\rho_{\beta_2.\gamma_2}$	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	-0.139	-0.121	-0.016	-0.072	0.009	0.002	-0.017	-0.021	-0.002	-0.002	0.002	-0.003
	$\widehat{M}$	-0.143	-0.148	0.024	-0.077	0.018	0.017	-0.009	-0.014	0.008	0.008	0.003	-0.009
B	$\widehat{\sigma}^2$	0.065	0.217	0.195	0.03	0.011	0.013	0.013	0.016	0.013	0.013	0.061	0.055
	2.5%	-0.631	-0.752	-0.756	-0.398	-0.219	-0.242	-0.237	-0.27	-0.238	-0.238	-0.497	-0.438
	97.5%	0.36	0.552	0.69	0.289	0.184	0.183	0.182	0.182	0.193	0.193	0.467	0.475
	$\widehat{\mu}$	-0.001	-0.003	< .001	-0.005	0.017	0.004	-0.057	-0.07	-0.005	-0.005	< .001	< .001
	$\widehat{M}$	-0.001	-0.004	0.001	-0.005	0.036	0.033	-0.029	-0.046	0.021	0.021	< .001	-0.001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.046	0.053	0.149	0.177	0.079	0.079	0.001	0.001
	2.5%	-0.006	-0.021	-0.017	-0.027	-0.438	-0.483	-0.792	-0.901	-0.594	-0.594	-0.062	-0.055
	97.5%	0.004	0.016	0.016	0.019	0.368	0.366	0.607	0.606	0.483	0.483	0.058	0.059
Skew		0.036	4.618	-3.539	0.214	-1.111	-1.247	-0.944	-1.269	-0.459	-0.459	-0.055	0.097
Kurtosis		0.094	47.831	37.343	0.207	4.398	5.051	4.879	7.332	0.426	0.426	-0.198	0.019

Condition 19; Scenario 1; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	-0.14	-0.144	0.009	-0.073	0.01	-0.014	< .001	< .001
	$\widehat{M}$	-0.145	-0.167	0.038	-0.076	0.021	-0.012	0.005	-0.006
B	$\widehat{\sigma}^2$	0.064	0.164	0.154	0.03	0.008	0.006	0.008	0.03
	2.5%	-0.637	-0.73	-0.655	-0.398	-0.172	-0.179	-0.188	-0.336
	97.5%	0.351	0.472	0.643	0.289	0.156	0.132	0.16	0.336
	$\widehat{\mu}$	-0.001	-0.004	< .001	-0.005	0.019	-0.048	< .001	< .001
	$\widehat{M}$	-0.001	-0.005	0.001	-0.005	0.042	-0.039	0.013	-0.001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.034	0.067	0.052	< .001
	2.5%	-0.006	-0.021	-0.015	-0.027	-0.345	-0.596	-0.47	-0.042
	97.5%	0.004	0.013	0.015	0.019	0.312	0.438	0.4	0.042
Skew		0.043	3.677	-2.521	0.22	-1.616	-0.117	-0.543	0.069
Kurtosis		0.114	32.56	26.391	0.215	9.822	-0.086	1.253	0.009

Condition 19; Scenario 1; Method 3.1

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha_1.\beta_1}$	$\rho_{\alpha_2.\beta_2}$	$\rho_{\alpha_1.\gamma_1}$	$\rho_{\alpha_2.\gamma_2}$	$\rho_{\beta_1.\gamma_1}$	$\rho_{\beta_2.\gamma_2}$	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
	$\widehat{\mu}$	-0.139	-0.125	-0.015	-0.072	0.009	0.002	-0.017	-0.021	-0.002	-0.004	-0.005	-0.002
	$\widehat{M}$	-0.143	-0.149	0.024	-0.077	0.018	0.017	-0.009	-0.014	0.009	0.005	-0.011	-0.01
B	$\widehat{\sigma}^2$	0.064	0.202	0.193	0.03	0.012	0.014	0.013	0.016	0.013	0.014	0.053	0.054
	2.5%	-0.63	-0.752	-0.723	-0.398	-0.222	-0.243	-0.237	-0.269	-0.238	-0.238	-0.437	-0.437
	97.5%	0.36	0.548	0.69	0.289	0.184	0.183	0.184	0.181	0.193	0.192	0.472	0.475
	$\widehat{\mu}$	-0.001	-0.004	< .001	-0.005	0.017	0.003	-0.056	-0.068	-0.004	-0.009	-0.001	< .001
	$\widehat{M}$	-0.001	-0.004	0.001	-0.005	0.037	0.034	-0.032	-0.048	0.021	0.013	-0.001	-0.001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.046	0.054	0.15	0.174	0.079	0.087	0.001	0.001
	2.5%	-0.006	-0.021	-0.017	-0.027	-0.443	-0.486	-0.792	-0.897	-0.594	-0.595	-0.055	-0.055
	97.5%	0.004	0.016	0.016	0.019	0.368	0.367	0.614	0.603	0.483	0.481	0.059	0.059
Skew		0.05	4.505	-3.548	0.216	-1.103	-1.242	-0.933	-1.257	-0.465	-1.145	0.129	0.157
Kurtosis		0.082	50.092	38.058	0.215	4.276	4.928	4.846	7.496	0.437	6.401	-0.209	-0.145

Condition 19; Scenario 1; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	-0.14	-0.16	0.014	-0.073	0.009	-0.015	0.001	< .001
	$\widehat{M}$	-0.144	-0.17	0.04	-0.076	0.021	-0.012	0.006	-0.006
B	$\widehat{\sigma}^2$	0.064	0.107	0.115	0.03	0.008	0.006	0.008	0.03
	2.5%	-0.637	-0.73	-0.645	-0.398	-0.172	-0.179	-0.184	-0.336
	97.5%	0.351	0.465	0.623	0.289	0.156	0.132	0.16	0.336
	$\widehat{\mu}$	-0.001	-0.005	< .001	-0.005	0.019	-0.048	0.003	< .001
	$\widehat{M}$	-0.001	-0.005	0.001	-0.005	0.042	-0.039	0.015	-0.001
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.033	0.067	0.05	< .001
	2.5%	-0.006	-0.021	-0.015	-0.027	-0.345	-0.596	-0.46	-0.042
	97.5%	0.004	0.013	0.014	0.019	0.312	0.438	0.4	0.042
Skew		0.042	0.876	-0.97	0.22	-1.62	-0.116	-0.351	0.069
Kurtosis		0.113	6.846	7.205	0.215	9.877	-0.086	0.025	0.009

Condition 19; Scenario 1; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
	$\widehat{\mu}$	0.048	0.022	0.009	0.009	0.034	0.056	0.011	–
	$\widehat{M}$	0.042	-0.011	0.043	0.005	0.04	0.057	0.017	–
B	$\widehat{\sigma}^2$	0.065	0.22	0.209	0.031	0.005	0.005	0.006	–
	2.5%	-0.446	-0.625	-0.722	-0.322	-0.115	-0.082	-0.147	–
	97.5%	0.544	0.703	0.72	0.377	0.153	0.184	0.147	–
	$\widehat{\mu}$	< .001	0.001	< .001	0.001	0.069	0.187	0.028	–
	$\widehat{M}$	< .001	< .001	0.001	< .001	0.081	0.19	0.042	–
P	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.02	0.05	0.037	–
	2.5%	-0.004	-0.018	-0.017	-0.021	-0.23	-0.275	-0.367	–
	97.5%	0.005	0.02	0.017	0.025	0.305	0.613	0.369	–
Skew		0.042	4.531	-3.69	0.22	-1.228	-0.446	-0.53	–
Kurtosis		0.106	45.883	35.97	0.221	5.151	1.346	1.07	–

TABLE C-56

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 19 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 19; Scenario 2; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.125	-0.031	-0.116	-0.078	-0.018	-0.01	-0.025	-0.033	-0.015	-0.015	0.013	0.011
	$\widehat{M}$	-0.126	-0.068	0.088	-0.084	0.004	0.026	-0.013	-0.014	0.001	0.001	0.009	0.005
	$\widehat{\sigma}^2$	0.059	0.243	1.635	0.035	0.016	0.042	0.016	0.037	0.012	0.012	0.041	0.111
	2.5%	-0.592	-0.679	-4.698	-0.425	-0.383	-0.529	-0.281	-0.44	-0.263	-0.263	-0.367	-0.626
	97.5%	0.353	0.802	1.171	0.275	0.147	0.249	0.162	0.285	0.146	0.146	0.427	0.684
P	$\widehat{\mu}$	-0.001	-0.001	-0.003	-0.005	-0.036	-0.02	-0.083	-0.109	-0.037	-0.037	0.002	0.001
	$\widehat{M}$	-0.001	-0.002	0.002	-0.006	0.008	0.051	-0.043	-0.047	0.003	0.003	0.001	0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	< .001	0.066	0.168	0.174	0.414	0.076	0.076	0.001	0.002
	2.5%	-0.006	-0.019	-0.108	-0.028	-0.768	-1.057	-0.937	-1.466	-0.657	-0.657	-0.046	-0.078
	97.5%	0.004	0.023	0.027	0.018	0.294	0.498	0.541	0.95	0.366	0.366	0.053	0.086
Skew		0.007	3.272	-3.729	0.084	-2.472	-2.4	-2.668	-1.143	-1.73	-1.73	0.064	0.158
Kurtosis		0.281	20.508	16.572	-0.22	9.613	10.269	17.41	4.564	7.488	7.488	-0.039	0.051

Condition 19; Scenario 2; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.127	-0.088	-0.047	-0.079	-0.012	-0.012	-0.011	0.014
	$\widehat{M}$	-0.13	-0.087	0.136	-0.085	0.012	-0.015	0.003	0.014
	$\widehat{\sigma}^2$	0.056	0.186	1.202	0.034	0.02	0.005	0.013	0.029
	2.5%	-0.592	-0.785	-3.922	-0.425	-0.379	-0.154	-0.256	-0.318
	97.5%	0.339	0.555	1.016	0.268	0.156	0.13	0.149	0.356
P	$\widehat{\mu}$	-0.001	-0.003	-0.001	-0.005	-0.024	-0.042	-0.027	0.002
	$\widehat{M}$	-0.001	-0.003	0.003	-0.006	0.024	-0.052	0.007	0.002
	$\widehat{\sigma}^2$	< .001	< .001	0.001	< .001	0.08	0.059	0.083	< .001
	2.5%	-0.006	-0.022	-0.09	-0.028	-0.758	-0.513	-0.64	-0.04
	97.5%	0.003	0.016	0.023	0.018	0.313	0.433	0.373	0.044
Skew		0.106	0.3	-3.881	0.098	-3.367	0.048	-2.328	0.045
Kurtosis		0.052	15.36	19.061	-0.251	16.539	-0.159	10.712	0.097

Condition 19; Scenario 2; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.123	-0.034	-0.108	-0.078	-0.018	-0.01	-0.024	-0.031	-0.013	-0.006	0.013	0.011
	$\widehat{M}$	-0.124	-0.069	0.083	-0.084	0.005	0.025	-0.012	-0.013	0.002	0.018	0.007	0.005
	$\widehat{\sigma}^2$	0.064	0.232	1.584	0.035	0.017	0.042	0.015	0.036	0.011	0.031	0.107	0.11
	2.5%	-0.59	-0.676	-4.67	-0.425	-0.401	-0.529	-0.261	-0.432	-0.242	-0.396	-0.601	-0.626
	97.5%	0.358	0.663	1.145	0.275	0.147	0.251	0.162	0.287	0.148	0.274	0.68	0.68
P	$\widehat{\mu}$	-0.001	-0.001	-0.002	-0.005	-0.036	-0.02	-0.081	-0.105	-0.033	-0.016	0.002	0.001
	$\widehat{M}$	-0.001	-0.002	0.002	-0.006	0.01	0.049	-0.041	-0.043	0.005	0.044	0.001	0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	< .001	0.069	0.169	0.171	0.396	0.071	0.196	0.002	0.002
	2.5%	-0.006	-0.019	-0.107	-0.028	-0.802	-1.057	-0.869	-1.44	-0.603	-0.992	-0.075	-0.078
	97.5%	0.004	0.019	0.026	0.018	0.294	0.501	0.541	0.959	0.369	0.684	0.085	0.085
Skew		0.004	3.395	-3.785	0.088	-2.633	-2.439	-2.66	-0.975	-1.352	-1.175	0.17	0.157
Kurtosis		4.603	22.075	17.232	-0.234	10.854	10.372	17.818	3.474	4.064	3.18	0.135	0.058

Condition 19; Scenario 2; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.127	-0.108	0.096	-0.079	-0.001	-0.013	-0.002	0.014
	$\widehat{M}$	-0.13	-0.087	0.154	-0.085	0.014	-0.016	0.007	0.014
	$\widehat{\sigma}^2$	0.057	0.113	0.385	0.034	0.014	0.005	0.01	0.029
	2.5%	-0.592	-0.651	-0.892	-0.425	-0.182	-0.154	-0.198	-0.318
	97.5%	0.339	0.408	1.004	0.268	0.156	0.13	0.149	0.356
P	$\widehat{\mu}$	-0.001	-0.003	0.002	-0.005	-0.002	-0.042	-0.005	0.002
	$\widehat{M}$	-0.001	-0.003	0.004	-0.006	0.028	-0.053	0.016	0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.054	0.059	0.065	< .001
	2.5%	-0.006	-0.019	-0.021	-0.028	-0.364	-0.513	-0.494	-0.04
	97.5%	0.003	0.012	0.023	0.018	0.313	0.433	0.373	0.044
Skew		0.102	-2.444	-2.786	0.096	-3.606	0.049	-2.061	0.044
Kurtosis		0.052	15.139	17.047	-0.252	24.417	-0.164	10.67	0.095

Condition 19; Scenario 2; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.049	0.069	-0.203	0.004	0.017	0.05	0.001	–
	$\widehat{M}$	0.05	0.021	0.013	-0.004	0.034	0.051	0.011	–
	$\widehat{\sigma}^2$	0.058	0.223	1.651	0.035	0.01	0.005	0.008	–
	2.5%	-0.415	-0.545	-4.874	-0.351	-0.256	-0.084	-0.22	–
	97.5%	0.525	0.929	1.056	0.357	0.146	0.176	0.134	–
P	$\widehat{\mu}$	0.001	0.002	-0.005	< .001	0.035	0.166	0.003	–
	$\widehat{M}$	0.001	0.001	< .001	< .001	0.068	0.169	0.028	–
	$\widehat{\sigma}^2$	< .001	< .001	0.001	< .001	0.04	0.059	0.052	–
	2.5%	-0.004	-0.016	-0.112	-0.023	-0.512	-0.279	-0.551	–
	97.5%	0.005	0.026	0.024	0.024	0.291	0.586	0.336	–
Skew		0.08	3.583	-3.605	0.096	-2.528	-0.766	-1.446	–
Kurtosis		0.038	20.71	15.087	-0.239	10.322	2.892	4.487	–

TABLE C-57

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 19 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition19; Scenario3; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.102	-0.078	-1.683	-0.076	-0.093	-0.134	-0.084	-0.09	-0.068	-0.068	-0.005	-0.03
	$\widehat{M}$	-0.102	-0.108	-0.137	-0.078	-0.031	-0.043	-0.027	-0.042	-0.027	-0.027	-0.009	-0.03
	$\widehat{\sigma}^2$	0.069	0.698	9.827	0.032	0.044	0.134	0.072	0.099	0.029	0.029	0.046	0.169
	2.5%	-0.596	-1.611	-8.31	-0.43	-0.697	-1.147	-0.877	-0.838	-0.527	-0.527	-0.402	-0.826
	97.5%	0.444	2.255	1.673	0.267	0.163	0.317	0.339	0.4	0.139	0.139	0.418	0.802
P	$\widehat{\mu}$	-0.001	-0.002	-0.039	-0.005	-0.186	-0.267	-0.279	-0.3	-0.17	-0.17	-0.001	-0.004
	$\widehat{M}$	-0.001	-0.003	-0.003	-0.005	-0.062	-0.086	-0.089	-0.141	-0.067	-0.067	-0.001	-0.004
	$\widehat{\sigma}^2$	< .001	0.001	0.005	< .001	0.177	0.535	0.803	1.103	0.18	0.18	0.001	0.003
	2.5%	-0.006	-0.046	-0.191	-0.029	-1.394	-2.293	-2.923	-2.795	-1.317	-1.317	-0.05	-0.103
	97.5%	0.004	0.064	0.038	0.018	0.327	0.634	1.131	1.334	0.348	0.348	0.052	0.1
Skew		0.069	0.864	-0.942	-0.035	-1.594	-1.337	-1.825	-0.825	-1.434	-1.434	-0.022	0.062
Kurtosis		0.223	4.603	-0.599	0.306	2.709	1.496	4.716	0.844	2.476	2.476	2.532	0.599

Condition 19; Scenario 3; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.123	-0.359	-1.473	-0.084	-0.097	-0.014	-0.07	-0.004
	$\widehat{M}$	-0.123	-0.151	-0.052	-0.082	-0.023	-0.01	-0.021	-0.004
	$\widehat{\sigma}^2$	0.062	0.689	7.959	0.03	0.063	0.006	0.038	0.028
	2.5%	-0.595	-2.628	-7.462	-0.412	-0.802	-0.168	-0.632	-0.301
	97.5%	0.38	0.875	1.472	0.255	0.189	0.137	0.173	0.348
P	$\widehat{\mu}$	-0.001	-0.01	-0.034	-0.006	-0.195	-0.045	-0.176	-0.001
	$\widehat{M}$	-0.001	-0.004	-0.001	-0.005	-0.046	-0.033	-0.052	-0.001
	$\widehat{\sigma}^2$	< .001	0.001	0.004	< .001	0.252	0.065	0.236	< .001
	2.5%	-0.006	-0.075	-0.172	-0.027	-1.603	-0.559	-1.58	-0.038
	97.5%	0.004	0.025	0.034	0.017	0.379	0.457	0.431	0.043
Skew		-0.005	-1.13	-0.972	-0.005	-1.642	-0.174	-1.68	0.214
Kurtosis		0.169	2.629	-0.518	-0.177	2.264	-0.072	3.325	0.02

Condition19; Scenario3; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.205	-0.17	-2.206	-0.159	-0.124	-0.184	-0.108	-0.114	-0.094	-0.127	-0.027	-0.024
	$\widehat{M}$	-0.134	-0.149	-0.47	-0.095	-0.044	-0.079	-0.04	-0.066	-0.04	-0.091	-0.032	-0.029
	$\widehat{\sigma}^2$	0.352	0.837	10.553	0.192	0.053	0.148	0.082	0.108	0.037	0.093	0.152	0.153
	2.5%	-2.027	-2.147	-8.348	-1.691	-0.743	-1.154	-0.839	-0.852	-0.617	-0.844	-0.796	-0.791
	97.5%	0.527	2.098	1.533	0.317	0.164	0.307	0.366	0.39	0.141	0.333	0.769	0.783
P	$\widehat{\mu}$	-0.002	-0.005	-0.051	-0.011	-0.249	-0.368	-0.36	-0.379	-0.236	-0.317	-0.003	-0.003
	$\widehat{M}$	-0.001	-0.004	-0.011	-0.006	-0.089	-0.157	-0.131	-0.221	-0.1	-0.229	-0.004	-0.004
	$\widehat{\sigma}^2$	< .001	0.001	0.006	0.001	0.212	0.593	0.907	1.195	0.233	0.581	0.002	0.002
	2.5%	-0.02	-0.061	-0.192	-0.113	-1.487	-2.308	-2.797	-2.839	-1.542	-2.111	-0.1	-0.099
	97.5%	0.005	0.06	0.035	0.021	0.329	0.613	1.219	1.3	0.353	0.832	0.096	0.098
Skew		-1.527	0.549	-0.588	-2.508	-1.212	-1.074	-1.336	-0.703	-1.261	-0.736	-0.012	-0.005
Kurtosis		8.234	2.749	-1.195	10.963	0.913	0.598	2.484	0.402	1.338	0.326	0.751	0.715

Condition 19; Scenario 3; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.123	-0.386	-0.696	-0.084	-0.081	-0.014	-0.059	-0.004
	$\widehat{M}$	-0.123	-0.141	0.069	-0.082	-0.007	-0.01	-0.013	-0.004
	$\widehat{\sigma}^2$	0.062	0.61	4.57	0.03	0.064	0.006	0.037	0.028
	2.5%	-0.595	-2.664	-6.035	-0.412	-0.848	-0.168	-0.696	-0.301
	97.5%	0.381	0.413	1.472	0.253	0.196	0.137	0.158	0.348
P	$\widehat{\mu}$	-0.001	-0.011	-0.016	-0.006	-0.161	-0.046	-0.146	-0.001
	$\widehat{M}$	-0.001	-0.004	0.002	-0.005	-0.015	-0.033	-0.033	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.002	< .001	0.257	0.065	0.233	< .001
	2.5%	-0.006	-0.076	-0.139	-0.027	-1.696	-0.559	-1.739	-0.038
	97.5%	0.004	0.012	0.034	0.017	0.392	0.457	0.394	0.043
Skew		-0.002	-1.898	-1.501	-0.006	-1.951	-0.18	-2.04	0.213
Kurtosis		0.153	3.42	1.098	-0.178	3.295	-0.07	4.721	0.02

Condition 19; Scenario 3; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.049	0.05	-1.883	-0.001	-0.044	0.03	-0.047	–
	$\widehat{M}$	0.049	-0.029	-0.346	-0.003	0.003	0.048	-0.014	–
	$\widehat{\sigma}^2$	0.063	0.475	9.622	0.031	0.022	0.013	0.017	–
	2.5%	-0.44	-1.14	-8.122	-0.34	-0.437	-0.24	-0.395	–
	97.5%	0.561	1.999	1.527	0.342	0.142	0.23	0.128	–
P	$\widehat{\mu}$	< .001	0.001	-0.043	< .001	-0.088	0.101	-0.117	–
	$\widehat{M}$	< .001	-0.001	-0.008	< .001	0.007	0.159	-0.034	–
	$\widehat{\sigma}^2$	< .001	< .001	0.005	< .001	0.089	0.148	0.107	–
	2.5%	-0.004	-0.033	-0.187	-0.023	-0.875	-0.8	-0.989	–
	97.5%	0.006	0.057	0.035	0.023	0.284	0.768	0.32	–
Skew		-0.001	1.434	-0.864	-0.007	-1.261	-0.746	-1.266	–
Kurtosis		0.175	4.567	-0.805	-0.185	1.149	1.573	1.824	–

TABLE C-58

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 20 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 20; Scenario 1; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.128	-0.132	0.01	-0.077	0.008	0.009	-0.014	-0.012	0.009	0.009	< .001	0.006
	$\widehat{M}$	-0.133	-0.131	0.009	-0.076	0.013	0.018	-0.011	-0.008	0.009	0.009	0.003	0.006
	$\widehat{\sigma}^2$	0.026	0.06	0.064	0.012	0.005	0.007	0.005	0.005	0.005	0.005	0.024	0.022
	2.5%	-0.432	-0.56	-0.426	-0.287	-0.123	-0.142	-0.157	-0.15	-0.135	-0.135	-0.293	-0.297
	97.5%	0.189	0.312	0.447	0.14	0.117	0.13	0.108	0.111	0.134	0.134	0.307	0.293
P	$\widehat{\mu}$	-0.001	-0.004	< .001	-0.005	0.017	0.018	-0.047	-0.04	0.021	0.021	< .001	0.001
	$\widehat{M}$	-0.001	-0.004	< .001	-0.005	0.027	0.036	-0.036	-0.026	0.022	0.022	< .001	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.02	0.027	0.052	0.054	0.031	0.031	< .001	< .001
	2.5%	-0.004	-0.016	-0.01	-0.019	-0.246	-0.284	-0.522	-0.501	-0.337	-0.337	-0.037	-0.037
	97.5%	0.002	0.009	0.01	0.009	0.234	0.261	0.359	0.368	0.336	0.336	0.038	0.037
Skew		0.054	2.472	-2.38	0.067	-3.026	-5.196	-0.271	-0.633	-0.429	-0.429	< .001	-0.066
Kurtosis		0.482	32.141	39.289	0.025	31.027	80.957	0.359	2.941	0.918	0.918	-0.133	-0.08

Condition 20; Scenario 1; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.128	-0.136	0.015	-0.077	0.01	-0.012	0.008	0.003
	$\widehat{M}$	-0.132	-0.14	0.014	-0.076	0.018	-0.011	0.01	0.002
	$\widehat{\sigma}^2$	0.026	0.05	0.056	0.012	0.003	0.002	0.003	0.012
	2.5%	-0.435	-0.512	-0.367	-0.286	-0.106	-0.111	-0.108	-0.21
	97.5%	0.182	0.274	0.441	0.14	0.101	0.077	0.119	0.215
P	$\widehat{\mu}$	-0.001	-0.004	< .001	-0.005	0.021	-0.041	0.021	< .001
	$\widehat{M}$	-0.001	-0.004	< .001	-0.005	0.035	-0.036	0.026	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.014	0.025	0.021	< .001
	2.5%	-0.004	-0.015	-0.008	-0.019	-0.213	-0.372	-0.271	-0.026
	97.5%	0.002	0.008	0.01	0.009	0.202	0.258	0.297	0.027
Skew		0.152	2.997	-2.923	0.063	-2.478	-0.136	-0.584	0.027
Kurtosis		0.016	41.777	54.723	0.03	22.725	0.175	3.026	-0.093

Condition 20; Scenario 1; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.127	-0.133	0.012	-0.077	0.01	0.011	-0.014	-0.012	0.009	0.008	0.006	0.006
	$\widehat{M}$	-0.133	-0.132	0.01	-0.076	0.013	0.018	-0.011	-0.008	0.009	0.013	0.006	0.006
	$\widehat{\sigma}^2$	0.026	0.048	0.048	0.012	0.004	0.005	0.005	0.005	0.005	0.005	0.022	0.022
	2.5%	-0.43	-0.553	-0.426	-0.287	-0.122	-0.136	-0.157	-0.15	-0.135	-0.131	-0.293	-0.297
	97.5%	0.186	0.312	0.446	0.14	0.117	0.13	0.107	0.111	0.135	0.136	0.297	0.293
P	$\widehat{\mu}$	-0.001	-0.004	< .001	-0.005	0.019	0.021	-0.046	-0.039	0.023	0.019	0.001	0.001
	$\widehat{M}$	-0.001	-0.004	< .001	-0.005	0.027	0.036	-0.036	-0.026	0.022	0.032	0.001	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.015	0.019	0.051	0.051	0.03	0.03	< .001	< .001
	2.5%	-0.004	-0.016	-0.01	-0.019	-0.243	-0.272	-0.522	-0.501	-0.336	-0.328	-0.037	-0.037
	97.5%	0.002	0.009	0.01	0.009	0.234	0.261	0.357	0.368	0.338	0.34	0.037	0.037
Skew		0.151	0.159	0.034	0.066	-0.46	-0.439	-0.238	-0.234	-0.287	-0.161	-0.047	-0.069
Kurtosis		0.012	0.187	-0.018	0.031	0.379	0.193	0.266	-0.088	0.236	-0.188	-0.064	-0.076

Condition 20; Scenario 1; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.128	-0.139	0.017	-0.077	0.011	-0.012	0.009	0.003
	$\widehat{M}$	-0.132	-0.139	0.015	-0.076	0.018	-0.011	0.01	0.002
	$\widehat{\sigma}^2$	0.026	0.039	0.04	0.012	0.003	0.002	0.003	0.012
	2.5%	-0.435	-0.512	-0.364	-0.286	-0.106	-0.111	-0.105	-0.21
	97.5%	0.182	0.272	0.427	0.14	0.101	0.077	0.119	0.215
P	$\widehat{\mu}$	-0.001	-0.004	< .001	-0.005	0.023	-0.041	0.023	< .001
	$\widehat{M}$	-0.001	-0.004	< .001	-0.005	0.035	-0.036	0.026	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.011	0.025	0.02	< .001
	2.5%	-0.004	-0.015	-0.008	-0.019	-0.212	-0.372	-0.263	-0.026
	97.5%	0.002	0.008	0.01	0.009	0.202	0.258	0.297	0.027
Skew		0.151	0.124	0.126	0.062	-0.487	-0.136	-0.128	0.027
Kurtosis		0.014	0.075	0.137	0.029	0.206	0.175	-0.104	-0.093

Condition 20; Scenario 1; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.06	0.018	0.032	0.005	0.036	0.058	0.017	–
	$\widehat{M}$	0.057	0.008	0.031	0.007	0.041	0.059	0.019	–
	$\widehat{\sigma}^2$	0.026	0.064	0.059	0.012	0.002	0.002	0.002	–
	2.5%	-0.245	-0.42	-0.391	-0.205	-0.056	-0.024	-0.077	–
	97.5%	0.38	0.459	0.465	0.225	0.106	0.133	0.112	–
P	$\widehat{\mu}$	0.001	< .001	0.001	< .001	0.072	0.195	0.043	–
	$\widehat{M}$	0.001	< .001	0.001	< .001	0.083	0.199	0.046	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.008	0.018	0.015	–
	2.5%	-0.002	-0.012	-0.009	-0.014	-0.113	-0.081	-0.193	–
	97.5%	0.004	0.013	0.011	0.015	0.211	0.445	0.281	–
Skew		0.151	2.737	-2.81	0.057	-2.245	-0.198	-0.455	–
Kurtosis		0.039	30.327	39.704	0.022	21.526	0.438	2.213	–

TABLE C-59

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 20 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 20; Scenario 2; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.11	-0.066	0.078	-0.071	-0.002	0.023	-0.014	-0.017	-0.002	-0.002	0.002	0.024
	$\widehat{M}$	-0.111	-0.065	0.129	-0.069	0.008	0.032	-0.01	-0.006	0.001	0.001	-0.001	0.018
	$\widehat{\sigma}^2$	0.026	0.055	0.398	0.012	0.006	0.012	0.005	0.014	0.005	0.005	0.016	0.045
	2.5%	-0.439	-0.428	-0.592	-0.284	-0.107	-0.181	-0.138	-0.236	-0.127	-0.127	-0.238	-0.384
	97.5%	0.196	0.267	0.751	0.14	0.094	0.176	0.095	0.167	0.113	0.113	0.257	0.455
P	$\widehat{\mu}$	-0.001	-0.002	0.002	-0.005	-0.003	0.046	-0.047	-0.056	-0.005	-0.005	< .001	0.003
	$\widehat{M}$	-0.001	-0.002	0.003	-0.005	0.016	0.064	-0.034	-0.021	0.002	0.002	< .001	0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.025	0.047	0.06	0.156	0.03	0.03	< .001	0.001
	2.5%	-0.004	-0.012	-0.014	-0.019	-0.213	-0.362	-0.459	-0.788	-0.316	-0.316	-0.03	-0.048
	97.5%	0.002	0.008	0.017	0.009	0.188	0.352	0.315	0.556	0.283	0.283	0.032	0.057
Skew		-0.126	1.646	-6.274	-0.048	-5.475	-2.783	-2.869	-1.953	-2.493	-2.493	0.072	0.098
Kurtosis		0.257	40.021	53.614	-0.139	51.037	18.005	20.892	13.975	18.609	18.609	0.091	-0.087

Condition 20; Scenario 2; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.109	-0.074	0.09	-0.071	0.006	-0.01	0.003	0.008
	$\widehat{M}$	-0.113	-0.07	0.15	-0.068	0.013	-0.009	0.007	0.005
	$\widehat{\sigma}^2$	0.026	0.053	0.337	0.012	0.006	0.002	0.005	0.012
	2.5%	-0.443	-0.4	-0.475	-0.283	-0.095	-0.106	-0.11	-0.192
	97.5%	0.189	0.242	0.647	0.14	0.097	0.079	0.108	0.237
P	$\widehat{\mu}$	-0.001	-0.002	0.002	-0.005	0.012	-0.033	0.008	0.001
	$\widehat{M}$	-0.001	-0.002	0.003	-0.005	0.026	-0.029	0.018	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.023	0.026	0.028	< .001
	2.5%	-0.004	-0.011	-0.011	-0.019	-0.19	-0.352	-0.275	-0.024
	97.5%	0.002	0.007	0.015	0.009	0.194	0.262	0.27	0.03
Skew		-0.106	-2.973	-6.49	-0.025	-5.626	-0.117	-3.343	0.122
Kurtosis		0.029	51.011	54.229	-0.194	52.397	-0.199	30.357	-0.1

Condition 20; Scenario 2; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.11	-0.069	0.082	-0.071	-0.001	0.023	-0.014	-0.016	-0.001	0.016	0.024	0.024
	$\widehat{M}$	-0.111	-0.065	0.127	-0.069	0.008	0.033	-0.01	-0.005	0.001	0.025	0.018	0.018
	$\widehat{\sigma}^2$	0.026	0.05	0.357	0.012	0.006	0.012	0.005	0.014	0.005	0.011	0.042	0.045
	2.5%	-0.439	-0.428	-0.583	-0.284	-0.105	-0.179	-0.136	-0.236	-0.13	-0.191	-0.357	-0.384
	97.5%	0.196	0.263	0.745	0.139	0.094	0.179	0.093	0.167	0.113	0.183	0.454	0.455
P	$\widehat{\mu}$	-0.001	-0.002	0.002	-0.005	-0.003	0.046	-0.045	-0.055	-0.004	0.041	0.003	0.003
	$\widehat{M}$	-0.001	-0.002	0.003	-0.005	0.016	0.066	-0.034	-0.017	0.003	0.062	0.002	0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.025	0.047	0.053	0.155	0.03	0.067	0.001	0.001
	2.5%	-0.004	-0.012	-0.013	-0.019	-0.209	-0.358	-0.454	-0.787	-0.325	-0.478	-0.045	-0.048
	97.5%	0.002	0.008	0.017	0.009	0.188	0.357	0.31	0.556	0.283	0.457	0.057	0.057
Skew		-0.102	0.948	-6.178	-0.049	-5.665	-2.837	-2.306	-1.972	-2.484	-1.41	0.126	0.097
Kurtosis		0.179	38.075	54.493	-0.138	54.559	18.585	15.154	14.208	18.303	6.572	-0.028	-0.078

Condition 20; Scenario 2; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.109	-0.073	0.14	-0.071	0.011	-0.01	0.007	0.008
	$\widehat{M}$	-0.113	-0.07	0.153	-0.068	0.013	-0.009	0.008	0.005
	$\widehat{\sigma}^2$	0.026	0.031	0.09	0.012	0.003	0.002	0.003	0.012
	2.5%	-0.438	-0.391	-0.439	-0.283	-0.086	-0.106	-0.101	-0.192
	97.5%	0.189	0.223	0.646	0.14	0.097	0.079	0.108	0.237
P	$\widehat{\mu}$	-0.001	-0.002	0.003	-0.005	0.022	-0.033	0.018	0.001
	$\widehat{M}$	-0.001	-0.002	0.004	-0.005	0.028	-0.029	0.02	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.012	0.026	0.02	< .001
	2.5%	-0.004	-0.011	-0.01	-0.019	-0.171	-0.352	-0.253	-0.024
	97.5%	0.002	0.006	0.015	0.009	0.194	0.262	0.269	0.03
Skew		-0.103	-2.362	-2.421	-0.025	-4.794	-0.118	-1.64	0.122
Kurtosis		0.031	29.165	30.169	-0.193	76.576	-0.197	18.059	-0.1

Condition 20; Scenario 2; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.067	0.032	-0.005	0.012	0.032	0.056	0.013	–
	$\widehat{M}$	0.066	0.029	0.058	0.014	0.036	0.057	0.015	–
	$\widehat{\sigma}^2$	0.026	0.046	0.428	0.013	0.002	0.002	0.003	–
	2.5%	-0.263	-0.329	-0.668	-0.202	-0.053	-0.028	-0.088	–
	97.5%	0.371	0.364	0.682	0.223	0.108	0.132	0.103	–
P	$\widehat{\mu}$	0.001	0.001	< .001	0.001	0.064	0.186	0.032	–
	$\widehat{M}$	0.001	0.001	0.001	0.001	0.072	0.191	0.038	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.01	0.022	0.017	–
	2.5%	-0.003	-0.009	-0.015	-0.013	-0.106	-0.094	-0.22	–
	97.5%	0.004	0.01	0.016	0.015	0.216	0.441	0.258	–
Skew		-0.108	3.531	-6.147	-0.029	-2.801	-0.572	-1.439	–
Kurtosis		0.061	39.947	49.595	-0.193	16.808	2.234	7.982	–

TABLE C-60

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 20 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 20; Scenario 3; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.099	-0.082	-1.181	-0.056	-0.074	-0.062	-0.059	-0.056	-0.048	-0.048	-0.001	0.007
	$\widehat{M}$	-0.1	-0.068	0.068	-0.055	-0.008	0.028	-0.015	-0.021	-0.015	-0.015	-0.007	-0.004
	$\widehat{\sigma}^2$	0.036	0.476	7.759	0.014	0.036	0.084	0.04	0.06	0.015	0.015	0.016	0.081
	2.5%	-0.497	-1.56	-8.037	-0.272	-0.658	-0.894	-0.619	-0.77	-0.426	-0.426	-0.26	-0.565
	97.5%	0.278	1.901	1.21	0.169	0.097	0.233	0.224	0.289	0.097	0.097	0.249	0.573
P	$\widehat{\mu}$	-0.001	-0.002	-0.027	-0.004	-0.147	-0.124	-0.196	-0.187	-0.121	-0.121	< .001	0.001
	$\widehat{M}$	-0.001	-0.002	0.002	-0.004	-0.016	0.055	-0.05	-0.071	-0.037	-0.037	-0.001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.004	< .001	0.143	0.337	0.445	0.663	0.096	0.096	< .001	0.001
	2.5%	-0.005	-0.045	-0.185	-0.018	-1.315	-1.79	-2.063	-2.566	-1.066	-1.066	-0.033	-0.071
	97.5%	0.003	0.054	0.028	0.011	0.194	0.465	0.748	0.964	0.243	0.243	0.031	0.072
Skew		-0.285	0.481	-1.436	-0.128	-1.985	-1.762	-2.729	-1.428	-1.957	-1.957	0.115	0.038
Kurtosis		1.441	6.12	0.512	1.233	3.133	2.914	10.337	3.706	4.144	4.144	0.134	0.309

Condition 20; Scenario 3; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.109	-0.257	-1.067	-0.063	-0.072	-0.008	-0.049	0.001
	$\widehat{M}$	-0.112	-0.084	0.133	-0.061	-0.002	-0.007	-0.01	0.003
	$\widehat{\sigma}^2$	0.024	0.4	6.443	0.012	0.047	0.002	0.023	0.012
	2.5%	-0.401	-2.105	-6.899	-0.25	-0.752	-0.102	-0.561	-0.211
	97.5%	0.193	0.596	1.043	0.153	0.121	0.089	0.105	0.225
P	$\widehat{\mu}$	-0.001	-0.007	-0.025	-0.004	-0.145	-0.026	-0.122	< .001
	$\widehat{M}$	-0.001	-0.002	0.003	-0.004	-0.004	-0.023	-0.024	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.003	< .001	0.19	0.027	0.141	< .001
	2.5%	-0.004	-0.06	-0.159	-0.017	-1.503	-0.34	-1.402	-0.026
	97.5%	0.002	0.017	0.024	0.01	0.241	0.298	0.264	0.028
Skew		-0.037	-1.853	-1.379	0.198	-2.079	-0.118	-2.419	0.033
Kurtosis		-0.321	3.777	0.206	-0.226	3.631	0.359	6.844	-0.018

Condition 20; Scenario 3; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.215	-0.174	-1.716	-0.126	-0.111	-0.12	-0.093	-0.088	-0.078	-0.082	0.007	0.013
	$\widehat{M}$	-0.111	-0.089	-0.038	-0.058	-0.015	0.006	-0.028	-0.039	-0.021	-0.026	-0.006	0.005
	$\widehat{\sigma}^2$	0.328	0.635	9.224	0.133	0.049	0.113	0.053	0.07	0.026	0.066	0.072	0.073
	2.5%	-2.041	-1.851	-8.115	-1.404	-0.678	-0.978	-0.706	-0.804	-0.509	-0.749	-0.496	-0.496
	97.5%	0.358	1.901	1.181	0.244	0.099	0.236	0.245	0.269	0.097	0.251	0.573	0.573
P	$\widehat{\mu}$	-0.002	-0.005	-0.039	-0.008	-0.222	-0.24	-0.31	-0.295	-0.195	-0.206	0.001	0.002
	$\widehat{M}$	-0.001	-0.003	-0.001	-0.004	-0.029	0.012	-0.092	-0.131	-0.054	-0.064	-0.001	0.001
	$\widehat{\sigma}^2$	< .001	0.001	0.005	0.001	0.194	0.452	0.594	0.782	0.16	0.41	0.001	0.001
	2.5%	-0.02	-0.053	-0.187	-0.094	-1.356	-1.957	-2.354	-2.682	-1.273	-1.873	-0.062	-0.062
	97.5%	0.004	0.054	0.027	0.016	0.198	0.472	0.818	0.899	0.243	0.627	0.072	0.072
Skew		-1.878	0.317	-0.949	-2.382	-1.452	-1.335	-1.862	-1.239	-1.601	-1.201	0.216	0.174
Kurtosis		9.81	3.279	-0.72	11.974	0.91	0.979	4.619	2.079	2.11	1.443	0.341	0.269

Condition 20; Scenario 3; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.109	-0.246	-0.396	-0.063	-0.054	-0.008	-0.037	0.001
	$\widehat{M}$	-0.112	-0.075	0.196	-0.061	0.002	-0.007	-0.003	0.003
	$\widehat{\sigma}^2$	0.024	0.331	3.188	0.012	0.044	0.002	0.021	0.012
	2.5%	-0.407	-2.113	-5.52	-0.25	-0.76	-0.103	-0.497	-0.211
	97.5%	0.192	0.253	1.043	0.153	0.115	0.089	0.102	0.225
P	$\widehat{\mu}$	-0.001	-0.007	-0.009	-0.004	-0.108	-0.026	-0.091	< .001
	$\widehat{M}$	-0.001	-0.002	0.004	-0.004	0.004	-0.023	-0.008	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.002	< .001	0.177	0.027	0.13	< .001
	2.5%	-0.004	-0.06	-0.127	-0.017	-1.519	-0.343	-1.243	-0.026
	97.5%	0.002	0.007	0.024	0.01	0.23	0.298	0.254	0.028
Skew		-0.023	-2.569	-2.159	0.199	-2.543	-0.114	-2.866	0.031
Kurtosis		-0.238	6.151	3.231	-0.226	5.756	0.357	9.2	-0.019

Condition 20; Scenario 3; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.064	0.074	-1.428	0.021	-0.015	0.044	-0.023	–
	$\widehat{M}$	0.067	0.011	-0.186	0.022	0.025	0.052	0.002	–
	$\widehat{\sigma}^2$	0.024	0.233	7.797	0.012	0.012	0.004	0.008	–
	2.5%	-0.232	-0.637	-7.91	-0.17	-0.314	-0.129	-0.258	–
	97.5%	0.364	1.639	1.007	0.238	0.1	0.156	0.1	–
P	$\widehat{\mu}$	0.001	0.002	-0.033	0.001	-0.031	0.145	-0.056	–
	$\widehat{M}$	0.001	< .001	-0.004	0.001	0.05	0.172	0.004	–
	$\widehat{\sigma}^2$	< .001	< .001	0.004	< .001	0.048	0.049	0.053	–
	2.5%	-0.002	-0.018	-0.182	-0.011	-0.629	-0.43	-0.644	–
	97.5%	0.004	0.047	0.023	0.016	0.201	0.519	0.25	–
Skew		-0.054	2.396	-1.331	0.21	-1.486	-1.102	-1.54	–
Kurtosis		-0.21	8.259	0.083	-0.235	1.3	2.712	2.819	–

TABLE C-61

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 21 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 21; Scenario 1; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.096	-0.076	-0.029	-0.062	0.009	0.003	-0.018	-0.009	0.005	0.005	< .001	0
	$\widehat{M}$	-0.093	-0.098	0.008	-0.071	0.022	0.014	-0.013	-0.003	0.011	0.011	< .001	-0.005
	$\widehat{\sigma}^2$	0.06	0.293	0.293	0.03	0.012	0.016	0.014	0.014	0.012	0.012	0.027	0.025
	2.5%	-0.585	-0.782	-0.738	-0.382	-0.226	-0.26	-0.236	-0.253	-0.218	-0.218	-0.327	-0.293
	97.5%	0.373	0.656	0.664	0.302	0.177	0.185	0.186	0.202	0.197	0.197	0.327	0.313
P	$\widehat{\mu}$	-0.001	-0.002	-0.001	-0.004	0.018	0.007	-0.059	-0.029	0.011	0.011	< .001	< .001
	$\widehat{M}$	-0.001	-0.003	< .001	-0.005	0.043	0.028	-0.044	-0.01	0.029	0.029	< .001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.048	0.062	0.15	0.156	0.073	0.073	0.002	0.002
	2.5%	-0.006	-0.022	-0.017	-0.025	-0.451	-0.521	-0.788	-0.845	-0.546	-0.546	-0.082	-0.073
	97.5%	0.004	0.019	0.015	0.02	0.355	0.371	0.621	0.673	0.492	0.492	0.082	0.078
Skew		0.058	5.001	-2.91	0.162	-1.823	-3.062	-0.944	-0.382	-0.527	-0.527	0.031	0.095
Kurtosis		-0.019	44.766	38.56	0.006	11.052	27.098	6.313	1.317	0.897	0.897	0.167	-0.081

Condition 21; Scenario 1; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.098	-0.117	< .001	-0.063	0.01	-0.009	0.007	0.001
	$\widehat{M}$	-0.096	-0.128	0.018	-0.071	0.019	-0.007	0.015	-0.001
	$\widehat{\sigma}^2$	0.059	0.176	0.177	0.03	0.009	0.006	0.009	0.013
	2.5%	-0.581	-0.763	-0.64	-0.384	-0.184	-0.167	-0.187	-0.22
	97.5%	0.367	0.522	0.613	0.297	0.156	0.129	0.166	0.218
P	$\widehat{\mu}$	-0.001	-0.003	< .001	-0.004	0.02	-0.031	0.017	< .001
	$\widehat{M}$	-0.001	-0.004	< .001	-0.005	0.037	-0.024	0.036	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.037	0.064	0.054	0.001
	2.5%	-0.006	-0.022	-0.015	-0.026	-0.369	-0.558	-0.467	-0.055
	97.5%	0.004	0.015	0.014	0.02	0.311	0.429	0.416	0.055
Skew		0.023	3.611	-2.964	0.145	-2.422	-0.294	-0.607	-0.033
Kurtosis		-0.12	30.054	25.115	-0.024	17.745	0.41	1.51	-0.096

Condition 21; Scenario 1; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.096	-0.087	-0.025	-0.062	0.009	0.005	-0.017	-0.009	0.006	0.006	0.001	0.001
	$\widehat{M}$	-0.093	-0.1	0.006	-0.071	0.022	0.015	-0.013	-0.003	0.011	0.016	-0.003	-0.004
	$\widehat{\sigma}^2$	0.06	0.26	0.22	0.03	0.012	0.014	0.013	0.014	0.011	0.013	0.025	0.025
	2.5%	-0.585	-0.782	-0.718	-0.383	-0.221	-0.255	-0.232	-0.247	-0.217	-0.226	-0.289	-0.291
	97.5%	0.373	0.6	0.636	0.302	0.177	0.185	0.186	0.198	0.197	0.199	0.312	0.313
P	$\widehat{\mu}$	-0.001	-0.002	-0.001	-0.004	0.019	0.01	-0.056	-0.028	0.014	0.014	< .001	< .001
	$\widehat{M}$	-0.001	-0.003	< .001	-0.005	0.043	0.029	-0.043	-0.01	0.028	0.039	-0.001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.047	0.056	0.145	0.152	0.07	0.082	0.002	0.002
	2.5%	-0.006	-0.022	-0.017	-0.026	-0.443	-0.511	-0.775	-0.825	-0.543	-0.565	-0.072	-0.073
	97.5%	0.004	0.017	0.015	0.02	0.355	0.371	0.62	0.661	0.492	0.497	0.078	0.078
Skew		0.058	5.174	-4.345	0.149	-1.814	-2.381	-0.921	-0.365	-0.395	-0.646	0.105	0.101
Kurtosis		-0.025	52.063	40.279	0.009	11.115	19.532	6.477	1.165	0.261	1.535	-0.111	-0.076

Condition 21; Scenario 1; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.097	-0.138	0.025	-0.063	0.014	-0.009	0.009	0.001
	$\widehat{M}$	-0.095	-0.13	0.022	-0.071	0.019	-0.007	0.017	-0.001
	$\widehat{\sigma}^2$	0.059	0.094	0.096	0.03	0.007	0.006	0.008	0.013
	2.5%	-0.581	-0.752	-0.579	-0.384	-0.172	-0.167	-0.178	-0.22
	97.5%	0.367	0.469	0.625	0.297	0.157	0.129	0.166	0.218
P	$\widehat{\mu}$	-0.001	-0.004	0.001	-0.004	0.029	-0.031	0.024	< .001
	$\widehat{M}$	-0.001	-0.004	0.001	-0.005	0.039	-0.024	0.042	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.027	0.064	0.048	0.001
	2.5%	-0.006	-0.021	-0.013	-0.026	-0.344	-0.558	-0.445	-0.055
	97.5%	0.004	0.013	0.014	0.02	0.315	0.429	0.416	0.055
Skew		0.023	-0.038	-0.033	0.145	-0.54	-0.294	-0.29	-0.033
Kurtosis		-0.121	0.037	0.006	-0.024	1.115	0.407	-0.025	-0.095

Condition 21; Scenario 1; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.053	0.053	-0.021	0.005	0.027	0.048	0.011	–
	$\widehat{M}$	0.058	0.033	0.021	-0.005	0.032	0.052	0.016	–
	$\widehat{\sigma}^2$	0.06	0.29	0.275	0.031	0.006	0.005	0.006	–
	2.5%	-0.438	-0.672	-0.707	-0.319	-0.134	-0.099	-0.159	–
	97.5%	0.51	0.774	0.647	0.364	0.144	0.173	0.154	–
P	$\widehat{\mu}$	0.001	0.002	< .001	< .001	0.054	0.161	0.029	–
	$\widehat{M}$	0.001	0.001	< .001	< .001	0.065	0.173	0.04	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.023	0.054	0.04	–
	2.5%	-0.004	-0.019	-0.016	-0.021	-0.267	-0.329	-0.397	–
	97.5%	0.005	0.022	0.015	0.024	0.287	0.575	0.384	–
Skew		0.015	4.793	-3.854	0.146	-1.593	-0.34	-0.47	–
Kurtosis		-0.114	41.916	34.669	-0.028	7.529	1.624	0.7	–

TABLE C-62

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 21 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 21; Scenario 2; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.101	-0.039	-0.101	-0.061	-0.019	-0.017	-0.025	-0.035	-0.019	-0.019	0.004	0.001
	$\widehat{M}$	-0.099	-0.057	0.092	-0.058	0.002	0.018	-0.015	-0.021	-0.009	-0.009	0.003	-0.011
	$\widehat{\sigma}^2$	0.065	0.181	1.36	0.033	0.017	0.04	0.015	0.03	0.014	0.014	0.018	0.052
	2.5%	-0.587	-0.609	-4.129	-0.417	-0.4	-0.541	-0.295	-0.435	-0.282	-0.282	-0.249	-0.418
	97.5%	0.363	0.585	1.099	0.264	0.146	0.23	0.16	0.252	0.162	0.162	0.268	0.462
P	$\widehat{\mu}$	-0.001	-0.001	-0.002	-0.004	-0.039	-0.034	-0.083	-0.117	-0.047	-0.047	0.001	< .001
	$\widehat{M}$	-0.001	-0.002	0.002	-0.004	0.004	0.037	-0.05	-0.07	-0.023	-0.023	0.001	-0.003
	$\widehat{\sigma}^2$	< .001	< .001	0.001	< .001	0.069	0.158	0.163	0.33	0.088	0.088	0.001	0.003
	2.5%	-0.006	-0.017	-0.095	-0.028	-0.801	-1.083	-0.984	-1.451	-0.705	-0.705	-0.062	-0.104
	97.5%	0.004	0.017	0.025	0.018	0.291	0.46	0.533	0.839	0.405	0.405	0.067	0.116
Skew		-0.386	2.268	-3.56	-0.066	-2.643	-2.365	-2.155	-0.646	-1.612	-1.612	0.084	0.161
Kurtosis		2.379	20.95	15.529	-0.19	11.482	9.404	14.201	0.913	6.171	6.171	0.155	-0.131

Condition 21; Scenario 2; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.102	-0.105	-0.02	-0.063	-0.017	-0.014	-0.017	0.003
	$\widehat{M}$	-0.102	-0.086	0.135	-0.061	0.009	-0.011	-0.005	0.001
	$\widehat{\sigma}^2$	0.061	0.175	0.891	0.033	0.022	0.005	0.018	0.014
	2.5%	-0.584	-0.845	-3.054	-0.419	-0.511	-0.158	-0.298	-0.222
	97.5%	0.369	0.461	1.039	0.272	0.146	0.118	0.172	0.217
P	$\widehat{\mu}$	-0.001	-0.003	< .001	-0.004	-0.034	-0.047	-0.043	0.001
	$\widehat{M}$	-0.001	-0.002	0.003	-0.004	0.018	-0.038	-0.012	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.088	0.058	0.112	0.001
	2.5%	-0.006	-0.024	-0.07	-0.028	-1.022	-0.527	-0.746	-0.055
	97.5%	0.004	0.013	0.024	0.018	0.292	0.392	0.431	0.054
Skew		-0.01	-1.09	-3.609	-0.056	-3.228	-0.202	-2.671	-0.011
Kurtosis		-0.087	13.805	17.791	-0.171	14.392	-0.09	13.268	-0.196

Condition 21; Scenario 2; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.098	-0.04	-0.077	-0.061	-0.017	-0.016	-0.025	-0.035	-0.018	-0.015	-0.001	0.002
	$\widehat{M}$	-0.1	-0.058	0.088	-0.058	0.003	0.018	-0.015	-0.021	-0.008	0.007	-0.011	-0.011
	$\widehat{\sigma}^2$	0.067	0.166	1.226	0.033	0.016	0.04	0.014	0.03	0.014	0.035	0.05	0.052
	2.5%	-0.587	-0.605	-3.9	-0.418	-0.341	-0.541	-0.295	-0.435	-0.279	-0.41	-0.416	-0.418
	97.5%	0.376	0.557	1.089	0.264	0.146	0.23	0.159	0.251	0.163	0.27	0.462	0.462
P	$\widehat{\mu}$	-0.001	-0.001	-0.002	-0.004	-0.034	-0.032	-0.084	-0.117	-0.044	-0.036	< .001	< .001
	$\widehat{M}$	-0.001	-0.002	0.002	-0.004	0.005	0.036	-0.049	-0.07	-0.019	0.015	-0.003	-0.003
	$\widehat{\sigma}^2$	< .001	< .001	0.001	< .001	0.064	0.159	0.159	0.329	0.088	0.221	0.003	0.003
	2.5%	-0.006	-0.017	-0.09	-0.028	-0.682	-1.083	-0.984	-1.451	-0.698	-1.026	-0.104	-0.104
	97.5%	0.004	0.016	0.025	0.018	0.291	0.46	0.529	0.837	0.407	0.675	0.115	0.116
Skew		0.374	2.887	-3.699	-0.081	-2.516	-2.419	-2.242	-0.652	-1.665	-1.541	0.164	0.152
Kurtosis		2.826	21.78	17.502	-0.165	10.719	9.674	14.544	0.965	6.537	5.186	-0.085	-0.138

Condition 21; Scenario 2; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.102	-0.096	0.082	-0.063	-0.007	-0.014	-0.009	0.003
	$\widehat{M}$	-0.102	-0.082	0.152	-0.061	0.009	-0.011	-0.003	0.001
	$\widehat{\sigma}^2$	0.061	0.131	0.443	0.033	0.016	0.005	0.014	0.014
	2.5%	-0.584	-0.651	-0.956	-0.42	-0.212	-0.158	-0.206	-0.222
	97.5%	0.369	0.44	1.034	0.272	0.141	0.117	0.172	0.217
P	$\widehat{\mu}$	-0.001	-0.003	0.002	-0.004	-0.015	-0.048	-0.023	0.001
	$\widehat{M}$	-0.001	-0.002	0.004	-0.004	0.019	-0.038	-0.006	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.064	0.058	0.088	0.001
	2.5%	-0.006	-0.019	-0.022	-0.028	-0.425	-0.527	-0.515	-0.055
	97.5%	0.004	0.013	0.024	0.018	0.281	0.389	0.431	0.054
Skew		-0.007	-1.373	-3.546	-0.058	-3.549	-0.202	-2.442	-0.011
Kurtosis		-0.093	17.435	27.264	-0.171	20.59	-0.088	14.248	-0.197

Condition 21; Scenario 2; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.039	0.05	-0.2	0.005	0.007	0.036	-0.009	–
	$\widehat{M}$	0.037	0.029	-0.009	0.007	0.025	0.039	-0.001	–
	$\widehat{\sigma}^2$	0.062	0.156	1.382	0.033	0.01	0.005	0.01	–
	2.5%	-0.455	-0.498	-4.398	-0.352	-0.309	-0.111	-0.241	–
	97.5%	0.511	0.695	1.04	0.338	0.131	0.163	0.15	–
P	$\widehat{\mu}$	< .001	0.001	-0.005	< .001	0.015	0.119	-0.021	–
	$\widehat{M}$	< .001	0.001	< .001	< .001	0.05	0.13	-0.002	–
	$\widehat{\sigma}^2$	< .001	< .001	0.001	< .001	0.041	0.059	0.065	–
	2.5%	-0.005	-0.014	-0.101	-0.023	-0.619	-0.371	-0.602	–
	97.5%	0.005	0.02	0.024	0.023	0.262	0.542	0.374	–
Skew		-0.014	3.207	-3.422	-0.056	-2.346	-0.658	-1.659	–
Kurtosis		-0.103	21.201	14.043	-0.174	8.711	2.13	6.417	–

TABLE C-63

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 21 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 21; Scenario 3; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.098	-0.141	-1.744	-0.062	-0.088	-0.124	-0.067	-0.079	-0.065	-0.065	< .001	-0.003
	$\widehat{M}$	-0.098	-0.11	-0.213	-0.062	-0.029	-0.045	-0.023	-0.03	-0.022	-0.022	0.004	-0.011
	$\widehat{\sigma}^2$	0.073	0.729	9.768	0.038	0.04	0.106	0.057	0.085	0.028	0.028	0.019	0.084
	2.5%	-0.618	-2.208	-8.067	-0.446	-0.643	-1.026	-0.691	-0.779	-0.515	-0.515	-0.276	-0.561
	97.5%	0.441	2.014	1.604	0.317	0.149	0.313	0.351	0.371	0.169	0.169	0.268	0.633
P	$\widehat{\mu}$	-0.001	-0.004	-0.04	-0.004	-0.175	-0.248	-0.223	-0.263	-0.163	-0.163	< .001	-0.001
	$\widehat{M}$	-0.001	-0.003	-0.005	-0.004	-0.057	-0.09	-0.076	-0.1	-0.056	-0.056	0.001	-0.003
	$\widehat{\sigma}^2$	< .001	0.001	0.005	< .001	0.16	0.426	0.634	0.948	0.173	0.173	0.001	0.005
	2.5%	-0.006	-0.063	-0.185	-0.03	-1.285	-2.052	-2.304	-2.597	-1.289	-1.289	-0.069	-0.14
	97.5%	0.004	0.058	0.037	0.021	0.297	0.626	1.17	1.236	0.424	0.424	0.067	0.158
Skew		0.041	0.319	-0.865	-0.049	-1.496	-1.108	-1.472	-0.782	-1.397	-1.397	0.072	0.237
Kurtosis		0.189	4.722	-0.816	1.194	2.025	1.04	4.054	0.704	2.129	2.129	1.301	0.43

Condition 21; Scenario 3; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.105	-0.38	-1.6	-0.066	-0.093	-0.007	-0.07	0.003
	$\widehat{M}$	-0.114	-0.148	-0.161	-0.062	-0.02	-0.007	-0.019	0.005
	$\widehat{\sigma}^2$	0.063	0.729	8.069	0.032	0.055	0.005	0.037	0.012
	2.5%	-0.604	-2.731	-7.299	-0.441	-0.75	-0.15	-0.635	-0.215
	97.5%	0.402	0.855	1.258	0.296	0.211	0.135	0.16	0.216
P	$\widehat{\mu}$	-0.001	-0.011	-0.037	-0.004	-0.186	-0.023	-0.176	0.001
	$\widehat{M}$	-0.001	-0.004	-0.004	-0.004	-0.04	-0.025	-0.05	0.001
	$\widehat{\sigma}^2$	< .001	0.001	0.004	< .001	0.22	0.058	0.229	0.001
	2.5%	-0.006	-0.078	-0.168	-0.029	-1.502	-0.499	-1.588	-0.054
	97.5%	0.004	0.024	0.029	0.02	0.423	0.451	0.399	0.054
Skew		0.075	-1.21	-0.9	0.017	-1.556	-0.114	-1.6	-0.05
Kurtosis		0.102	2.195	-0.758	0.224	2.108	0.129	2.802	0.013

Condition 21; Scenario 3; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.162	-0.198	-2.318	-0.122	-0.111	-0.16	-0.095	-0.097	-0.088	-0.123	-0.009	-0.003
	$\widehat{M}$	-0.114	-0.136	-0.539	-0.073	-0.037	-0.08	-0.037	-0.047	-0.037	-0.073	-0.017	-0.007
	$\widehat{\sigma}^2$	0.417	0.843	10.696	0.176	0.048	0.119	0.068	0.091	0.035	0.085	0.074	0.077
	2.5%	-1.882	-2.259	-8.249	-1.439	-0.678	-1.031	-0.726	-0.781	-0.56	-0.803	-0.536	-0.536
	97.5%	0.789	2.043	1.565	0.459	0.153	0.311	0.353	0.376	0.173	0.308	0.629	0.632
P	$\widehat{\mu}$	-0.002	-0.006	-0.053	-0.008	-0.223	-0.32	-0.315	-0.324	-0.219	-0.307	-0.002	-0.001
	$\widehat{M}$	-0.001	-0.004	-0.012	-0.005	-0.074	-0.16	-0.123	-0.156	-0.093	-0.183	-0.004	-0.002
	$\widehat{\sigma}^2$	< .001	0.001	0.006	0.001	0.193	0.475	0.76	1.014	0.216	0.533	0.005	0.005
	2.5%	-0.019	-0.065	-0.19	-0.096	-1.356	-2.062	-2.42	-2.605	-1.401	-2.008	-0.134	-0.134
	97.5%	0.008	0.058	0.036	0.031	0.306	0.621	1.178	1.253	0.432	0.769	0.157	0.158
Skew		-0.833	0.287	-0.486	-1.866	-1.278	-0.932	-1.12	-0.631	-1.169	-0.776	0.283	0.243
Kurtosis		9.867	2.952	-1.374	10.726	1.347	0.363	1.911	0.194	0.969	0.26	0.823	0.643

Condition 21; Scenario 3; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.105	-0.331	-0.668	-0.066	-0.064	-0.007	-0.045	0.003
	$\widehat{M}$	-0.113	-0.121	0.106	-0.062	-0.005	-0.008	-0.009	0.006
	$\widehat{\sigma}^2$	0.063	0.534	4.361	0.032	0.048	0.005	0.029	0.012
	2.5%	-0.599	-2.667	-6.136	-0.441	-0.749	-0.151	-0.613	-0.214
	97.5%	0.399	0.424	1.358	0.296	0.188	0.133	0.155	0.217
P	$\widehat{\mu}$	-0.001	-0.009	-0.015	-0.004	-0.128	-0.023	-0.112	0.001
	$\widehat{M}$	-0.001	-0.003	0.002	-0.004	-0.011	-0.026	-0.023	0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.002	< .001	0.192	0.058	0.183	0.001
	2.5%	-0.006	-0.076	-0.141	-0.029	-1.499	-0.503	-1.533	-0.053
	97.5%	0.004	0.012	0.031	0.02	0.376	0.444	0.387	0.054
Skew		0.067	-2.055	-1.606	0.016	-2.057	-0.115	-1.98	-0.047
Kurtosis		0.08	3.888	1.416	0.22	4.123	0.14	4.92	0.019

Condition 21; Scenario 3; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.034	0.006	-1.962	0.002	-0.049	0.023	-0.048	–
	$\widehat{M}$	0.028	-0.048	-0.422	0.005	-0.005	0.032	-0.02	–
	$\widehat{\sigma}^2$	0.063	0.463	9.461	0.033	0.02	0.011	0.016	–
	2.5%	-0.462	-1.376	-7.991	-0.374	-0.395	-0.232	-0.382	–
	97.5%	0.534	1.956	1.399	0.362	0.124	0.215	0.132	–
P	$\widehat{\mu}$	< .001	< .001	-0.045	< .001	-0.098	0.078	-0.119	–
	$\widehat{M}$	< .001	-0.001	-0.01	< .001	-0.009	0.106	-0.05	–
	$\widehat{\sigma}^2$	< .001	< .001	0.005	< .001	0.08	0.127	0.1	–
	2.5%	-0.005	-0.039	-0.184	-0.025	-0.79	-0.772	-0.955	–
	97.5%	0.005	0.056	0.032	0.024	0.248	0.715	0.331	–
Skew		0.083	1.322	-0.799	0.022	-1.254	-0.782	-1.15	–
Kurtosis		0.081	5.547	-0.973	0.218	1.22	1.677	1.519	–

TABLE C-64

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 22 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 22; Scenario 1; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.107	-0.116	0.012	-0.062	0.01	0.013	-0.014	-0.012	0.004	0.004	0.005	-0.003
	$\widehat{M}$	-0.116	-0.123	0.015	-0.062	0.015	0.017	-0.014	-0.011	0.007	0.007	0.006	-0.003
	$\widehat{\sigma}^2$	0.023	0.052	0.045	0.013	0.004	0.004	0.004	0.005	0.005	0.005	0.011	0.01
	2.5%	-0.401	-0.562	-0.425	-0.275	-0.134	-0.124	-0.157	-0.143	-0.138	-0.138	-0.201	-0.195
	97.5%	0.179	0.281	0.45	0.169	0.115	0.121	0.108	0.116	0.125	0.125	0.214	0.191
P	$\widehat{\mu}$	-0.001	-0.003	< .001	-0.004	0.02	0.026	-0.047	-0.038	0.01	0.01	0.001	-0.001
	$\widehat{M}$	-0.001	-0.004	< .001	-0.004	0.031	0.034	-0.046	-0.037	0.018	0.018	0.001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.017	0.018	0.049	0.055	0.028	0.028	0.001	0.001
	2.5%	-0.004	-0.016	-0.01	-0.018	-0.267	-0.248	-0.523	-0.476	-0.344	-0.344	-0.05	-0.049
	97.5%	0.002	0.008	0.01	0.011	0.23	0.242	0.36	0.387	0.313	0.313	0.053	0.048
Skew		-0.026	1.455	0.061	0.072	-1.557	-2.326	-0.2	-2.008	-0.763	-0.763	0.044	0.081
Kurtosis		0.045	16.191	0.335	-0.154	12.492	24.649	0.255	24.643	3.965	3.965	0.071	-0.003

Condition 22; Scenario 1; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.107	-0.129	0.025	-0.062	0.013	-0.012	0.005	0.001
	$\widehat{M}$	-0.116	-0.133	0.028	-0.062	0.018	-0.01	0.007	0.002
	$\widehat{\sigma}^2$	0.023	0.041	0.038	0.013	0.003	0.002	0.003	0.005
	2.5%	-0.401	-0.511	-0.349	-0.275	-0.098	-0.101	-0.1	-0.145
	97.5%	0.18	0.243	0.404	0.171	0.098	0.071	0.102	0.141
P	$\widehat{\mu}$	-0.001	-0.004	0.001	-0.004	0.025	-0.039	0.013	< .001
	$\widehat{M}$	-0.001	-0.004	0.001	-0.004	0.037	-0.033	0.018	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.012	0.022	0.019	< .001
	2.5%	-0.004	-0.015	-0.008	-0.018	-0.196	-0.337	-0.249	-0.036
	97.5%	0.002	0.007	0.009	0.011	0.196	0.237	0.254	0.035
Skew		-0.02	1.151	0.088	0.071	-3.026	-0.102	-1.614	0.034
Kurtosis		0.048	11.848	0.17	-0.159	37.713	0.033	16.705	0.203

Condition 22; Scenario 1; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.107	-0.116	0.011	-0.062	0.01	0.013	-0.014	-0.012	0.004	0.005	-0.003	-0.002
	$\widehat{M}$	-0.116	-0.123	0.014	-0.062	0.016	0.017	-0.014	-0.011	0.008	0.008	-0.003	-0.003
	$\widehat{\sigma}^2$	0.023	0.052	0.046	0.013	0.004	0.004	0.004	0.005	0.005	0.005	0.01	0.01
	2.5%	-0.401	-0.562	-0.425	-0.275	-0.134	-0.124	-0.157	-0.143	-0.138	-0.128	-0.196	-0.195
	97.5%	0.179	0.281	0.45	0.169	0.117	0.121	0.108	0.116	0.125	0.127	0.192	0.191
P	$\widehat{\mu}$	-0.001	-0.003	< .001	-0.004	0.02	0.027	-0.047	-0.039	0.011	0.013	-0.001	-0.001
	$\widehat{M}$	-0.001	-0.004	< .001	-0.004	0.031	0.034	-0.046	-0.038	0.021	0.021	-0.001	-0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.018	0.018	0.049	0.055	0.028	0.029	0.001	0.001
	2.5%	-0.004	-0.016	-0.01	-0.018	-0.267	-0.248	-0.523	-0.476	-0.344	-0.319	-0.049	-0.049
	97.5%	0.002	0.008	0.01	0.011	0.233	0.242	0.36	0.386	0.313	0.316	0.048	0.048
Skew		-0.026	1.451	0.057	0.072	-1.551	-2.319	-0.198	-2.021	-0.762	-1.229	0.065	0.08
Kurtosis		0.045	16.153	0.333	-0.154	12.422	24.571	0.243	24.792	3.963	10.189	0.058	-0.003

Condition 22; Scenario 1; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.107	-0.131	0.025	-0.062	0.014	-0.012	0.006	0.001
	$\widehat{M}$	-0.116	-0.133	0.026	-0.062	0.018	-0.01	0.007	0.002
	$\widehat{\sigma}^2$	0.023	0.036	0.038	0.013	0.002	0.002	0.003	0.005
	2.5%	-0.401	-0.511	-0.349	-0.275	-0.096	-0.101	-0.1	-0.145
	97.5%	0.18	0.242	0.404	0.171	0.098	0.071	0.102	0.141
P	$\widehat{\mu}$	-0.001	-0.004	0.001	-0.004	0.027	-0.039	0.015	< .001
	$\widehat{M}$	-0.001	-0.004	0.001	-0.004	0.037	-0.033	0.018	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	0	0.01	0.022	0.017	< .001
	2.5%	-0.004	-0.015	-0.008	-0.018	-0.192	-0.337	-0.249	-0.036
	97.5%	0.002	0.007	0.009	0.011	0.196	0.237	0.254	0.035
Skew		-0.019	-0.003	0.09	0.071	-0.427	-0.102	-0.157	0.034
Kurtosis		0.05	0.021	0.167	-0.159	0.32	0.033	-0.074	0.203

Condition 22; Scenario 1; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.044	0.016	0.018	0.006	0.03	0.046	0.011	–
	$\widehat{M}$	0.036	0.01	0.026	0.005	0.033	0.047	0.011	–
	$\widehat{\sigma}^2$	0.023	0.053	0.048	0.013	0.002	0.002	0.002	–
	2.5%	-0.252	-0.417	-0.427	-0.208	-0.058	-0.03	-0.076	–
	97.5%	0.336	0.439	0.444	0.24	0.096	0.12	0.097	–
P	$\widehat{\mu}$	< .001	< .001	< .001	< .001	0.06	0.154	0.026	–
	$\widehat{M}$	< .001	< .001	0.001	< .001	0.066	0.157	0.029	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.007	0.017	0.013	–
	2.5%	-0.003	-0.012	-0.01	-0.014	-0.116	-0.101	-0.189	–
	97.5%	0.003	0.013	0.01	0.016	0.193	0.4	0.243	–
Skew		-0.023	1.649	-0.329	0.077	-1.513	-0.101	-0.703	–
Kurtosis		0.032	19.632	2.633	-0.155	12.989	0.18	5.086	–

TABLE C-65

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 22 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 22; Scenario 2; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.093	-0.058	0.099	-0.056	< .001	0.019	-0.012	-0.017	-0.003	-0.003	0.001	0.005
	$\widehat{M}$	-0.097	-0.056	0.135	-0.055	0.003	0.032	-0.01	-0.007	0.003	0.003	-0.001	0.009
	$\widehat{\sigma}^2$	0.023	0.038	0.265	0.013	0.004	0.011	0.004	0.015	0.005	0.005	0.007	0.022
	2.5%	-0.378	-0.377	-0.513	-0.282	-0.104	-0.165	-0.119	-0.224	-0.125	-0.125	-0.168	-0.285
	97.5%	0.192	0.271	0.739	0.162	0.092	0.16	0.104	0.168	0.103	0.103	0.173	0.292
P	$\widehat{\mu}$	-0.001	-0.002	0.002	-0.004	-0.001	0.037	-0.039	-0.055	-0.007	-0.007	< .001	0.001
	$\widehat{M}$	-0.001	-0.002	0.003	-0.004	0.006	0.064	-0.032	-0.026	0.007	0.007	< .001	0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.018	0.044	0.046	0.165	0.029	0.029	< .001	0.001
	2.5%	-0.004	-0.011	-0.012	-0.019	-0.208	-0.33	-0.397	-0.747	-0.312	-0.312	-0.042	-0.071
	97.5%	0.002	0.008	0.017	0.011	0.184	0.32	0.345	0.559	0.257	0.257	0.043	0.073
Skew		0.111	-1.622	-5.332	-0.037	-4.11	-3.123	-2.457	-3.016	-2.781	-2.781	0.131	0.034
Kurtosis		0.236	32.047	45.55	0.044	35.447	22.822	19.849	22.945	24.7	24.7	0.047	-0.064

Condition 22; Scenario 2; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.095	-0.061	0.099	-0.057	0.005	-0.009	0.003	0.002
	$\widehat{M}$	-0.098	-0.062	0.143	-0.056	0.01	-0.01	0.007	< .001
	$\widehat{\sigma}^2$	0.022	0.05	0.278	0.013	0.005	0.002	0.004	0.005
	2.5%	-0.374	-0.385	-0.406	-0.281	-0.094	-0.092	-0.122	-0.134
	97.5%	0.183	0.249	0.635	0.161	0.095	0.084	0.102	0.154
P	$\widehat{\mu}$	-0.001	-0.002	0.002	-0.004	0.011	-0.029	0.007	0.001
	$\widehat{M}$	-0.001	-0.002	0.003	-0.004	0.02	-0.034	0.018	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.019	0.022	0.026	< .001
	2.5%	-0.004	-0.011	-0.009	-0.019	-0.187	-0.307	-0.306	-0.034
	97.5%	0.002	0.007	0.015	0.011	0.189	0.279	0.254	0.038
Skew		0.076	-1.192	-7.191	-0.03	-5.688	0.139	-2.525	0.091
Kurtosis		-0.125	49.646	70.133	-0.291	61.508	0.172	18.658	-0.244

Condition 22; Scenario 2; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.093	-0.056	0.105	-0.056	< .001	0.019	-0.011	-0.017	-0.001	0.017	0.003	0.005
	$\widehat{M}$	-0.096	-0.056	0.133	-0.055	0.003	0.031	-0.009	-0.008	0.003	0.024	0.007	0.01
	$\widehat{\sigma}^2$	0.023	0.032	0.211	0.013	0.004	0.011	0.004	0.015	0.004	0.01	0.021	0.022
	2.5%	-0.378	-0.376	-0.512	-0.28	-0.102	-0.16	-0.119	-0.224	-0.123	-0.191	-0.281	-0.289
	97.5%	0.192	0.271	0.739	0.162	0.092	0.16	0.104	0.168	0.103	0.182	0.29	0.293
P	$\widehat{\mu}$	-0.001	-0.002	0.002	-0.004	0.001	0.038	-0.037	-0.055	-0.002	0.043	0.001	0.001
	$\widehat{M}$	-0.001	-0.002	0.003	-0.004	0.007	0.062	-0.03	-0.026	0.008	0.06	0.002	0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.017	0.043	0.041	0.165	0.023	0.062	0.001	0.001
	2.5%	-0.004	-0.011	-0.012	-0.019	-0.205	-0.32	-0.395	-0.747	-0.308	-0.478	-0.07	-0.072
	97.5%	0.002	0.008	0.017	0.011	0.184	0.32	0.345	0.559	0.257	0.456	0.072	0.073
Skew		0.15	0.778	-4.615	0.022	-4.127	-3.119	-1.629	-3.032	-1.016	-0.663	0.027	0.028
Kurtosis		0.116	5.289	39.99	-0.165	37.603	23.508	11.293	23.088	4.286	1.627	0.018	-0.07

Condition 22; Scenario 2; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.094	-0.062	0.136	-0.057	0.009	-0.009	0.006	0.002
	$\widehat{M}$	-0.098	-0.062	0.143	-0.054	0.011	-0.01	0.008	< .001
	$\widehat{\sigma}^2$	0.022	0.025	0.085	0.013	0.003	0.002	0.003	0.005
	2.5%	-0.374	-0.378	-0.37	-0.281	-0.082	-0.092	-0.105	-0.134
	97.5%	0.183	0.229	0.635	0.161	0.095	0.084	0.102	0.154
P	$\widehat{\mu}$	-0.001	-0.002	0.003	-0.004	0.018	-0.029	0.015	0.001
	$\widehat{M}$	-0.001	-0.002	0.003	-0.004	0.022	-0.034	0.02	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.011	0.022	0.019	< .001
	2.5%	-0.004	-0.011	-0.009	-0.019	-0.163	-0.307	-0.263	-0.034
	97.5%	0.002	0.007	0.015	0.011	0.19	0.279	0.254	0.038
Skew		0.078	-0.046	-2.337	-0.03	-3.283	0.139	-0.752	0.091
Kurtosis		-0.125	0.094	30.428	-0.291	45.632	0.174	4.21	-0.243

Condition 22; Scenario 2; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.048	0.023	-0.005	0.011	0.024	0.045	0.009	–
	$\widehat{M}$	0.045	0.017	0.028	0.014	0.027	0.045	0.012	–
	$\widehat{\sigma}^2$	0.022	0.033	0.308	0.013	0.002	0.002	0.003	–
	2.5%	-0.236	-0.282	-0.606	-0.215	-0.057	-0.033	-0.102	–
	97.5%	0.326	0.356	0.661	0.229	0.097	0.127	0.095	–
P	$\widehat{\mu}$	0.001	0.001	< .001	0.001	0.048	0.149	0.022	–
	$\widehat{M}$	< .001	< .001	0.001	0.001	0.054	0.151	0.029	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.01	0.019	0.017	–
	2.5%	-0.002	-0.008	-0.014	-0.014	-0.114	-0.109	-0.255	–
	97.5%	0.003	0.01	0.015	0.015	0.194	0.423	0.238	–
Skew		0.075	1.36	-5.745	-0.027	-2.978	-0.522	-1.038	–
Kurtosis		-0.125	8.709	48.967	-0.286	20.936	3.337	3.948	–

TABLE C-66

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 22 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 22; Scenario 3; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.093	-0.16	-1.319	-0.062	-0.084	-0.091	-0.074	-0.077	-0.066	-0.066	-0.004	0.001
	$\widehat{M}$	-0.089	-0.092	0.018	-0.056	-0.014	0.016	-0.022	-0.036	-0.021	-0.021	-0.004	0.003
	$\widehat{\sigma}^2$	0.036	0.473	7.807	0.016	0.035	0.093	0.038	0.054	0.021	0.021	0.008	0.036
	2.5%	-0.501	-1.799	-8.016	-0.308	-0.603	-0.959	-0.632	-0.632	-0.507	-0.507	-0.181	-0.348
	97.5%	0.261	1.641	1.079	0.185	0.086	0.239	0.189	0.258	0.082	0.082	0.179	0.37
P	$\widehat{\mu}$	-0.001	-0.005	-0.03	-0.004	-0.168	-0.183	-0.245	-0.257	-0.164	-0.164	-0.001	< .001
	$\widehat{M}$	-0.001	-0.003	< .001	-0.004	-0.029	0.032	-0.074	-0.121	-0.052	-0.052	-0.001	0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.004	< .001	0.142	0.373	0.425	0.597	0.132	0.132	0.001	0.002
	2.5%	-0.005	-0.051	-0.184	-0.021	-1.207	-1.919	-2.108	-2.105	-1.268	-1.268	-0.045	-0.087
	97.5%	0.003	0.047	0.025	0.012	0.173	0.478	0.629	0.859	0.204	0.204	0.045	0.093
Skew		-0.475	0.352	-1.293	-0.485	-1.748	-1.672	-1.901	-1.258	-1.934	-1.934	0.029	-0.011
Kurtosis		2.77	5.445	0.073	2.049	2.168	2.364	4.927	2.911	3.501	3.501	0.375	0.635

Condition 22; Scenario 3; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.096	-0.284	-1.209	-0.061	-0.086	-0.011	-0.066	-0.001
	$\widehat{M}$	-0.094	-0.1	0.073	-0.059	-0.005	-0.011	-0.018	-0.002
	$\widehat{\sigma}^2$	0.024	0.403	6.691	0.012	0.051	0.002	0.027	0.006
	2.5%	-0.392	-1.962	-7.041	-0.272	-0.724	-0.113	-0.555	-0.147
	97.5%	0.224	0.409	0.935	0.174	0.104	0.077	0.094	0.147
P	$\widehat{\mu}$	-0.001	-0.008	-0.028	-0.004	-0.173	-0.037	-0.165	< .001
	$\widehat{M}$	-0.001	-0.003	0.002	-0.004	-0.011	-0.038	-0.045	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.004	< .001	0.202	0.024	0.166	< .001
	2.5%	-0.004	-0.056	-0.162	-0.018	-1.448	-0.375	-1.387	-0.037
	97.5%	0.002	0.012	0.021	0.012	0.209	0.257	0.236	0.037
Skew		-0.031	-1.473	-1.263	0.056	-1.858	-0.162	-2.07	0.088
Kurtosis		0.046	3.584	-0.102	0.192	2.453	0.126	4.263	-0.093

Condition 22; Scenario 3; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.237	-0.258	-1.924	-0.152	-0.126	-0.152	-0.11	-0.105	-0.098	-0.104	0.001	0.002
	$\widehat{M}$	-0.103	-0.117	-0.116	-0.072	-0.026	-0.026	-0.029	-0.053	-0.032	-0.048	-0.003	0.003
	$\widehat{\sigma}^2$	0.429	0.66	9.303	0.161	0.048	0.118	0.053	0.06	0.029	0.059	0.032	0.032
	2.5%	-2.333	-1.954	-8.11	-1.405	-0.653	-1.01	-0.69	-0.657	-0.55	-0.701	-0.331	-0.341
	97.5%	0.313	1.762	1.049	0.199	0.088	0.228	0.201	0.265	0.078	0.224	0.358	0.368
P	$\widehat{\mu}$	-0.002	-0.007	-0.044	-0.01	-0.252	-0.304	-0.367	-0.35	-0.244	-0.261	< .001	0.001
	$\widehat{M}$	-0.001	-0.003	-0.003	-0.005	-0.052	-0.051	-0.096	-0.176	-0.079	-0.12	-0.001	0.001
	$\widehat{\sigma}^2$	< .001	0.001	0.005	0.001	0.192	0.471	0.589	0.661	0.183	0.369	0.002	0.002
	2.5%	-0.023	-0.056	-0.186	-0.094	-1.306	-2.021	-2.301	-2.189	-1.375	-1.753	-0.083	-0.085
	97.5%	0.003	0.05	0.024	0.013	0.177	0.455	0.671	0.883	0.195	0.56	0.09	0.092
Skew		-1.787	0.42	-0.782	-2.646	-1.236	-1.195	-1.399	-0.873	-1.407	-0.956	0.067	0.036
Kurtosis		8.576	3.409	-1.053	12.654	0.315	0.502	2.08	1.041	1.066	0.511	0.82	0.794

Condition 22; Scenario 3; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.095	-0.249	-0.449	-0.061	-0.053	-0.011	-0.039	-0.001
	$\widehat{M}$	-0.093	-0.088	0.149	-0.059	0.001	-0.011	-0.005	-0.002
	$\widehat{\sigma}^2$	0.024	0.307	3.191	0.012	0.042	0.002	0.022	0.006
	2.5%	-0.392	-2.005	-5.661	-0.272	-0.735	-0.113	-0.55	-0.148
	97.5%	0.223	0.218	0.949	0.174	0.108	0.077	0.1	0.148
P	$\widehat{\mu}$	-0.001	-0.007	-0.01	-0.004	-0.107	-0.037	-0.098	< .001
	$\widehat{M}$	-0.001	-0.003	0.003	-0.004	0.003	-0.037	-0.013	< .001
	$\widehat{\sigma}^2$	< .001	< .001	0.002	< .001	0.167	0.024	0.138	< .001
	2.5%	-0.004	-0.057	-0.13	-0.018	-1.47	-0.375	-1.374	-0.037
	97.5%	0.002	0.006	0.022	0.012	0.216	0.257	0.25	0.037
Skew		-0.034	-2.56	-2.132	0.052	-2.533	-0.157	-2.707	0.087
Kurtosis		0.018	6.292	3.156	0.198	5.686	0.126	8.181	-0.093

Condition 22; Scenario 3; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.043	0.017	-1.597	0.007	-0.029	0.025	-0.035	–
	$\widehat{M}$	0.042	-0.021	-0.227	0.009	0.013	0.035	-0.011	–
	$\widehat{\sigma}^2$	0.025	0.215	7.968	0.012	0.013	0.005	0.009	–
	2.5%	-0.257	-0.758	-7.864	-0.207	-0.317	-0.156	-0.268	–
	97.5%	0.359	1.497	0.886	0.239	0.094	0.131	0.086	–
P	$\widehat{\mu}$	< .001	< .001	-0.037	0.001	-0.059	0.084	-0.086	–
	$\widehat{M}$	< .001	-0.001	-0.005	0.001	0.025	0.116	-0.028	–
	$\widehat{\sigma}^2$	< .001	< .001	0.004	< .001	0.052	0.055	0.054	–
	2.5%	-0.003	-0.022	-0.181	-0.014	-0.635	-0.521	-0.67	–
	97.5%	0.004	0.043	0.02	0.016	0.187	0.435	0.215	–
Skew		-0.043	2.275	-1.196	0.054	-1.427	-0.978	-1.349	–
Kurtosis		0.046	9.761	-0.305	0.206	1.238	2.406	1.924	–

TABLE C-67

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 23 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 23; Scenario 1; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.185	-0.124	-0.021	-0.114	-0.002	-0.013	-0.034	-0.03	-0.007	-0.007	0.001	< .001
	$\widehat{M}$	-0.192	-0.168	-0.005	-0.12	0.014	0.01	-0.024	-0.022	0.001	0.001	-0.002	-0.003
	$\widehat{\sigma}^2$	0.076	0.305	0.262	0.037	0.016	0.023	0.017	0.024	0.017	0.017	0.111	0.109
	2.5%	-0.734	-0.86	-0.729	-0.493	-0.283	-0.314	-0.285	-0.302	-0.279	-0.279	-0.639	-0.655
	97.5%	0.326	0.642	0.698	0.271	0.203	0.19	0.176	0.191	0.213	0.213	0.71	0.646
P	$\widehat{\mu}$	-0.002	-0.004	< .001	-0.008	-0.003	-0.026	-0.113	-0.099	-0.017	-0.017	< .001	< .001
	$\widehat{M}$	-0.002	-0.005	< .001	-0.008	0.028	0.021	-0.08	-0.072	0.001	0.001	< .001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.066	0.091	0.187	0.268	0.105	0.105	0.002	0.002
	2.5%	-0.007	-0.025	-0.017	-0.033	-0.565	-0.629	-0.951	-1.008	-0.698	-0.698	-0.08	-0.082
	97.5%	0.003	0.018	0.016	0.018	0.406	0.381	0.585	0.637	0.533	0.533	0.089	0.081
Skew		0.021	3.621	-2.04	-0.003	-1.367	-2.604	-1.468	-2.812	-0.795	-0.795	0.198	-0.021
Kurtosis		-0.051	23.644	37.731	-0.106	5.428	15.728	11.625	17.916	2.209	2.209	0.122	0.041

Condition 23; Scenario 1; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.192	-0.146	-0.014	-0.116	-0.001	-0.023	-0.002	0.002
	$\widehat{M}$	-0.193	-0.187	-0.009	-0.123	0.013	-0.021	0.005	-0.002
	$\widehat{\sigma}^2$	0.076	0.26	0.215	0.037	0.012	0.007	0.013	0.054
	2.5%	-0.742	-0.809	-0.67	-0.493	-0.268	-0.2	-0.22	-0.448
	97.5%	0.347	0.564	0.669	0.275	0.161	0.136	0.188	0.452
P	$\widehat{\mu}$	-0.002	-0.004	< .001	-0.008	-0.003	-0.076	-0.004	< .001
	$\widehat{M}$	-0.002	-0.005	< .001	-0.008	0.026	-0.07	0.012	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.049	0.077	0.078	0.001
	2.5%	-0.007	-0.023	-0.015	-0.033	-0.535	-0.665	-0.551	-0.056
	97.5%	0.003	0.016	0.015	0.018	0.322	0.455	0.47	0.056
Skew		0.013	3.817	-2.435	-0.008	-1.91	-0.256	-1.303	-0.007
Kurtosis		-0.082	26.872	33.175	-0.081	8.524	0.1	5.877	0.08

Condition 23; Scenario 1; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.186	-0.13	-0.029	-0.114	-0.001	-0.011	-0.034	-0.028	-0.006	-0.001	0.008	< .001
	$\widehat{M}$	-0.192	-0.169	-0.007	-0.119	0.014	0.01	-0.024	-0.022	0.001	0.015	0.005	-0.003
	$\widehat{\sigma}^2$	0.075	0.284	0.236	0.037	0.016	0.021	0.017	0.023	0.016	0.018	0.108	0.108
	2.5%	-0.734	-0.86	-0.729	-0.493	-0.268	-0.317	-0.285	-0.3	-0.277	-0.281	-0.622	-0.65
	97.5%	0.321	0.614	0.672	0.271	0.203	0.191	0.179	0.194	0.213	0.214	0.646	0.646
P	$\widehat{\mu}$	-0.002	-0.004	-0.001	-0.008	-0.002	-0.023	-0.112	-0.093	-0.015	-0.003	0.001	< .001
	$\widehat{M}$	-0.002	-0.005	< .001	-0.008	0.029	0.021	-0.081	-0.072	0.001	0.036	0.001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.065	0.086	0.187	0.258	0.103	0.113	0.002	0.002
	2.5%	-0.007	-0.025	-0.017	-0.033	-0.536	-0.634	-0.951	-0.999	-0.692	-0.705	-0.078	-0.081
	97.5%	0.003	0.018	0.015	0.018	0.406	0.383	0.596	0.647	0.533	0.534	0.081	0.081
Skew		0.009	3.603	-3.432	-0.007	-1.348	-2.353	-1.466	-2.781	-0.753	-1.166	-0.019	-0.004
Kurtosis		-0.06	25.059	36.285	-0.096	5.43	14.143	11.61	18.574	2.148	4.469	0.012	0.016

Condition 23; Scenario 1; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.192	-0.183	-0.003	-0.116	0.003	-0.023	0.003	0.003
	$\widehat{M}$	-0.192	-0.191	-0.01	-0.123	0.013	-0.021	0.006	-0.002
	$\widehat{\sigma}^2$	0.076	0.123	0.112	0.037	0.01	0.007	0.011	0.054
	2.5%	-0.733	-0.809	-0.631	-0.497	-0.214	-0.2	-0.203	-0.448
	97.5%	0.347	0.489	0.634	0.275	0.16	0.136	0.188	0.452
P	$\widehat{\mu}$	-0.002	-0.005	< .001	-0.008	0.006	-0.076	0.007	< .001
	$\widehat{M}$	-0.002	-0.005	< .001	-0.008	0.027	-0.069	0.015	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.04	0.076	0.069	0.001
	2.5%	-0.007	-0.023	-0.015	-0.033	-0.429	-0.665	-0.508	-0.056
	97.5%	0.003	0.014	0.015	0.018	0.32	0.455	0.47	0.056
Skew		0.014	0.551	-0.356	-0.01	-1.397	-0.256	-0.989	-0.007
Kurtosis		-0.075	3.353	2.085	-0.081	6.406	0.101	5.157	0.079

Condition 23; Scenario 1; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.079	0.06	0.033	0.005	0.045	0.077	0.018	–
	$\widehat{M}$	0.076	0.009	0.062	-0.001	0.054	0.081	0.025	–
	$\widehat{\sigma}^2$	0.078	0.291	0.245	0.038	0.007	0.005	0.008	–
	2.5%	-0.467	-0.678	-0.684	-0.374	-0.128	-0.064	-0.156	–
	97.5%	0.617	0.821	0.758	0.397	0.165	0.201	0.172	–
P	$\widehat{\mu}$	0.001	0.002	0.001	< .001	0.091	0.258	0.044	–
	$\widehat{M}$	0.001	< .001	0.001	< .001	0.108	0.269	0.061	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.026	0.056	0.051	–
	2.5%	-0.005	-0.019	-0.016	-0.025	-0.257	-0.214	-0.39	–
	97.5%	0.006	0.023	0.017	0.026	0.33	0.669	0.429	–
Skew		0.033	3.499	-2.961	-0.002	-1.82	-1.11	-1.141	–
Kurtosis		-0.025	21.972	35.345	-0.081	7.413	6.657	4.436	–

TABLE C-68

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 23 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 23; Scenario 2; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.15	-0.006	-0.128	-0.089	-0.019	-0.017	-0.031	-0.033	-0.02	-0.02	0.01	-0.017
	$\widehat{M}$	-0.15	-0.085	0.097	-0.095	0.003	0.031	-0.013	-0.003	-0.003	-0.003	0.012	-0.024
	$\widehat{\sigma}^2$	0.07	0.38	1.761	0.036	0.021	0.056	0.023	0.053	0.017	0.017	0.075	0.229
	2.5%	-0.688	-0.647	-5.35	-0.442	-0.382	-0.534	-0.308	-0.562	-0.274	-0.274	-0.496	-0.962
	97.5%	0.367	1.466	1.172	0.304	0.165	0.275	0.169	0.307	0.159	0.159	0.599	0.918
P	$\widehat{\mu}$	-0.001	< .001	-0.003	-0.006	-0.037	-0.034	-0.105	-0.109	-0.049	-0.049	0.001	-0.002
	$\widehat{M}$	-0.002	-0.002	0.002	-0.006	0.006	0.061	-0.044	-0.008	-0.007	-0.007	0.002	-0.003
	$\widehat{\sigma}^2$	< .001	< .001	0.001	< .001	0.084	0.225	0.261	0.594	0.104	0.104	0.001	0.004
	2.5%	-0.007	-0.018	-0.123	-0.029	-0.764	-1.067	-1.027	-1.875	-0.686	-0.686	-0.062	-0.12
	97.5%	0.004	0.042	0.027	0.02	0.331	0.549	0.564	1.024	0.398	0.398	0.075	0.115
Skew		-0.052	4.625	-3.479	0.08	-3.065	-2.578	-3.294	-1.887	-2.803	-2.803	0.188	0.056
Kurtosis		0.238	34.543	13.782	0.219	18.373	10.694	20.986	7.125	19.125	19.125	0.765	0.096

Condition 23; Scenario 2; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.16	-0.062	-0.107	-0.095	-0.01	-0.012	-0.011	0.009
	$\widehat{M}$	-0.163	-0.097	0.114	-0.1	0.013	-0.012	0.003	0.009
	$\widehat{\sigma}^2$	0.068	0.257	1.648	0.035	0.02	0.006	0.013	0.054
	2.5%	-0.683	-0.678	-4.957	-0.442	-0.318	-0.172	-0.231	-0.429
	97.5%	0.357	1.215	1.073	0.273	0.161	0.139	0.17	0.473
P	$\widehat{\mu}$	-0.002	-0.002	-0.002	-0.006	-0.019	-0.039	-0.027	0.001
	$\widehat{M}$	-0.002	-0.003	0.003	-0.007	0.026	-0.04	0.009	0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.001	< .001	0.079	0.072	0.083	0.001
	2.5%	-0.007	-0.019	-0.114	-0.029	-0.635	-0.574	-0.578	-0.054
	97.5%	0.004	0.035	0.025	0.018	0.322	0.463	0.425	0.059
Skew		-0.069	2.358	-3.569	0.052	-2.982	-0.167	-1.515	0.186
Kurtosis		0.054	16.429	14.026	0.05	14.49	-0.209	6.434	0.156

Condition 23; Scenario 2; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.153	-0.029	-0.104	-0.09	-0.018	-0.015	-0.028	-0.031	-0.016	-0.007	-0.019	-0.016
	$\widehat{M}$	-0.15	-0.087	0.096	-0.096	0.004	0.032	-0.013	-0.002	-0.002	0.013	-0.027	-0.023
	$\widehat{\sigma}^2$	0.073	0.268	1.59	0.037	0.021	0.056	0.02	0.051	0.013	0.036	0.229	0.229
	2.5%	-0.709	-0.653	-4.518	-0.442	-0.383	-0.534	-0.275	-0.516	-0.251	-0.466	-0.962	-0.962
	97.5%	0.363	1.157	1.172	0.299	0.165	0.275	0.169	0.307	0.159	0.292	0.918	0.953
P	$\widehat{\mu}$	-0.002	-0.001	-0.002	-0.006	-0.035	-0.031	-0.092	-0.103	-0.039	-0.018	-0.002	-0.002
	$\widehat{M}$	-0.002	-0.002	0.002	-0.006	0.007	0.065	-0.042	-0.007	-0.006	0.031	-0.003	-0.003
	$\widehat{\sigma}^2$	< .001	< .001	0.001	< .001	0.084	0.223	0.22	0.569	0.082	0.222	0.004	0.004
	2.5%	-0.007	-0.019	-0.104	-0.029	-0.767	-1.067	-0.914	-1.72	-0.627	-1.165	-0.12	-0.12
	97.5%	0.004	0.033	0.027	0.02	0.331	0.549	0.564	1.024	0.398	0.731	0.115	0.119
Skew		-0.121	3.48	-3.555	-0.008	-3.085	-2.537	-2.742	-1.797	-1.482	-0.976	0.065	0.055
Kurtosis		0.681	21.444	14.96	1.4	18.437	10.417	17.897	6.841	6.157	2.118	0.11	0.091

Condition 23; Scenario 2; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.16	-0.113	0.121	-0.095	0.004	-0.012	< .001	0.009
	$\widehat{M}$	-0.162	-0.099	0.144	-0.1	0.016	-0.013	0.009	0.009
	$\widehat{\sigma}^2$	0.068	0.099	0.394	0.035	0.013	0.006	0.01	0.054
	2.5%	-0.683	-0.641	-0.961	-0.442	-0.2	-0.172	-0.201	-0.429
	97.5%	0.348	0.431	1.074	0.281	0.163	0.139	0.17	0.473
P	$\widehat{\mu}$	-0.002	-0.003	0.003	-0.006	0.008	-0.039	0.001	0.001
	$\widehat{M}$	-0.002	-0.003	0.003	-0.007	0.033	-0.042	0.024	0.001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.051	0.072	0.061	0.001
	2.5%	-0.007	-0.018	-0.022	-0.029	-0.4	-0.574	-0.502	-0.054
	97.5%	0.003	0.012	0.025	0.019	0.325	0.463	0.425	0.059
Skew		-0.073	-1.385	-2.895	0.052	-2.94	-0.169	-0.957	0.185
Kurtosis		0.061	10.325	19.907	0.05	19.144	-0.209	4.357	0.158

Condition 23; Scenario 2; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.095	0.119	-0.197	0.027	0.033	0.077	0.007	–
	$\widehat{M}$	0.095	0.041	0.047	0.021	0.051	0.081	0.016	–
	$\widehat{\sigma}^2$	0.069	0.319	1.782	0.036	0.01	0.005	0.009	–
	2.5%	-0.443	-0.499	-5.389	-0.328	-0.241	-0.077	-0.208	–
	97.5%	0.619	1.596	1.111	0.409	0.163	0.199	0.148	–
P	$\widehat{\mu}$	0.001	0.003	-0.005	0.002	0.066	0.256	0.017	–
	$\widehat{M}$	0.001	0.001	0.001	0.001	0.102	0.271	0.04	–
	$\widehat{\sigma}^2$	< .001	< .001	0.001	< .001	0.039	0.06	0.054	–
	2.5%	-0.004	-0.014	-0.124	-0.022	-0.482	-0.257	-0.52	–
	97.5%	0.006	0.046	0.026	0.027	0.325	0.664	0.37	–
Skew		-0.059	4.185	-3.304	0.055	-2.314	-0.915	-1.007	–
Kurtosis		0.073	27.229	12.196	0.049	8.544	2.612	2.228	–

TABLE C-69

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 23 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 23; Scenario 3; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.118	-0.077	-1.772	-0.067	-0.113	-0.158	-0.09	-0.095	-0.075	-0.075	-0.016	0.001
	$\widehat{M}$	-0.115	-0.111	-0.211	-0.067	-0.033	-0.064	-0.036	-0.019	-0.034	-0.034	-0.015	-0.009
	$\widehat{\sigma}^2$	0.098	0.934	10.293	0.04	0.067	0.167	0.083	0.145	0.03	0.03	0.079	0.346
	2.5%	-0.723	-1.992	-8.242	-0.472	-0.777	-1.246	-0.935	-1.168	-0.516	-0.516	-0.577	-1.139
	97.5%	0.477	2.53	1.729	0.321	0.197	0.363	0.392	0.476	0.159	0.159	0.517	1.2
P	$\widehat{\mu}$	-0.001	-0.002	-0.041	-0.004	-0.225	-0.316	-0.3	-0.316	-0.187	-0.187	-0.002	< .001
	$\widehat{M}$	-0.001	-0.003	-0.005	-0.004	-0.066	-0.127	-0.121	-0.066	-0.085	-0.085	-0.002	-0.001
	$\widehat{\sigma}^2$	< .001	0.001	0.005	< .001	0.269	0.666	0.928	1.615	0.188	0.188	0.001	0.005
	2.5%	-0.007	-0.057	-0.19	-0.031	-1.555	-2.492	-3.117	-3.892	-1.291	-1.291	-0.072	-0.142
	97.5%	0.005	0.072	0.04	0.021	0.393	0.725	1.307	1.588	0.396	0.396	0.065	0.15
Skew		-0.513	0.401	-0.798	-0.074	-1.749	-1.159	-1.702	-1.04	-1.318	-1.318	-0.194	-0.157
Kurtosis		3.415	6.048	-0.719	0.428	4.013	0.935	4.797	1.328	2.466	2.466	0.597	1.762

Condition 23; Scenario 3; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.151	-0.356	-1.645	-0.085	-0.114	-0.016	-0.083	0.001
	$\widehat{M}$	-0.151	-0.161	-0.119	-0.092	-0.023	-0.013	-0.035	0.001
	$\widehat{\sigma}^2$	0.069	0.749	8.548	0.035	0.073	0.006	0.039	0.051
	2.5%	-0.676	-2.417	-7.591	-0.461	-0.865	-0.178	-0.667	-0.436
	97.5%	0.345	1.057	1.573	0.281	0.25	0.129	0.164	0.449
P	$\widehat{\mu}$	-0.002	-0.01	-0.038	-0.006	-0.228	-0.055	-0.208	< .001
	$\widehat{M}$	-0.002	-0.005	-0.003	-0.006	-0.047	-0.044	-0.085	< .001
	$\widehat{\sigma}^2$	< .001	0.001	0.005	< .001	0.293	0.071	0.245	0.001
	2.5%	-0.007	-0.069	-0.175	-0.031	-1.731	-0.593	-1.67	-0.055
	97.5%	0.003	0.03	0.036	0.019	0.5	0.429	0.409	0.056
Skew		-0.096	-1.004	-0.843	0.003	-1.514	-0.221	-1.599	0.098
Kurtosis		0.252	3.047	-0.778	-0.111	1.982	-0.081	2.82	-0.04

Condition 23; Scenario 3; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.171	-0.112	-2.248	-0.104	-0.132	-0.186	-0.112	-0.109	-0.093	-0.121	-0.013	0.001
	$\widehat{M}$	-0.121	-0.134	-0.521	-0.079	-0.042	-0.093	-0.042	-0.037	-0.043	-0.073	-0.023	-0.011
	$\widehat{\sigma}^2$	0.372	1.036	11.181	0.174	0.072	0.172	0.099	0.159	0.037	0.108	0.316	0.324
	2.5%	-2.136	-2.115	-8.425	-1.025	-0.84	-1.232	-0.935	-1.182	-0.599	-0.972	-1.068	-1.068
	97.5%	0.712	2.53	1.682	0.449	0.197	0.363	0.416	0.494	0.162	0.352	1.134	1.161
P	$\widehat{\mu}$	-0.002	-0.003	-0.052	-0.007	-0.264	-0.372	-0.374	-0.363	-0.232	-0.303	-0.002	< .001
	$\widehat{M}$	-0.001	-0.004	-0.012	-0.005	-0.084	-0.186	-0.141	-0.123	-0.108	-0.183	-0.003	-0.001
	$\widehat{\sigma}^2$	< .001	0.001	0.006	0.001	0.289	0.688	1.1	1.764	0.233	0.675	0.005	0.005
	2.5%	-0.021	-0.06	-0.194	-0.068	-1.68	-2.464	-3.118	-3.94	-1.5	-2.431	-0.133	-0.133
	97.5%	0.007	0.072	0.039	0.03	0.393	0.725	1.387	1.647	0.405	0.88	0.142	0.145
Skew		-1.253	0.646	-0.524	-2.224	-1.376	-0.994	-1.351	-0.923	-1.251	-0.969	-0.18	-0.189
Kurtosis		11.051	3.109	-1.14	17.232	2.014	0.487	2.693	0.873	1.933	0.994	2.255	2.101

Condition 23; Scenario 3; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.152	-0.433	-0.861	-0.085	-0.103	-0.017	-0.075	0.002
	$\widehat{M}$	-0.152	-0.168	0.075	-0.092	-0.009	-0.014	-0.021	0.001
	$\widehat{\sigma}^2$	0.069	0.643	5.285	0.035	0.08	0.006	0.045	0.051
	2.5%	-0.685	-2.597	-6.253	-0.461	-0.895	-0.178	-0.7	-0.436
	97.5%	0.345	0.411	1.647	0.281	0.236	0.129	0.167	0.449
P	$\widehat{\mu}$	-0.002	-0.012	-0.02	-0.006	-0.205	-0.055	-0.189	< .001
	$\widehat{M}$	-0.002	-0.005	0.002	-0.006	-0.018	-0.045	-0.051	< .001
	$\widehat{\sigma}^2$	< .001	0.001	0.003	< .001	0.319	0.071	0.278	0.001
	2.5%	-0.007	-0.074	-0.144	-0.031	-1.791	-0.595	-1.75	-0.055
	97.5%	0.003	0.012	0.038	0.019	0.474	0.429	0.416	0.056
Skew		-0.087	-1.623	-1.195	< .001	-1.704	-0.214	-1.776	0.099
Kurtosis		0.273	2.501	0.097	-0.11	2.27	-0.083	3.058	-0.04

Condition 23; Scenario 3; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.098	0.097	-1.971	0.037	-0.038	0.054	-0.047	–
	$\widehat{M}$	0.103	0.007	-0.465	0.029	0.015	0.064	-0.014	–
	$\widehat{\sigma}^2$	0.07	0.588	10.026	0.036	0.023	0.013	0.018	–
	2.5%	-0.437	-1.25	-8.166	-0.345	-0.417	-0.217	-0.378	–
	97.5%	0.61	2.344	1.613	0.407	0.15	0.265	0.135	–
P	$\widehat{\mu}$	0.001	0.003	-0.045	0.002	-0.076	0.179	-0.117	–
	$\widehat{M}$	0.001	< .001	-0.011	0.002	0.03	0.212	-0.035	–
	$\widehat{\sigma}^2$	< .001	< .001	0.005	< .001	0.093	0.141	0.11	–
	2.5%	-0.004	-0.036	-0.188	-0.023	-0.835	-0.723	-0.946	–
	97.5%	0.006	0.067	0.037	0.027	0.299	0.882	0.337	–
Skew		-0.093	1.41	-0.703	-0.007	-1.191	-0.808	-1.084	–
Kurtosis		0.278	4.341	-0.932	-0.088	0.866	2.508	1.014	–

TABLE C-70

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 24 SCENARIO 1 FOR EACH OF THE FIVE METHODS

Condition 24; Scenario 1; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.171	-0.146	-0.017	-0.107	0.015	0.014	-0.016	-0.015	0.007	0.007	0.002	0.007
	$\widehat{M}$	-0.165	-0.152	-0.011	-0.105	0.019	0.016	-0.011	-0.013	0.012	0.012	-0.006	0.002
	$\widehat{\sigma}^2$	0.029	0.068	0.083	0.014	0.004	0.005	0.005	0.006	0.005	0.005	0.04	0.04
	2.5%	-0.499	-0.571	-0.461	-0.342	-0.121	-0.134	-0.165	-0.168	-0.148	-0.148	-0.38	-0.388
	97.5%	0.168	0.28	0.411	0.136	0.13	0.133	0.115	0.122	0.137	0.137	0.417	0.413
P	$\widehat{\mu}$	-0.002	-0.004	< .001	-0.007	0.031	0.027	-0.053	-0.051	0.018	0.018	< .001	0.001
	$\widehat{M}$	-0.002	-0.004	< .001	-0.007	0.038	0.033	-0.036	-0.042	0.031	0.031	-0.001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.018	0.019	0.058	0.065	0.034	0.034	0.001	0.001
	2.5%	-0.005	-0.016	-0.011	-0.023	-0.242	-0.269	-0.55	-0.56	-0.369	-0.369	-0.048	-0.048
	97.5%	0.002	0.008	0.009	0.009	0.261	0.267	0.382	0.405	0.342	0.342	0.052	0.052
Skew		-0.009	5.043	-8.828	0.016	-0.458	-0.423	-0.345	-1.287	-0.437	-0.437	0.107	0.107
Kurtosis		0.037	86.044	180.493	0.167	0.702	0.161	0.473	11.269	1.055	1.055	0.161	0.051

Condition 24; Scenario 1; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.169	-0.166	0.006	-0.107	0.016	-0.014	0.01	0.004
	$\widehat{M}$	-0.167	-0.172	0.013	-0.106	0.02	-0.011	0.013	0.001
	$\widehat{\sigma}^2$	0.028	0.056	0.052	0.014	0.003	0.002	0.003	0.02
	2.5%	-0.492	-0.554	-0.392	-0.334	-0.098	-0.117	-0.113	-0.269
	97.5%	0.165	0.243	0.396	0.133	0.111	0.077	0.119	0.275
P	$\widehat{\mu}$	-0.002	-0.005	< .001	-0.007	0.032	-0.046	0.025	0.001
	$\widehat{M}$	-0.002	-0.005	< .001	-0.007	0.04	-0.037	0.034	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.012	0.027	0.022	< .001
	2.5%	-0.005	-0.016	-0.009	-0.022	-0.195	-0.389	-0.282	-0.034
	97.5%	0.002	0.007	0.009	0.009	0.223	0.255	0.298	0.034
Skew		0.048	4.801	-3.923	0.049	-0.401	-0.219	-0.476	-0.036
Kurtosis		-0.129	80.691	59.989	0.062	0.094	0.071	0.875	-0.099

Condition 24; Scenario 1; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.171	-0.146	-0.017	-0.107	0.015	0.014	-0.016	-0.015	0.007	0.011	0.009	0.007
	$\widehat{M}$	-0.165	-0.152	-0.011	-0.105	0.019	0.016	-0.011	-0.013	0.012	0.015	0.004	0.002
	$\widehat{\sigma}^2$	0.029	0.068	0.083	0.014	0.004	0.005	0.005	0.006	0.005	0.005	0.04	0.04
	2.5%	-0.499	-0.571	-0.461	-0.342	-0.123	-0.136	-0.165	-0.168	-0.144	-0.151	-0.387	-0.387
	97.5%	0.168	0.28	0.414	0.136	0.13	0.133	0.115	0.122	0.137	0.146	0.416	0.413
P	$\widehat{\mu}$	-0.002	-0.004	< .001	-0.007	0.031	0.027	-0.053	-0.051	0.019	0.027	0.001	0.001
	$\widehat{M}$	-0.002	-0.004	< .001	-0.007	0.038	0.033	-0.036	-0.044	0.031	0.039	0.001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.018	0.019	0.058	0.065	0.034	0.032	0.001	0.001
	2.5%	-0.005	-0.016	-0.011	-0.023	-0.245	-0.271	-0.55	-0.56	-0.36	-0.377	-0.048	-0.048
	97.5%	0.002	0.008	0.01	0.009	0.261	0.267	0.382	0.405	0.342	0.366	0.052	0.052
Skew		-0.009	5.043	-8.819	0.016	-0.462	-0.429	-0.343	-1.282	-0.437	-0.394	0.128	0.108
Kurtosis		0.037	86.061	180.216	0.168	0.699	0.158	0.492	11.214	1.061	0.401	0.082	0.047

Condition 24; Scenario 1; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.169	-0.17	0.009	-0.107	0.016	-0.014	0.01	0.004
	$\widehat{M}$	-0.167	-0.174	0.013	-0.106	0.02	-0.011	0.013	0.001
	$\widehat{\sigma}^2$	0.028	0.04	0.039	0.014	0.003	0.002	0.003	0.02
	2.5%	-0.492	-0.554	-0.391	-0.334	-0.098	-0.117	-0.113	-0.269
	97.5%	0.165	0.24	0.396	0.133	0.111	0.077	0.119	0.275
P	$\widehat{\mu}$	-0.002	-0.005	< .001	-0.007	0.033	-0.046	0.026	0.001
	$\widehat{M}$	-0.002	-0.005	< .001	-0.007	0.04	-0.037	0.034	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.012	0.027	0.021	< .001
	2.5%	-0.005	-0.016	-0.009	-0.022	-0.195	-0.389	-0.282	-0.034
	97.5%	0.002	0.007	0.009	0.009	0.223	0.255	0.298	0.034
Skew		0.048	-0.019	-0.094	0.049	-0.379	-0.219	-0.401	-0.036
Kurtosis		-0.129	0.187	0.032	0.062	0.024	0.07	0.474	-0.099

Condition 24; Scenario 1; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.101	0.044	0.044	0.014	0.058	0.087	0.027	–
	$\widehat{M}$	0.102	0.041	0.048	0.013	0.06	0.089	0.03	–
	$\widehat{\sigma}^2$	0.03	0.067	0.076	0.015	0.002	0.001	0.002	–
	2.5%	-0.227	-0.394	-0.405	-0.216	-0.028	0.011	-0.075	–
	97.5%	0.437	0.478	0.483	0.263	0.128	0.158	0.117	–
P	$\widehat{\mu}$	0.001	0.001	0.001	0.001	0.115	0.289	0.068	–
	$\widehat{M}$	0.001	0.001	0.001	0.001	0.12	0.296	0.076	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.007	0.016	0.015	–
	2.5%	-0.002	-0.011	-0.009	-0.014	-0.055	0.035	-0.187	–
	97.5%	0.004	0.014	0.011	0.018	0.257	0.528	0.292	–
Skew		0.046	4.695	-6.951	0.055	-0.635	-0.268	-0.68	–
Kurtosis		-0.168	78.335	130.46	0.068	2.089	0.115	2.901	–

TABLE C-71

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 24 SCENARIO 2 FOR EACH OF THE FIVE METHODS

Condition 24; Scenario 2; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.166	-0.076	0.07	-0.105	-0.006	0.018	-0.021	-0.02	-0.006	-0.006	0.023	0.01
	$\widehat{M}$	-0.168	-0.088	0.134	-0.102	0.003	0.03	-0.016	-0.011	-0.005	-0.005	0.019	0.003
	$\widehat{\sigma}^2$	0.03	0.104	0.484	0.014	0.006	0.013	0.007	0.021	0.005	0.005	0.03	0.091
	2.5%	-0.502	-0.459	-0.608	-0.342	-0.132	-0.212	-0.138	-0.291	-0.128	-0.128	-0.323	-0.558
	97.5%	0.182	0.28	0.777	0.118	0.105	0.194	0.096	0.203	0.107	0.107	0.349	0.594
P	$\widehat{\mu}$	-0.002	-0.002	0.002	-0.007	-0.012	0.036	-0.069	-0.067	-0.015	-0.015	0.003	0.001
	$\widehat{M}$	-0.002	-0.003	0.003	-0.007	0.007	0.059	-0.053	-0.038	-0.011	-0.011	0.002	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.025	0.053	0.077	0.228	0.029	0.029	< .001	0.001
	2.5%	-0.005	-0.013	-0.014	-0.023	-0.263	-0.423	-0.459	-0.971	-0.32	-0.32	-0.04	-0.07
	97.5%	0.002	0.008	0.018	0.008	0.21	0.388	0.319	0.676	0.267	0.267	0.044	0.074
Skew		0.092	5.899	-6.43	-0.112	-4.288	-1.899	-5.363	-2.87	-1.796	-1.796	0.051	0.091
Kurtosis		-0.037	76.084	53.969	-0.051	38.858	10.209	70.4	22.831	12.926	12.926	0.017	0.277

Condition 24; Scenario 2; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.169	-0.09	0.069	-0.106	0.003	-0.015	0.001	0.021
	$\widehat{M}$	-0.173	-0.09	0.143	-0.103	0.01	-0.014	0.003	0.019
	$\widehat{\sigma}^2$	0.028	0.072	0.492	0.014	0.005	0.003	0.004	0.023
	2.5%	-0.499	-0.444	-0.499	-0.342	-0.111	-0.11	-0.114	-0.272
	97.5%	0.178	0.243	0.698	0.119	0.104	0.078	0.105	0.333
P	$\widehat{\mu}$	-0.002	-0.003	0.002	-0.007	0.005	-0.049	0.003	0.003
	$\widehat{M}$	-0.002	-0.003	0.003	-0.007	0.02	-0.048	0.007	0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.021	0.028	0.024	< .001
	2.5%	-0.005	-0.013	-0.011	-0.023	-0.223	-0.368	-0.284	-0.034
	97.5%	0.002	0.007	0.016	0.008	0.209	0.259	0.262	0.042
Skew		0.097	0.191	-6.671	-0.087	-4.199	-0.085	-2.11	0.08
Kurtosis		-0.09	33.84	53.527	-0.048	41.056	-0.326	20.373	-0.035

Condition 24; Scenario 2; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.166	-0.078	0.069	-0.105	-0.006	0.018	-0.02	-0.019	-0.005	0.019	0.01	0.011
	$\widehat{M}$	-0.168	-0.091	0.129	-0.102	0.003	0.029	-0.015	-0.012	-0.004	0.03	0.005	0.002
	$\widehat{\sigma}^2$	0.029	0.101	0.482	0.014	0.006	0.013	0.007	0.019	0.005	0.011	0.088	0.09
	2.5%	-0.502	-0.467	-0.572	-0.342	-0.131	-0.21	-0.138	-0.29	-0.127	-0.211	-0.544	-0.544
	97.5%	0.177	0.275	0.777	0.118	0.105	0.194	0.096	0.202	0.107	0.203	0.588	0.594
P	$\widehat{\mu}$	-0.002	-0.002	0.002	-0.007	-0.011	0.037	-0.068	-0.063	-0.013	0.047	0.001	0.001
	$\widehat{M}$	-0.002	-0.003	0.003	-0.007	0.007	0.058	-0.051	-0.041	-0.01	0.075	0.001	< .001
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.025	0.052	0.077	0.21	0.029	0.07	0.001	0.001
	2.5%	-0.005	-0.013	-0.013	-0.023	-0.262	-0.419	-0.459	-0.968	-0.318	-0.527	-0.068	-0.068
	97.5%	0.002	0.008	0.018	0.008	0.21	0.389	0.319	0.672	0.268	0.508	0.074	0.074
Skew		0.077	6.065	-6.466	-0.109	-4.202	-1.812	-5.356	-2.481	-1.842	-0.766	0.107	0.127
Kurtosis		-0.066	80.386	54.538	-0.053	37.415	9.963	70.042	20.211	13.341	2.251	0.23	0.195

Condition 24; Scenario 2; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.169	-0.097	0.134	-0.106	0.005	-0.015	0.004	0.021
	$\widehat{M}$	-0.173	-0.09	0.143	-0.103	0.01	-0.015	0.004	0.019
	$\widehat{\sigma}^2$	0.028	0.038	0.107	0.014	0.004	0.003	0.003	0.023
	2.5%	-0.499	-0.44	-0.437	-0.341	-0.107	-0.11	-0.104	-0.272
	97.5%	0.178	0.228	0.698	0.119	0.104	0.078	0.107	0.334
P	$\widehat{\mu}$	-0.002	-0.003	0.003	-0.007	0.011	-0.05	0.011	0.003
	$\widehat{M}$	-0.002	-0.003	0.003	-0.007	0.021	-0.049	0.01	0.002
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.017	0.028	0.019	< .001
	2.5%	-0.005	-0.013	-0.01	-0.023	-0.213	-0.368	-0.261	-0.034
	97.5%	0.002	0.007	0.016	0.008	0.209	0.259	0.268	0.042
Skew		0.098	-2.603	-2.471	-0.086	-4.479	-0.084	-0.208	0.08
Kurtosis		-0.094	29.999	25.675	-0.047	56.427	-0.328	0.174	-0.035

Condition 24; Scenario 2; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.088	0.053	< .001	0.017	0.045	0.08	0.018	–
	$\widehat{M}$	0.089	0.035	0.063	0.018	0.05	0.08	0.021	–
	$\widehat{\sigma}^2$	0.029	0.086	0.509	0.014	0.002	0.002	0.002	–
	2.5%	-0.243	-0.323	-0.649	-0.221	-0.051	0.005	-0.078	–
	97.5%	0.434	0.403	0.719	0.244	0.118	0.157	0.105	–
P	$\widehat{\mu}$	0.001	0.002	< .001	0.001	0.09	0.268	0.045	–
	$\widehat{M}$	0.001	0.001	0.001	0.001	0.099	0.267	0.052	–
	$\widehat{\sigma}^2$	< .001	< .001	< .001	< .001	0.01	0.02	0.016	–
	2.5%	-0.002	-0.009	-0.015	-0.015	-0.103	0.016	-0.195	–
	97.5%	0.004	0.012	0.017	0.016	0.237	0.525	0.264	–
Skew		0.107	6.235	-6.214	-0.088	-2.128	-0.776	-0.951	–
Kurtosis		-0.077	69.039	49.347	-0.046	11.377	4.913	3.9	–

TABLE C-72

SUMMARY OF THE DISCREPANCIES BETWEEN THE ESTIMATED POPULATION VALUES AND THE TRUE POPULATION VALUES FOR EACH OF THE MODEL PARAMETERS FROM CONDITION 24 SCENARIO 3 FOR EACH OF THE FIVE METHODS

Condition 24; Scenario 3; Method 1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.142	-0.043	-1.17	-0.09	-0.072	-0.064	-0.073	-0.074	-0.05	-0.05	0.004	-0.005
	$\widehat{M}$	-0.139	-0.084	0.086	-0.093	-0.009	0.028	-0.021	-0.036	-0.018	-0.018	0.001	-0.003
	$\widehat{\sigma}^2$	0.052	0.635	8.009	0.022	0.038	0.093	0.066	0.074	0.018	0.018	0.03	0.153
	2.5%	-0.563	-1.509	-8.015	-0.336	-0.632	-1.039	-0.957	-0.829	-0.458	-0.458	-0.344	-0.8
	97.5%	0.302	2.5	1.21	0.21	0.111	0.252	0.327	0.326	0.106	0.106	0.34	0.779
P	$\widehat{\mu}$	-0.001	-0.001	-0.027	-0.006	-0.145	-0.127	-0.243	-0.246	-0.125	-0.125	0.001	-0.001
	$\widehat{M}$	-0.001	-0.002	0.002	-0.006	-0.018	0.055	-0.071	-0.12	-0.043	-0.043	< .001	< .001
	$\widehat{\sigma}^2$	< .001	0.001	0.004	< .001	0.151	0.371	0.73	0.826	0.115	0.115	< .001	0.002
	2.5%	-0.006	-0.043	-0.184	-0.022	-1.264	-2.078	-3.191	-2.764	-1.144	-1.144	-0.043	-0.1
	97.5%	0.003	0.071	0.028	0.014	0.222	0.504	1.09	1.087	0.264	0.264	0.043	0.097
Skew		-0.997	1.704	-1.426	-0.893	-2.201	-2.103	-2.466	-1.696	-1.905	-1.905	-0.179	0.059
Kurtosis		7.607	11.133	0.606	8.725	5.623	5.11	8.642	4.945	5.193	5.193	1.161	0.269

Condition 24; Scenario 3; Method 2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.161	-0.219	-1.102	-0.101	-0.062	-0.014	-0.043	0.011
	$\widehat{M}$	-0.159	-0.103	0.121	-0.105	-0.003	-0.011	-0.012	0.005
	$\widehat{\sigma}^2$	0.026	0.347	6.939	0.014	0.038	0.003	0.02	0.02
	2.5%	-0.484	-2.082	-7.387	-0.32	-0.685	-0.118	-0.48	-0.257
	97.5%	0.138	0.805	0.992	0.137	0.144	0.08	0.119	0.289
P	$\widehat{\mu}$	-0.002	-0.006	-0.025	-0.007	-0.124	-0.048	-0.108	0.001
	$\widehat{M}$	-0.002	-0.003	0.003	-0.007	-0.005	-0.037	-0.029	0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.004	< .001	0.154	0.028	0.122	< .001
	2.5%	-0.005	-0.059	-0.17	-0.021	-1.371	-0.394	-1.2	-0.032
	97.5%	0.001	0.023	0.023	0.009	0.289	0.266	0.298	0.036
Skew		-0.121	-1.749	-1.421	0.142	-2.134	-0.166	-2.207	0.01
Kurtosis		-0.154	4.925	0.357	-0.112	4.474	-0.04	6.982	-0.047

Condition 24; Scenario 3; Method 3.1

D	Stat.	α	β_1	β_2	γ	ρ_{α_1,β_1}	ρ_{α_2,β_2}	ρ_{α_1,γ_1}	ρ_{α_2,γ_2}	ρ_{β_1,γ_1}	ρ_{β_2,γ_2}	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
B	$\widehat{\mu}$	-0.21	-0.103	-1.401	-0.138	-0.09	-0.095	-0.087	-0.089	-0.063	-0.063	-0.002	-0.001
	$\widehat{M}$	-0.15	-0.101	0.018	-0.097	-0.011	0.013	-0.024	-0.045	-0.02	-0.022	-0.003	-0.005
	$\widehat{\sigma}^2$	0.234	0.655	8.445	0.104	0.046	0.11	0.072	0.08	0.023	0.055	0.14	0.147
	2.5%	-1.758	-1.842	-8.032	-1.117	-0.7	-1.116	-0.91	-0.861	-0.49	-0.697	-0.791	-0.791
	97.5%	0.318	2.378	1.14	0.211	0.114	0.245	0.333	0.326	0.108	0.258	0.757	0.764
P	$\widehat{\mu}$	-0.002	-0.003	-0.032	-0.009	-0.181	-0.191	-0.29	-0.296	-0.158	-0.157	< .001	< .001
	$\widehat{M}$	-0.002	-0.003	< .001	-0.006	-0.022	0.026	-0.082	-0.152	-0.05	-0.055	< .001	-0.001
	$\widehat{\sigma}^2$	< .001	0.001	0.004	< .001	0.185	0.441	0.796	0.89	0.145	0.344	0.002	0.002
	2.5%	-0.018	-0.053	-0.185	-0.074	-1.4	-2.233	-3.034	-2.87	-1.225	-1.742	-0.099	-0.099
	97.5%	0.003	0.068	0.026	0.014	0.228	0.49	1.11	1.087	0.269	0.646	0.095	0.095
Skew		-2.347	1.375	-1.226	-4.258	-1.905	-1.816	-2.113	-1.562	-1.679	-1.36	0.051	0.132
Kurtosis		21.564	7.519	-0.102	26.545	3.599	3.274	6.291	3.853	3.034	2.462	0.438	0.556

Condition 24; Scenario 3; Method 3.2

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	-0.159	-0.258	-0.367	-0.1	-0.039	-0.014	-0.028	0.012
	$\widehat{M}$	-0.156	-0.108	0.193	-0.104	0.005	-0.011	-0.003	0.005
	$\widehat{\sigma}^2$	0.026	0.304	3.117	0.014	0.036	0.003	0.018	0.02
	2.5%	-0.483	-2.091	-5.709	-0.32	-0.711	-0.118	-0.518	-0.257
	97.5%	0.138	0.222	1.034	0.141	0.195	0.08	0.123	0.291
P	$\widehat{\mu}$	-0.002	-0.007	-0.008	-0.007	-0.078	-0.048	-0.07	0.001
	$\widehat{M}$	-0.002	-0.003	0.004	-0.007	0.01	-0.037	-0.007	0.001
	$\widehat{\sigma}^2$	< .001	< .001	0.002	< .001	0.145	0.028	0.114	< .001
	2.5%	-0.005	-0.06	-0.131	-0.021	-1.423	-0.394	-1.296	-0.032
	97.5%	0.001	0.006	0.024	0.009	0.392	0.266	0.308	0.036
Skew		-0.138	-2.555	-2.233	0.139	-2.682	-0.169	-2.818	0.011
Kurtosis		-0.16	6.079	3.645	-0.115	7.544	-0.048	10.597	-0.05

Condition 24; Scenario 3; Method 4

D	Stat.	α	β_1	β_2	γ	$\rho_{\alpha,\beta}$	$\rho_{\alpha,\gamma}$	$\rho_{\beta,\gamma}$	σ^2_{ε}
B	$\widehat{\mu}$	0.089	0.137	-1.415	0.022	0.003	0.067	-0.015	–
	$\widehat{M}$	0.09	0.03	-0.147	0.019	0.04	0.074	0.007	–
	$\widehat{\sigma}^2$	0.027	0.328	8.033	0.015	0.01	0.005	0.007	–
	2.5%	-0.244	-0.475	-7.891	-0.2	-0.267	-0.131	-0.235	–
	97.5%	0.401	2.19	0.953	0.261	0.114	0.179	0.1	–
P	$\widehat{\mu}$	0.001	0.004	-0.033	0.001	0.007	0.224	-0.037	–
	$\widehat{M}$	0.001	0.001	-0.003	0.001	0.079	0.247	0.017	–
	$\widehat{\sigma}^2$	< .001	< .001	0.004	< .001	0.041	0.054	0.046	–
	2.5%	-0.002	-0.014	-0.181	-0.013	-0.534	-0.437	-0.588	–
	97.5%	0.004	0.063	0.022	0.017	0.228	0.595	0.251	–
Skew		-0.097	3.389	-1.321	0.142	-1.505	-1.334	-1.407	–
Kurtosis		-0.145	14.558	0.176	-0.105	1.617	4.222	2.816	–

APPENDIX D

The following tables give a two-by-two description of the classification success based on the true class and the predicted class based on the results of the FMM (in terms of the proportion correctly classified and those misclassified). The tables also give information on the effectiveness of the FMM at recovering π_1 (and π_2). As can be seen, even though the πs were not always recovered very accurately, the classification success was generally very effective. This illustrates that it is sometimes difficult to accurately estimate the precise value of the proportion of the population belonging to the particular class, but that individual entities can be accurately classified into the class in which they belong.

TABLE D-1

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 1

Condition 1		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.865	0.909	0.819	0.135	0.091	0.181
	$\widehat{M}$	0.88	0.94	0.931	0.12	0.06	0.069
	$\widehat{\sigma}$	0.083	0.114	0.222	0.083	0.114	0.222
True LC 2	$\widehat{\mu}$	0.111	0.198	0.227	0.889	0.802	0.773
	$\widehat{M}$	0.1	0.18	0.2	0.9	0.82	0.8
	$\widehat{\sigma}$	0.069	0.108	0.158	0.069	0.108	0.158
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.488	0.718	0.734	0.512	0.282	0.266
$\widehat{M}$		0.49	0.733	0.814	0.51	0.267	0.186
$\widehat{\sigma}$		0.056	0.086	0.182	0.056	0.086	0.182

TABLE D-2

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 2

Condition 2		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.883	0.943	0.867	0.117	0.057	0.133
	$\widehat{M}$	0.888	0.952	0.966	0.112	0.048	0.034
	$\widehat{\sigma}$	0.038	0.052	0.214	0.038	0.052	0.214
True LC 2	$\widehat{\mu}$	0.1	0.2	0.246	0.9	0.8	0.754
	$\widehat{M}$	0.096	0.192	0.242	0.904	0.808	0.758
	$\widehat{\sigma}$	0.033	0.067	0.127	0.033	0.067	0.127
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.491	0.739	0.777	0.509	0.261	0.223
$\widehat{M}$		0.493	0.743	0.851	0.507	0.257	0.149
$\widehat{\sigma}$		0.029	0.043	0.166	0.029	0.043	0.166

TABLE D-3

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 3

Condition 3		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.848	0.892	0.791	0.152	0.108	0.209
	$\widehat{M}$	0.865	0.93	0.909	0.135	0.07	0.091
	$\widehat{\sigma}$	0.088	0.12	0.234	0.088	0.12	0.234
True LC 2	$\widehat{\mu}$	0.115	0.206	0.229	0.885	0.794	0.771
	$\widehat{M}$	0.1	0.18	0.2	0.9	0.82	0.8
	$\widehat{\sigma}$	0.073	0.117	0.167	0.073	0.117	0.167
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.482	0.705	0.71	0.518	0.295	0.29
$\widehat{M}$		0.482	0.725	0.786	0.518	0.275	0.214
$\widehat{\sigma}$		0.06	0.094	0.193	0.06	0.094	0.193

TABLE D-4

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 4

Condition 4		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.866	0.931	0.824	0.134	0.069	0.176
	$\widehat{M}$	0.872	0.947	0.957	0.128	0.053	0.043
	$\widehat{\sigma}$	0.047	0.073	0.238	0.047	0.073	0.238
True LC 2	$\widehat{\mu}$	0.1	0.208	0.244	0.9	0.792	0.756
	$\widehat{M}$	0.096	0.2	0.242	0.904	0.8	0.758
	$\widehat{\sigma}$	0.038	0.074	0.144	0.038	0.074	0.144
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.483	0.728	0.74	0.517	0.272	0.26
$\widehat{M}$		0.483	0.737	0.835	0.517	0.263	0.165
$\widehat{\sigma}$		0.034	0.057	0.184	0.034	0.057	0.184

TABLE D-5

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 5

Condition 5		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.855	0.89	0.791	0.145	0.11	0.209
	$\widehat{M}$	0.87	0.933	0.909	0.13	0.067	0.091
	$\widehat{\sigma}$	0.084	0.135	0.234	0.084	0.135	0.234
True LC 2	$\widehat{\mu}$	0.112	0.199	0.22	0.888	0.801	0.78
	$\widehat{M}$	0.1	0.18	0.2	0.9	0.82	0.8
	$\widehat{\sigma}$	0.072	0.115	0.155	0.072	0.115	0.155
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
	$\widehat{\mu}$	0.484	0.703	0.711	0.516	0.297	0.289
	$\widehat{M}$	0.488	0.727	0.795	0.512	0.273	0.205
	$\widehat{\sigma}$	0.058	0.103	0.191	0.058	0.103	0.191

TABLE D-6

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 6

Condition 6		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.872	0.935	0.836	0.128	0.065	0.164
	$\widehat{M}$	0.876	0.947	0.961	0.124	0.053	0.039
	$\widehat{\sigma}$	0.044	0.069	0.235	0.044	0.069	0.235
True LC 2	$\widehat{\mu}$	0.1	0.202	0.245	0.9	0.798	0.755
	$\widehat{M}$	0.096	0.192	0.242	0.904	0.808	0.758
	$\widehat{\sigma}$	0.038	0.071	0.138	0.038	0.071	0.138
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
	$\widehat{\mu}$	0.486	0.731	0.749	0.514	0.269	0.251
	$\widehat{M}$	0.486	0.739	0.843	0.514	0.261	0.157
	$\widehat{\sigma}$	0.033	0.054	0.183	0.033	0.054	0.183

TABLE D-7

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 7

Condition 7		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.817	0.857	0.728	0.183	0.143	0.272
	$\widehat{M}$	0.84	0.913	0.806	0.16	0.087	0.194
	$\widehat{\sigma}$	0.105	0.159	0.256	0.105	0.159	0.256
True LC 2	$\widehat{\mu}$	0.114	0.21	0.213	0.886	0.79	0.787
	$\widehat{M}$	0.1	0.18	0.2	0.9	0.82	0.8
	$\widehat{\sigma}$	0.083	0.129	0.173	0.083	0.129	0.173
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.467	0.679	0.656	0.533	0.321	0.344
$\widehat{M}$		0.468	0.712	0.699	0.532	0.288	0.301
$\widehat{\sigma}$		0.07	0.12	0.208	0.07	0.12	0.208

TABLE D-8

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 8

Condition 8		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.834	0.901	0.792	0.166	0.099	0.208
	$\widehat{M}$	0.842	0.936	0.934	0.158	0.064	0.066
	$\widehat{\sigma}$	0.063	0.124	0.251	0.063	0.124	0.251
True LC 2	$\widehat{\mu}$	0.096	0.209	0.242	0.904	0.791	0.758
	$\widehat{M}$	0.092	0.2	0.242	0.908	0.8	0.758
	$\widehat{\sigma}$	0.045	0.087	0.155	0.045	0.087	0.155
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.467	0.704	0.709	0.533	0.296	0.291
$\widehat{M}$		0.468	0.724	0.807	0.532	0.276	0.193
$\widehat{\sigma}$		0.042	0.088	0.193	0.042	0.088	0.193

TABLE D-9

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 9

Condition 9		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.765	0.78	0.689	0.235	0.22	0.311
	$\widehat{M}$	0.79	0.833	0.737	0.21	0.167	0.263
	$\widehat{\sigma}$	0.143	0.182	0.231	0.143	0.182	0.231
True LC 2	$\widehat{\mu}$	0.199	0.27	0.24	0.801	0.73	0.76
	$\widehat{M}$	0.18	0.24	0.2	0.82	0.76	0.8
	$\widehat{\sigma}$	0.125	0.166	0.184	0.125	0.166	0.184
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.484	0.639	0.623	0.516	0.361	0.377
$\widehat{M}$		0.49	0.663	0.644	0.51	0.337	0.356
$\widehat{\sigma}$		0.103	0.143	0.188	0.103	0.143	0.188

TABLE D-10

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 10

Condition 10		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.784	0.827	0.71	0.216	0.173	0.29
	$\widehat{M}$	0.8	0.88	0.808	0.2	0.12	0.192
	$\widehat{\sigma}$	0.1	0.158	0.25	0.1	0.158	0.25
True LC 2	$\widehat{\mu}$	0.176	0.263	0.241	0.824	0.737	0.759
	$\widehat{M}$	0.156	0.264	0.226	0.844	0.736	0.774
	$\widehat{\sigma}$	0.091	0.121	0.173	0.091	0.121	0.173
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.482	0.661	0.638	0.518	0.339	0.362
$\widehat{M}$		0.481	0.689	0.681	0.519	0.311	0.319
$\widehat{\sigma}$		0.072	0.114	0.185	0.072	0.114	0.185

TABLE D-11

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 11

Condition 11		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.724	0.74	0.639	0.276	0.26	0.361
	$\widehat{M}$	0.75	0.793	0.649	0.25	0.207	0.351
	$\widehat{\sigma}$	0.161	0.201	0.233	0.161	0.201	0.233
True LC 2	$\widehat{\mu}$	0.195	0.253	0.224	0.805	0.747	0.776
	$\widehat{M}$	0.17	0.22	0.2	0.83	0.78	0.8
	$\widehat{\sigma}$	0.141	0.169	0.18	0.141	0.169	0.18
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.464	0.606	0.579	0.536	0.394	0.421
$\widehat{M}$		0.467	0.625	0.576	0.533	0.375	0.424
$\widehat{\sigma}$		0.115	0.154	0.188	0.115	0.154	0.188

TABLE D-12

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 12

Condition 12		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.741	0.778	0.677	0.259	0.222	0.323
	$\widehat{M}$	0.764	0.843	0.742	0.236	0.157	0.258
	$\widehat{\sigma}$	0.126	0.186	0.252	0.126	0.186	0.252
True LC 2	$\widehat{\mu}$	0.168	0.251	0.233	0.832	0.749	0.767
	$\widehat{M}$	0.144	0.24	0.21	0.856	0.76	0.79
	$\widehat{\sigma}$	0.11	0.138	0.178	0.11	0.138	0.178
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.461	0.623	0.607	0.539	0.377	0.393
$\widehat{M}$		0.459	0.654	0.629	0.541	0.346	0.371
$\widehat{\sigma}$		0.087	0.132	0.183	0.087	0.132	0.183

TABLE D-13

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 13

Condition 13		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.74	0.764	0.67	0.26	0.236	0.33
	$\widehat{M}$	0.765	0.813	0.714	0.235	0.187	0.286
	$\widehat{\sigma}$	0.15	0.188	0.237	0.15	0.188	0.237
True LC 2	$\widehat{\mu}$	0.202	0.272	0.238	0.798	0.728	0.762
	$\widehat{M}$	0.17	0.24	0.2	0.83	0.76	0.8
	$\widehat{\sigma}$	0.137	0.172	0.191	0.137	0.172	0.191
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.473	0.626	0.607	0.527	0.374	0.393
$\widehat{M}$		0.472	0.65	0.627	0.528	0.35	0.373
$\widehat{\sigma}$		0.111	0.148	0.192	0.111	0.148	0.192

TABLE D-14

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 14

Condition 14		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.755	0.791	0.696	0.244	0.209	0.304
	$\widehat{M}$	0.772	0.864	0.774	0.228	0.136	0.226
	$\widehat{\sigma}$	0.117	0.188	0.244	0.117	0.188	0.244
True LC 2	$\widehat{\mu}$	0.17	0.259	0.234	0.83	0.741	0.766
	$\widehat{M}$	0.152	0.248	0.21	0.848	0.752	0.79
	$\widehat{\sigma}$	0.101	0.143	0.173	0.101	0.143	0.173
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.468	0.636	0.622	0.532	0.364	0.378
$\widehat{M}$		0.467	0.675	0.65	0.533	0.325	0.35
$\widehat{\sigma}$		0.08	0.133	0.181	0.08	0.133	0.181

TABLE D-15

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 15

Condition 15		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.675	0.689	0.609	0.325	0.311	0.391
	$\widehat{M}$	0.7	0.733	0.629	0.3	0.267	0.371
	$\widehat{\sigma}$	0.177	0.217	0.245	0.177	0.217	0.245
True LC 2	$\widehat{\mu}$	0.19	0.247	0.222	0.81	0.753	0.778
	$\widehat{M}$	0.15	0.22	0.2	0.85	0.78	0.8
	$\widehat{\sigma}$	0.145	0.181	0.193	0.145	0.181	0.193
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.44	0.57	0.556	0.56	0.43	0.444
$\widehat{M}$		0.439	0.593	0.565	0.561	0.407	0.435
$\widehat{\sigma}$		0.126	0.168	0.194	0.126	0.168	0.194

TABLE D-16

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 16

Condition 16		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.695	0.717	0.616	0.305	0.283	0.384
	$\widehat{M}$	0.716	0.779	0.644	0.284	0.221	0.356
	$\widehat{\sigma}$	0.141	0.209	0.249	0.141	0.209	0.249
True LC 2	$\widehat{\mu}$	0.159	0.231	0.204	0.841	0.769	0.796
	$\widehat{M}$	0.128	0.208	0.161	0.872	0.792	0.839
	$\widehat{\sigma}$	0.114	0.159	0.174	0.114	0.159	0.174
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.438	0.579	0.554	0.562	0.421	0.446
$\widehat{M}$		0.435	0.603	0.559	0.565	0.397	0.441
$\widehat{\sigma}$		0.095	0.143	0.175	0.095	0.143	0.175

TABLE D-17

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 17

Condition 17		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.957	0.964	0.843	0.043	0.036	0.157
	$\widehat{M}$	0.96	0.98	0.983	0.04	0.02	0.017
	$\widehat{\sigma}$	0.045	0.089	0.236	0.045	0.089	0.236
True LC 2	$\widehat{\mu}$	0.044	0.082	0.141	0.956	0.918	0.859
	$\widehat{M}$	0.04	0.06	0.12	0.96	0.94	0.88
	$\widehat{\sigma}$	0.043	0.073	0.133	0.043	0.073	0.133
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.5	0.74	0.754	0.5	0.26	0.246
$\widehat{M}$		0.5	0.749	0.863	0.5	0.251	0.137
$\widehat{\sigma}$		0.023	0.058	0.191	0.023	0.058	0.191

TABLE D-18

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 18

Condition 18		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.961	0.978	0.875	0.039	0.022	0.125
	$\widehat{M}$	0.964	0.981	0.986	0.036	0.019	0.014
	$\widehat{\sigma}$	0.015	0.042	0.219	0.015	0.042	0.219
True LC 2	$\widehat{\mu}$	0.037	0.072	0.133	0.963	0.928	0.867
	$\widehat{M}$	0.036	0.072	0.113	0.964	0.928	0.887
	$\widehat{\sigma}$	0.014	0.037	0.101	0.014	0.037	0.101
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.499	0.748	0.782	0.501	0.252	0.218
$\widehat{M}$		0.5	0.75	0.871	0.5	0.25	0.129
$\widehat{\sigma}$		0.01	0.028	0.174	0.01	0.028	0.174

TABLE D-19

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 19

Condition 19		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.956	0.958	0.846	0.044	0.042	0.154
	$\widehat{M}$	0.96	0.98	0.983	0.04	0.02	0.017
	$\widehat{\sigma}$	0.038	0.096	0.23	0.038	0.096	0.23
True LC 2	$\widehat{\mu}$	0.045	0.092	0.155	0.955	0.908	0.845
	$\widehat{M}$	0.04	0.08	0.12	0.96	0.92	0.88
	$\widehat{\sigma}$	0.046	0.089	0.138	0.046	0.089	0.138
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.5	0.738	0.757	0.5	0.262	0.243
$\widehat{M}$		0.5	0.748	0.864	0.5	0.252	0.136
$\widehat{\sigma}$		0.021	0.062	0.188	0.021	0.062	0.188

TABLE D-20

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 20

Condition 20		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.96	0.975	0.884	0.04	0.025	0.116
	$\widehat{M}$	0.964	0.981	0.986	0.036	0.019	0.014
	$\widehat{\sigma}$	0.028	0.053	0.208	0.028	0.053	0.208
True LC 2	$\widehat{\mu}$	0.041	0.076	0.15	0.959	0.924	0.85
	$\widehat{M}$	0.04	0.072	0.113	0.96	0.928	0.887
	$\widehat{\sigma}$	0.027	0.046	0.127	0.027	0.046	0.127
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.5	0.746	0.79	0.5	0.254	0.21
$\widehat{M}$		0.5	0.75	0.871	0.5	0.25	0.129
$\widehat{\sigma}$		0.016	0.032	0.167	0.016	0.032	0.167

TABLE D-21

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 21

Condition 21		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.953	0.959	0.831	0.047	0.041	0.169
	$\widehat{M}$	0.96	0.98	0.983	0.04	0.02	0.017
	$\widehat{\sigma}$	0.057	0.093	0.249	0.057	0.093	0.249
True LC 2	$\widehat{\mu}$	0.045	0.086	0.155	0.955	0.914	0.845
	$\widehat{M}$	0.04	0.08	0.12	0.96	0.92	0.88
	$\widehat{\sigma}$	0.057	0.075	0.136	0.057	0.075	0.136
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.499	0.738	0.746	0.501	0.262	0.254
$\widehat{M}$		0.499	0.749	0.863	0.501	0.251	0.137
$\widehat{\sigma}$		0.031	0.059	0.202	0.031	0.059	0.202

TABLE D-22

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 22

Condition 22		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.96	0.977	0.87	0.04	0.023	0.13
	$\widehat{M}$	0.96	0.981	0.986	0.04	0.019	0.014
	$\widehat{\sigma}$	0.016	0.041	0.217	0.016	0.041	0.217
True LC 2	$\widehat{\mu}$	0.04	0.076	0.145	0.96	0.924	0.855
	$\widehat{M}$	0.04	0.072	0.113	0.96	0.928	0.887
	$\widehat{\sigma}$	0.022	0.049	0.107	0.022	0.049	0.107
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.5	0.748	0.778	0.5	0.252	0.222
$\widehat{M}$		0.5	0.749	0.869	0.5	0.251	0.131
$\widehat{\sigma}$		0.011	0.025	0.172	0.011	0.025	0.172

TABLE D-23

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 23

Condition 23		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.951	0.953	0.832	0.049	0.047	0.168
	$\widehat{M}$	0.96	0.98	0.977	0.04	0.02	0.023
	$\widehat{\sigma}$	0.045	0.111	0.241	0.045	0.111	0.241
True LC 2	$\widehat{\mu}$	0.055	0.104	0.175	0.945	0.896	0.825
	$\widehat{M}$	0.04	0.08	0.12	0.96	0.92	0.88
	$\widehat{\sigma}$	0.075	0.108	0.165	0.075	0.108	0.165
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.502	0.736	0.748	0.498	0.264	0.252
$\widehat{M}$		0.5	0.748	0.862	0.5	0.252	0.138
$\widehat{\sigma}$		0.034	0.072	0.197	0.034	0.072	0.197

TABLE D-24

SUMMARY OF THE CLASSIFICATION SUCCESS FOR EACH OF THE THREE SCENARIOS WITHIN CONDITION 24

Condition 24		Predicted LC 1			Predicted LC 2		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
True LC 1	$\widehat{\mu}$	0.956	0.973	0.879	0.044	0.027	0.121
	$\widehat{M}$	0.956	0.981	0.986	0.044	0.019	0.014
	$\widehat{\sigma}$	0.016	0.061	0.221	0.016	0.061	0.221
True LC 2	$\widehat{\mu}$	0.044	0.087	0.159	0.956	0.913	0.841
	$\widehat{M}$	0.04	0.08	0.129	0.96	0.92	0.871
	$\widehat{\sigma}$	0.029	0.055	0.128	0.029	0.055	0.128
Classification Recovery							
True π		.50	.75	.875	.50	.25	.125
$\widehat{\mu}$		0.5	0.747	0.787	0.5	0.253	0.213
$\widehat{M}$		0.499	0.75	0.87	0.501	0.25	0.13
$\widehat{\sigma}$		0.014	0.038	0.174	0.014	0.038	0.174

REFERENCES

Allport, G. W. (1937). *Personality: A psychological interpretation.* New York, NY: Henry Hold and Company.

Anderson, T. W. (1984). *An introduction to multivariate statistical analysis* (2nd ed.). New York, NY: John Wiley & Sons, Inc.

Angrist, J. D., Imbens, G. W., & Rubin, D. B. (1996). Identification of causal effects using instrumental variables. *Journal of the American Statistical Association*, *91*, 444–445.

Arminger, G., & Stein, P. (1997). Finite mixtures of covariance structure models with regressors. *Sociological Methods and Research*, *26*, 148–182.

Arminger, G., Stein, P., & Wittenberg, J. (1998). Mixtures of conditional mean- and covariance-structure models. *Psychometrika*, *64*, 475–494.

Baltes, P. B., & Nesselroade, J. R. (1979). History and rationale of longitudinal research. In J. R. Nesselroade & P. B. Baltes (Eds.), *Longitudinal research in the study of behavior and development* (pp. 1–39). New York, NY: Academic Press.

Bamber, D., & van Santen, J. P. H. (2000). How to assess a model's testability and identifiability. *Journal of Mathematical Psychology*, *44*, 20–40.

Bandeen-Roche, K., Miglioretti, D. L., Zeger, S. L., & Rathouz, P. J. (1997). Latent variable regression for multiple discrete outcomes. *Journal of the American Statistical Association*, *92*, 1375–1386.

Bartholomew, D. J. (1987). *Latent variable models and factor analysis.* New York: Oxford University Press.

Basford, K. E., & McLachlan, G. J. (1985). Estimation of allocation rates in a cluster analysis context. *Journal of the American Statistical Association*, *80*, 286–293.

Bates, D. M., & Watts, D. G. (1988). *Nonlinear regression analysis and its applications.* New York, NY: Wiley.

Bauer, D. J. (2003). Estimating multilevel linear models as structural equation models. *Journal of Educational and Behavioral Statistics*, *28*(2), 135–167.

Bayne, C. K., Beauchamp, J. J., Begovich, C. L., & Kane, V. E. (1980). Monte Carlo comparisons of selected clustering procedures. *Pattern Recognition*, *12*, 51–62.

Bentler, P. M., & Weeks, D. G. (1980). Linear structural equations with latent variables. *Psychometrika*, *45*(3), 289–308.

Berger, J. O., & Berry, D. A. (1988). Statistical analysis and the illusion of objectivity. *American Scientist*, *76*, 159–165.

Bollen, K. A. (1996). An alternative two stage least squares (2SLS) estimator for latent variable equations. *Psychometrika*, *61*(1), 109–121.

Bollen, K. A. (2001). Two-stage least squares and latent variable models: Simultaneous estimation and robustness to misspecifications. In R. Cudeck, S. du Toit, & D. Sörbom (Eds.), *Structural equation modeling: Present and future. A festschrift in honor of Karl Joreskog* (pp. 119–138). Lincolnwood, IL: Scientific Software International, Inc.

Box, G. (1979a). Robustness in the strategy of scientific model building. In R. L. Launder & G. N. Williamson (Eds.), *Robustness in statistics* (pp. 201–236). New York, NY: Academic Press.

Box, G. (1979b). Some problems of statistics and everyday life. *Journal of American Statistical Association*, *74*, 1–4.

Browne, M. W., & Cudeck, R. (1993). Alternative ways of assessing model fit. In K. A. Bollen & J. S. Long (Eds.), *Testing structural equation models* (pp. 136–162). Newbury Park, CA: Sage Publications.

Browne, M. W., & du Toit, S. H. C. (1991). Models for learning data. In L. M. Collins & J. L. Horn (Eds.), *Best methods for the analysis of change: Recent advances, unanswered questions, future directions.* Washington, DC: American Psychological Association.

Bryk, A. S., & Raudenbush, S. W. (1987). Application of hierarchical linear models to assessing change. *Psychological Bulletin*, *101*(1), 147-158.

Burchinal, M., & Appelbaum, M. I. (1991). Estimating individual developmental functions: Methods and their assumptions. *Child Development*, *62*, 23–43.

Causton, D. R., & Venus, J. C. (1981). *The biometry of plant growth.* London: Edward Arnold.

Chatterjee, N. (2004). A two-stage regression model for epidemiological studies with multivariate disease classification data. *Journal of the American Statistical Association*, *99*(465), 127–138.

Chen, H., Chen, J., & Kalbfleisch, J. D. (2004). Testing for a finite mixture model with two components. *Journal of the Royal Statistical Society. Series B*, *66*, 95–115.

Christoffersson, A. (1975). Factor analysis of dichotomized variables. *Psychometrika*, *40*(1), 5–32.

Clogg, C. C. (1995). Latent class models. In G. Arminger, C. C. Clogg, & M. E. Sobel (Eds.), *Handbook of statistical modeling for the social and behavioral sciences* (pp. 311–353). New York, NY: Plenum Press.

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Hillsdale, NJ: Lawrence Erlbaum Associates.

Collins, L. M. (1996). Measurement of change in research on aging: Old and new issues from an individual growth perspective. In J. E. Birren & K. Schaie (Eds.), *Handbook of the psychology of the aging* (4th ed., pp. 38–58). San Diego, CA: Academic Press.

Corder, E. H., Woodbury, M. A., Volkmann, I., Madsen, D. K., Bogdanovic, N., & Winblad, B. (2000). Density profiles of Alzheimer disease regional brain pathology for the Huddinge Brain Bank: Pattern recognition emulates and expands upon Braak staging. *Experimental Gerontology*, *35*, 851–864.

Cronbach, L. J., & Furby, L. (1970). How we should measure "change": Or should we? *Psychological Bulletin*, *74*, 68–80.

Croudace, T. J., Jarvelin, M. R., Wadsworth, M. E., & Jones, P. B. (2003). Developmental typology of trajectories to nighttime bladder control: Epidemiologic application of longitudinal latent class analysis. *American Journal of Epidemiology*, *157*(9), 834–842.

Cudeck, R. (1996). Mixed-effects models in the study of individual differences with repeated measures data. *Multivariate Behavioral Research*, *31*(3), 371–403.

Cudeck, R., & du Toit, S. H. C. (2002). A version of quadratic regreession with interpretable parameters. *Multivariate Behavioral Research*, *37*(4), 501–519.

Curran, P. J. (2003). Have multilevel models been structural equation models all along? *Multivariate Behavioral Research*, *38*(4), 529–569.

Datta, G. S., & Ghosh, M. (1995). Some remarks on noninformative priors. *Journal of the American Statistical Association*, *90*(432), 1357–1363.

Davidian, M., & Giltinan, D. M. (1995). *Nonlinear models for repeated measurement data*. New York, NY: Chapman & Hall.

Davidian, M., & Giltinan, D. M. (2003). Nonlinear models for repeated measurement data: An overview and update. *Journal of Agricultural, Biological, and Environmental Statistics*, *8*(4), 387–419.

Davies, A. E. (1900). The concept of change. *The Philosophical Review*, *9*(5), 502–517.

Dawid, A. P., Stone, M., & Zidek, J. V. (1973). Marginalization paradoxes in Bayesian and structural inference. *Journal of the Royal Statistical Society. Series B*, *189–223*(2), 189.

Dempster, A., Laird, N., & Rubin, D. (1977). Maximum likelihood from incomplete data via the EM algorithm. *Journal of the Royal Statistical Society. Series B*, *39*, 1–8.

Dolan, C. V., & van der Maas, H. L. J. (1998). Fitting multivariate normal mixtures subject to structural equation modeling. *Psychometrika*, *63*, 227–253.

du Toit, S. H. C., & Cudeck, R. (2001). The analysis of nonlinear random coefficient regression models with LISREL using constraints. In R. Cudeck, S. H. C. du Toit, & D. Sörbom (Eds.), *Structural equation modeling: Present and future. A festschrift in honor of Karl Jöreskog* (pp. 259–278). Lincolnwood, IL: Scientific Software International.

Dumenci, L., & Windle, M. (2001). Cluster analysis as a method of recovering types of intraindividual growth trajectories: A Monte Carlo study. *Multivariate Behavioral Research*, *36*(4), 501–5522.

D'Unger, A. V., Land, K. C., McCall, P. L., & Nagin, D. S. (1998). How many latent classes of delinquent/criminal careers? Results from mixed Poisson regression analyses. *American Journal of Sociology*, *103*(6), 1593–1630.

Dunn, J. G. H. (1994). Toward the combined use of nomothetic and idiographic methodologies in sport psychology: An empirical example. *The Sport Psychologist*, *8*, 376–392.

Eckart, C., & Young, G. (1936). The approximation of one matrix by another of lower rank. *Psychometrika*, *1*(3), 211–218.

Efron, B. (1986). Why isn't everyone a Bayesian? *The American Statistician*, *40*(1), 1–5.

Efron, B., & Hinkley, D. V. (1978). Assessing the accuracy of the maximum likelihood estimator: Observed versus expected Fisher information. *Biometrika*, *65*(3), 457–482.

Efron, B., & Tibshirani, R. J. (1993). *An introduction to the bootstrap.* New York, NY: Chapman & Hall/CRC.

Einstein, A. (1977). "Quotable quotes". *Reader's Digest*, *111*(663), 42.

Everitt, B. S., & Hand, D. J. (1981). *Finite mixture distributions.* New York, NY: Chapman and Hall.

Finney, R. L., Weir, M. D., & Giordano, F. R. (2001). *Thomas' calculus* (10th ed.). New York, NY: Addison Wesley.

Fisher, R. A. (1922). On the mathematical foundations of theoretical statistics. *Philosophical Transaction of the Royal Society of London. Series A*, *222*, 309–368.

Follmann, D. A., & Lambert, D. (1989). Generalizing logistic regression by nonparametric mixing. *Journal of the American Statistical Association*, *84*, 295–300.

Formann, A. K. (1992). Linear logistic latent class analysis for polytomous data. *Journal of the American Statistical Association*, *87*, 476–486.

Fraley, C., & Raftery, A. E. (2002). Model-based clustering, discriminant analysis, and density estimation. *Journal of the American Statistical Association*, *97*(458), 611–631.

Fraley, C., & Raftery, A. E. (2004). MCLUST: Software for model based clustering (version 2.1-5) [computer software and manual]. Retrieved from http://www.cran.r-project.org/.

Frangakis, C. E., & Baker, S. G. (2001). Compliance subsampling designs for comparative research: Estimation and optimal planning. *Biometrics*, *57*, 899–908.

Gallant, A. R. (1987). *Nonlinear statistical models.* New York, NY: John Wiley & Sons, Inc.

Geman, S., & Geman, D. J. (1984). Stochastic relaxation, Gibbs distributions and the Bayesian restoration of images. *IEEE Transactions on Pattern Analysis and Machine Intelligence, 6*, 721–741.

Gilks, W. R., Richardson, S., & Spiegelhalter, D. J. (1996). Introducing Markov chain Monte Carlo. In W. R. Gilks, S. Richardson, & D. J. Spiegelhalter (Eds.), *Markov chain Monte Carlo in practice* (pp. 1–19). London: Chapman and Hall.

Glass, G. V. (1976). Primary, secondary, and meta-analysis of research. *Educational Researcher*, 3–8.

Goodman, L. A. (1974). Exploratory latent structure analysis using both identifiable and unidentifiable models. *Biometrika*, *61*, 215–231.

Goodman, L. A. (2002). Latent class analysis: The empirical study of latent types, latent variables, and latent structures. In J. A. Hagenaars & A. L. McCutcheon (Eds.), *Applied latent class analysis* (pp. 3–54). New York, NY: Cambridge University Press.

Graham, J. W., Collins, L. M., Wugalter, S. E., Chung, N. K., & Hansen, W. B. (1991). Modeling transitions in latent stage-sequential processes: A substance use prevention example. *Journal of Consulting and Clinical Psychology*, *59*, 48–57.

Graybill, F. A. (1976). *Theory and application of the linear model.* Pacific Grove, CA: Brooks/Cole.

Green, P. J. (1995). Reversible jump Markov chain Monte Carlo computation and Bayesian model determination. *Biometrika*, *82*(2), 711–732.

Hampel, F. (2002). Some thoughts about classification. In K. Jajuga, A. Sokołowski, & H.-H. Bock (Eds.), *Classification, clustering, and data analysis: Recent advances and applications* (pp. 5–26). New York, NY: Springer.

Harris, C. W. (Ed.). (1963). *Problems in measuring change.* Madison, WI: University of Wisconsin Press.

Hartley, H. O. (1961). The modified Gauss-Newton method for fitting non-linear regression functions by least squares. *Technometrics*, *3*(2), 269–280.

Hastings, W. K. (1970). Monte Carlo sampling methods using Markov chains and their applications. *Biometrika*, *57*(1), 97–109.

Hawkins, D. M., Muller, M. W., & ten Krooden, J. A. (1982). Cluster analysis. In D. M. Hawkins (Ed.), *Topics in applied multivariate analysis* (pp. 303–356). Cambridge, UK: Cambridge.

Hedges, L. V. (1981). Distribution theory for Glass's estimator of effect size and related estimators. *Journal of Educational Statistics*, *6*(2), 107–128.

Heijden, P. G. M., Dressens, J., & Bockenholt, U. (1996). Estimating the concomitant variable latent class model with the EM algorithm. *Journal of Educational and Behavioral Statistics*, *21*, 215–229.

Hogg, R. V., & Craig, A. T. (1978). *Introduction to mathematical statistics* (4th ed.). New York, NY: Macmillan Publishing Company.

Howard, G. S., Maxwell, S. E., & Fleming, K. J. (2000). The proof of the pudding: An illustration of the relative strengths of null hypothesis, meta-analysis, and Bayesian analysis. *Psychological Methods*, *5*(3), 315–332.

Hunt, R. (1982). *Plant growth curves: The functional approach to plant growth analysis.* Baltimore, MD: University Park Press.

Jackman, S. (2000). Estimation and inference via Bayesian simulation: An introduction to Markov chain Monte Carlo. *American Journal of Political Science*, *44*(2), 369–398.

Jones, B. L., Nagin, D. S., & Roeder, K. (2001). A SAS procedure based on mixture models for estimating developmental trajectories. *Sociological Methods & Research*, *29*(3), 374–393.

Jöreskog, K. (1970). A general method for analysis of covariance structures. *Biometrika*, *57*, 239–251.

Jöreskog, K., & Sörbom, D. (1996). *LISREL 8: User's reference guide* (2nd ed.). Chicago, IL: Scientific Software International.

Jöreskog, K. G. (1973). A general method for estimating a linear structural equation system. In A. S. Goldberger & O. Duncan (Eds.), *Structural equation models in the social sciences.* New York, NY: Seminar Press.

Jöreskog, K. G., & Sörbom, D. (1979). *Advances in factor analysis and structural equation modeling.* Cambridge, MA: Abt Books.

Kuiper, F. K., & Fisher, L. (1975). A Monte Carlo comparison of six clustering procedures. *Biometrics*, *31*, 777-783.

Laird, N. M., & Ware, H. (1982). Random-effects models for longitudinal data. *Biometrics*, *38*, 963–974.

Lamiell, J. T. (1998). 'Nomothetic' and 'idiographic:' Contrasting Windelband's understanding with contemporary usage. *Theory & Psychology*, *8*(1), 23–38.

Land, K. C., & Nagin, D. S. (1996). Micro-models of criminal careers: A synthesis of the criminal careers and life course approaches via semiparametric mixed Poisson models with empirical applications. *Journal of Quantitative Criminology*, *12*, 163–191.

Lazarfeld, P. F., & Henry, N. W. (1968). *Latent structure analysis.* New York, NY: Houghton Mifflin Company.

Lee, S.-Y., & Zhu, H.-T. (2002). Maximum likelihood estimation of nonlinear structural equation models. *Psychometrika, 67*(2), 189–210.

Lindstrom, M. J., & Bates, D. M. (1990). Nonlinear mixed effects models for repeated measures data. *Biometrics, 46*, 673–687.

Little, R., & Rubin, D. (1987). *Statistical analysis with missing data.* New York, NY: John Wiley & Sons, Inc.

Loehlin, J. C. (1998). *Latent variable models: An introduction to factor, path, and structural analysis* (3rd ed.). Mahwah, NJ: Lawrence Erlbaum Associates.

López, S., France, J., Dhanoa, M. S., Mould, F., & Dijkstra, J. (1999). Comparison of mathematical models to describe disappearance curves obtained using the polyester bag technique for incubating feeds in the rumen. *Journal of Animal Science*, *77*, 1875–1888.

Lord, F. M. (1956). The measurement of growth. *Educational and Psychological Measurement*, *16*, 421–437.

Lord, F. M. (1958). Further problems in the measurement of growth. *Educational and Psychological Measurement*, *18*(3), 437–451.

Magnusson, D., & Bergmen, L. R. (1988). Individual and variable-based approaches to longitudinal research on early risk factors. In M. Rutter (Ed.), *Studies of psychosocial risk: The power of longitudinal data* (pp. 45–61). New York, NY: Cambridge University Press.

Mahalanobis, P. (1936). On the generalized distance in statistics. *Proceedings of the National Institute of Science of India*, *12*, 49-55.

Manton, K. G., Woodbury, M. A., Stallard, E., & Corder, L. S. (1992). The use of grade-of-membership techniques to estimate regression relationships. *Sociological Methodology*, *22,* 321–381.

Manton, K. G., Woodbury, M. A., & Tolley, H. D. (1994). *Statistical applications using fuzzy sets.* New York, NY: John Wiley & Sons, Inc.

Mayo, D. G. (1996). *Error and the growth of experimental knowledge.* Chicago, IL: University of Chicago Press.

McArdle, J. J., & Boker, S. M. (1990). *RAMpath: Path diagram software.* Denver, CO: Data Transforms, Inc.

McArdle, J. J., & Epstein, D. (1987). Latent growth curves with developmental structural equation models. *Child Development*, *58*, 110–133.

McArdle, J. J., & Hamagami, F. (1991). Modeling incomplete longitudinal and cross-sectional data using latent growth structural models. In L. M. Collins & J. L. Horn (Eds.), *Best methods for the analysis of change* (pp. 276–309). Washington, DC: American Psychological Association.

McArdle, J. J., & Hamagami, F. (2001). Latent difference score structural models for linear dynamic analyses with incomplete longitudinal data. In L. M. Collins & A. G. Sayer (Eds.), *New methods for the analysis of change* (pp. 139–175). Washington, DC: American Psychological Association.

McArdle, J. J., & McDonald, R. P. (1984). Some algebraic properties of the reticular action model for moment structures. *British Journal of Mathematical and Statistical Psychology*, *37*, 234–251.

McArdle, J. J., & Nesselroade, J. R. (1994). Structuring data to study development and change. In S. H. Cohen & H. W. Reese (Eds.), *Life-span developmental psychology: Methodological innovations* (pp. 223–267). Mahwah, NJ: Erlbaum.

McCall, R. B., Appelbaum, M. I., & Hogarty, P. S. (1973). Developmental changes in mental performance. *Monographs of the society for research in child development*, *38*(3), 1–83.

McCullagh, P., & Nelder, J. A. (1989). *Generalized linear models* (2nd ed.). London: Chapman and Hall.

McDonald, R. P. (1999). *Test theory: A unified treatment.* Mahwah, NJ: Lawrence Erlbaum Associates.

McLachlan, G., & Peel, D. (2001). *Finite mixture models.* New York, NY: John Wiley & Sons, Inc.

McLachlan, G. J., & Basford, K. E. (1988). *Mixture models: Inference and applications to clustering.* New York, NY: Marcel Dekker, Inc.

Mehta, P. D., & West, S. G. (2000). Putting the individual back into individual growth curves. *Psychological Methods*, *5*(1), 23–43.

Meredith, W., & Tisak, J. (1984). *"Tuckerizing" curves.* Paper presented at the Psychometric Society annual meeting, Santa Barbara, CA.

Meredith, W., & Tisak, J. (1990). Latent curve analysis. *Psychometrika*, *55*(1), 107–122.

Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., & Teller, E. (1953). Equations of state calculations by fast computing machines. *Journal of Chemistry in Physics*, *21*, 1087–1092.

Moffitt, T. E. (1993). Adolescence-limited and life-course-persistent antisocial behavior: A development taxonomy. *Psychological Review*, *100*(4), 674–701.

Muthén, B. (2001a). Latent variable mixture modeling. In G. A. Marcoulides & R. E. Schumacker (Eds.), *New developments and techniques in structural equation modeling* (pp. 1–33). Hillsdale, NJ: Lawrence Erlbaum Associates.

Muthén, B. (2001b). Second-generation structural equation modeling with a combination of categorical and continuous latent variables: New opportunities for latent class/latent growth modeling. In L. M. Collins & A. Sayer (Eds.), *New methods for the analysis of change* (p. 291-322). Washington, DC: American Psychological Association.

Muthén, B. (2002). Beyond SEM: General latent variable modeling. *Behaviormetrika*, *29*, 81–117.

Muthén, B., Brown, C. H., Masyn, K., Jo, B., Khoo, S.-T., Yang, C.-C., Wang, C.-P., Kellam, S. G., Carlin, J. B., & Liao, J. (2002). General growth mixture modeling for randomized preventive interventions. *Biostatistics*, *3*(4), 459–475.

Muthén, B., Khoo, S. T., Francis, D., & Boscardin, C. K. (2002). Analysis of reading skills development from kindergarten through first grade: An application of growth mixture modeling to sequential processes. In S. R. Reise & N. Duan (Eds.), *Multilevel modeling: Methodological advances, issues, and applications* (pp. 71–89). Mahaw, NJ: Lawrence Erlbaum Associates.

Muthén, B., & Muthén, L. K. (2000). Integrating person-centered and variables-centered analyses: Growth mixture modeling with latent trajectory classes. *Alcoholism: Clinical and Experimental Research*, *24*(6), 882–891.

Muthén, B., & Shedden, K. (1999). Finite mixture modeling with mixture outcomes using the EM algorithm. *Biometrics*, *55*, 463–469.

Muthén, B. O. (1989). Latent variable modeling in heterogeneous populations. *Psychometrika*, *54*(4), 557–585.

Muthén, L. K., & Muthén, B. O. (1998). Mplus user's guide (version 2) [computer software and manual]. Los Angeles, CA: Muthén and Muthén.

Nagin, D. S. (1999). Analyzing developmental trajectories: A semi-parametric, group-based approach. *Psychological Methods*, *4*, 137–157.

Nagin, D. S., & Tremblay, R. E. (2001). Analyzing developmental trajectories of distinct but related behaviors: A group-based method. *Psychological Methods*, *6*, 18–34.

Oxford, M. L., Gilchrist, L. D., Morrison, D. M., Gillmore, M. R., J., M., & Lewis, S. M. (2003). Alcohol use among adolescent mothers: Heterogeneity in growth curves. *Prevention Science*, *4*, 15–26.

Pauler, D. K., & Laird, N. M. (2000). A mixture model for longitudinal data with application to assessment of noncompliance. *Biometrics*, *56*, 464–472.

Pawitan, Y. (2001). *In all likelihood: Statistical modelling and inference using likelihood.* New York, NY: Oxford University Press.

Pearson, K. (1894). Contributions to the mathematical theory of evolution. *Philosophical Transactions of the Royal Society of London. Series A*, *185*, 71–110.

Pearson, K. (1895). Contributions to the mathematical theory of evolution.–II: Skew variation in homogeneous material. *Philosophical Transactions of the Royal Society of London. Series A*, *186*, 343–414.

Pearson, K. (1906). Walter Frank Raphael Weldon, 1860–1906: A memoir. *Biometrika*, *5*, 1–52.

Pinheiro, J., & Bates, D. (2000). *Mixed-effects models in S and S-Plus.* New York, NY: Springer.

Pinheiro, J., Bates, D., DebRoy, S., & Sarkar, D. (2004). NLME: Linear and nonlinear mixed effects models (version 3.3-48) [computer software and manual]. Retrieved from http://www.cran.r-project.org/.

R. (2004). (Version 1.9) [computer software and manual]. Retrieved from http://www.cran.r-project.org/.

Ratkowsky, D. A. (1983). *Nonlinear regression modeling: A unified practical approach.* New York, NY: Marcel Dekker, Inc.

Raudenbush, S. W. (2001a). Comparing personal trajectories and drawing causal inferences from longitudinal data. *Annual Review of Psychology*, *52*, 501–525.

Raudenbush, S. W. (2001b). Toward a coherent framework for comparing trajectories of individual change. In L. M. Collins & A. G. Sayer (Eds.), *New methods for the analysis of change* (pp. 35–64). Washington, DC: American Psychological Association.

Raudenbush, S. W., & Bryk, A. S. (2002). *Hierarchical linear models: Applications and data analysis methods* (2nd ed.). Thousand Oaks, CA: Sage.

Reboussin, B. A., Reboussin, D. M., Liang, K. Y., & Anthony, J. C. (1998). Latent transition modeling of progression of health-risk behavior. *Multivariate Behavioral Research*, *33*, 457–478.

Rencher, A. C. (2000). *Linear models in statistics.* New York, NY: John Wiley & Sons, Inc.

Richards, F. J. (1959). A flexible growth function for empirical use. *Journal of Experimental Botany*, *10*(29), 290–300.

Rindskopf, D. (1990). Testing developmental models using latent class analysis. In A. von Eye (Ed.), *Statistical methods in longitudinal research: Time series and categorical longitudinal data* (Vol. 2, pp. 443–469). Boston, MA: Academic Press.

Rogosa, D., Brandt, D., & Zimowski, M. (1982). A growth curve approach to the measurement of change. *Psychological Bulletin*, *92*(3), 726–748.

Rogosa, D., & Willett, J. B. (1985). Understanding correlates of change by modeling individual differences in growth. *Psychometrika*, *50*(2), 203–228.

Rozeboom, W. W. (1966). *Foundations of the theory of prediction.* Homewood, IL: The Dorsey Press.

S-Plus. (2002). (Version 6.1) [computer software and manual]. Seattle, WA: Insightful Corporation.

SAS. (2004). (Version 9.0) [computer software and manual]. Cary, NC: SAS Institute.

Schafer, J. (1997). *Analysis of incomplete multivariate data.* London: Chapman & Hall.

Schafer, J. L. (2001). Multiple imputation with PAN. In L. M. Collins & A. G. Sayer (Eds.), *New methods for the analysis of change* (pp. 357–383). Washington, DC: American Psychological Association.

Schafer, J. L., & Graham, J. W. (2002). Missing data: Our view of the state of the art. *Psychological Methods*, *7*(2), 147–177.

Schulenberg, J., O'Malley, P. M., Bachman, J. G., Wadsworth, K. N., & Johnston, L. D. (1996a). Adolescent risk factors for binge drinking during the transition to young adulthood: Variable- and pattern-centered approaches to change. *Developmental Psychology*, *32*(4), 659–673.

Schulenberg, J., O'Malley, P. M., Bachman, J. G., Wadsworth, K. N., & Johnston, L. D. (1996b). Getting drunk and growing up: Trajectories of frequent binge drinking during the transition to young adulthood. *Journal of Studies on Alcohol, 57*, 289–304.

Seber, G. A. F., & Wild, C. J. (1989). *Nonlinear regression.* New York, NY: John Wiley & Sons, Inc.

Sheiner, L., & Beal, S. (1980). Evaluation of methods for estimating population pharmacokinetic parameters. I. Michaelis-Menton model: Routine clinical pharmacokinetic data. *Journal of Pharmacokinetics and Biopharmaceutics, 8*(6), 553–571.

Sokal, R. R. (1977). Cluster and classification: Background and current directions. In J. Van Ryzin (Ed.), *Classification and clustering.* New York, NY: Academic Press, Inc.

Sörbom, D. (1974). A general method for studying differences in factor means and factor structure between groups. *British Journal of Mathematical and statistical Psychology*, *27*, 229–239.

Steinley, D. (2003). Local optima in K-means clustering: What you don't know may hurt you. *Psychological Methods, 8*(3), 294–304.

Stevens, W. (1951). Asymptotic regression. *Biometrics*, *7*(3), 247–267.

Stuart, A., & Ord, J. K. (1994). *Kendall's advanced theory of statistics: Distribution theory* (Vol. 1, 6th ed.). New York, NY: John Wiley & Sons, Inc.

Stuart, A., Ord, J. K., & Arnold, S. (1999). *Kendall's advanced theory of statistics: Classical inference and the linear model* (Vol. 2A, 6th ed.). New York, NY: Oxford University Press.

Stukel, T. A., & Demidenko, E. (1997). Two-stage method of estimation for general linear growth curve models. *Biometrics*, *53*, 720–728.

Terrell, G. R., & Scott, D. W. (1985). Oversmoothed nonparametric density estimation. *Journal of the American Statistical Association*, *80*(389), 209–214.

Thomson, G. H. (1924). A formula to correct for the effect of errors of measurement on the correlation of initial values with gains. *Journal of Experimental Psychology*, *7*, 321–324.

Titterington, D. M. (1996). Mixture distributions (update). In S. Kotz, N. L. Johnson, & D. Banks (Eds.), *Encyclopedia of statistical sciences* (Vol. 5, pp. 399–407). New York, NY: Wiley.

Tsoularis, A., & Wallace, J. (2002). Analysis of logistic growth models. *Mathematical Biosciences*, *179*, 21–55.

Tubman, J. G., Vicary, R. J., von Eye, A., & Lerner, J. V. (1990). Longitudinal substance use and adult development. *Journal of substance abuse*, *2*, 317–334.

Twisk, J., & de Vente, W. (2002). Attrition in longitudinal studies: How to deal with missing data. *Journal of Clinical Epidemiology*, *55*, 329–337.

van Geert, P. (1991). A dynamic systems model of cognitive and language growth. *Psychological Review*, *98*(1), 3–52.

Vonesh, E. F., & Chinchilli, V. M. (1997). *Linear and nonlinear models for the analysis of repeated measurements.* New York, NY: Marcel Dekker.

Weldon, W. F. R. (1892). Certain correlated variations in *Crangon vulgaris*. *Proceedings of the Royal Society of London*, *51*, 2–21.

Weldon, W. F. R. (1893). On certain correlated variations in *Carcinus moenas*. *Proceedings of the Royal Society of London*, *54*, 318–329.

White, H. (1982). Maximum likelihood of misspecified models. *Econometrica*, *50*(1), 1–25.

Willett, J. B. (1988). Questions and answers in the measurement of change. In E. Z. Rothkopf (Ed.), *Review of research in education* (Vol. 15, pp. 345–422). Washington, DC: American Educational Research Association.

Willett, J. B., & Sayer, A. G. (1994). Using covariance structure analysis to detect correlates and predictors of individual change over time. *Psychological Bulletin*, *116*(2), 363–381.

Windelband, W. (1921). *An introduction to philosophy.* London: Unwin.

Winsor, C. P. (1932). The Gompertz curve as a growth curve. *The Proceedings of the National Academy of Sciences of the United States of America*, *18*(1), 1–8.

Wohlwill, J. (1973). *The study of behavioral development.* New York, NY: Academic Press.

Woodbury, M. A., Clive, J., & Arther Garson, J. (1978). Mathematical typology: A grade of membership technique for obtaining disease definition. *Computers and Biomedical Research*, *11*, 277–298.

Yalcin, I., & Amemiya, Y. (2001). Nonlinear factor analysis as a statistical method. *Statistical Science*, *16*(3), 275–294.

Yin, X., Goudriaan, J., Lantinga, E. A., Vos, J., & Spiertz, H. J. (2003). A flexible sigmoid function of deterministic growth. *Annuals of Botany*, *91*, 361–371.

Yuan, K.-H., & Bentler, P. M. (in press). Stepwise analysis of multilevel covariance structure models.

Yung, Y.-F. (1997). Finite mixtures in confirmatory factor-analysis models. *Psychometrika*, *62*(3), 297–330.