Name: ________________________________

Math 20610: Linear Algebra
Spring **Semester 2025**-26
Practice **Exam 1**

No calculators or notes allowed.

Scores

Question	Possible	Actual
1	12	
2	10	
3 & 4	10	
5	10	
6	10	
7	15	
Total	67	

GOOD LUCK

Note: our exam will be out of 100 points. This practice exam just collects sample problems and is not intended to be an exact parallel to the real exam.

1. (10 points) Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$ or explain why none exists.

2. (5 points) For which numbers $t \in \mathbf{R}$ (if any) is $\begin{pmatrix} t \\ t+1 \\ t \end{pmatrix}$ a linear combination of the vectors $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$?

3. (5 points) Find the reflection of the vector $\begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix}$ about the line L through $\mathbf{0}$ and $\begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$. (Half-credit for finding the *projection* onto L) Remember $\mathrm{ref}_\ell(\vec{x}) = 2\,\mathrm{proj}_\ell(\vec{x}) - \vec{x}$.

(This formula will be provided in the exam.)

4. Suppose that A is a 3×6 matrix with columns $\mathbf{a}_1, \ldots, \mathbf{a}_6 \in \mathbf{R}^3$ and that one can perform row operations on A to arrive at the matrix

$$\begin{bmatrix} 1 & 0 & 2 & 0 & -4 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a) (5 points) What are the solutions of $A\mathbf{x} = \mathbf{0}$?

(b) (3 points) What can you say about the number of solutions of the linear system $A\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$? Explain briefly.

5. Suppose that $T : \mathbf{R}^2 \to \mathbf{R}^3$ is a linear transformation such that $T\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix}$ and $T\begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$.

(a) (4 points) Find $T\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(b) (4 points) Find the matrix for T.

(c) (2 points) Compute $T\begin{pmatrix} e \\ \pi \end{pmatrix}$.

6. Two of the following six assertions are false (meaning 'not always true'). Identify them and give **specific** counterexamples at the bottom (and, if needed, back) of this page (go to the back if necessary). Note that you do not have to justify your counterexamples—only present them. (5 points each)

(a) If A is a 3×4 matrix with rank 3 and $\mathbf{b} \in \mathbf{R}^3$ is a vector, then the linear system $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

(b) If A is an invertible $n \times n$ matrix, then A is row equivalent to the identity matrix.

(c) If A and B are 2×2 matrices, then $AB = BA$.

(d) If A is an 3×2 matrix with linearly independent columns and $\mathbf{b} \in \mathbf{R}^m$ is a vector, then $A\mathbf{x} = \mathbf{b}$ is consistent. Two columns are linearly independent if neither is a multiple of the other.

(e) If $T : \mathbf{R}^n \to \mathbf{R}^m$ is a linear transformation, then $T(\mathbf{0}) = \mathbf{0}$.

(g) If $\mathbf{v} \in \mathbf{R}^3$ is a non-zero vector and L is the line joining $\mathbf{0}$ to $\mathbf{v}$, then $\text{proj}_L(\mathbf{v}) = \mathbf{v}$.