

A Supplementary Material for “Robust Numerical Algebraic Geometry”

The following provides tables associated with Sections 8.2 and 8.3.

Table S1: Solutions to (9) on a randomly selected affine patch for $\hat{p}$ where $i = \sqrt{-1}$

Solution	x_0			x_1			x_2		
1	0.0000	+	0.0000i	0.0000	+	0.0000i	-0.2235	+	0.8253i
2	0.0020	-	0.0072i	0.0010	-	0.0037i	-0.2263	+	0.8289i
3	-0.0368	+	2.7475i	-0.0682	+	5.0964i	0.0198	-	1.4776i

Table S2: Initial (exact), perturbed (8 decimals), and recovered (8 decimals) parameter values

Parameter	Initial ($\tilde{p}$)	Perturbed ($\hat{p}$)	Recovered (p^*)
p_1	-2.37160000	-2.37284227	-2.36891717
p_2	0.98608803	0.96067280	0.96814820
p_3	-0.53770000	-0.53492792	-0.52506952

Table S3: Initial (exact), perturbed (4 decimals), and recovered (4 decimals) parameter values

Parameter	Initial ($\tilde{p}$)	Perturbed ($\hat{p}$)	Recovered (p^*)
p_1	1	0.9876	1.0992
p_2	-2	-2.2542	-2.1984

Table S4: Summary of solutions (8 decimals) satisfying $f(x; \hat{p}) = 2x_1 - 3x_2 - 1 = 0$

j	w_j	$\dot{w}_j$	$\ddot{w}_j$
1	1.15384590	0.08241763	0.00546655
	0.43589727	-0.27838824	0.00364437
2	-0.99999993	0.07142858	-0.00546648
	-0.99999995	-0.28571428	-0.00364432
3	1.64589862	-0.17082057	0.13416408
	0.76393241	-0.44721371	0.08944272
4	8.35410056	1.17082044	-0.13416415
	5.23606704	0.44721363	-0.08944277

Table S5: Summary for different component systems

Index a	Dimension	System Size	Recovered Components
1	9	35	One component of degree 4
2	8	60	One component of degree 4
3	7	85	One component of degree 4
4	6	110	Two components of degree 2
5	6	135	Two components of degree 2

Table S6: Initial (exact), perturbed (exact), and recovered (7 decimals) parameter values

Parameter	Initial ($\tilde{p}$)	Perturbed ($\hat{p}$)	Recovered (p^*)
p_1	-30	-30.0000000	-30.0000003
p_2	20	20.0000000	19.9999994
p_3	18	18.0000000	18.0000003
p_4	-12	-12.0000000	-11.9999997
p_5	12	12.0000070	12.0000057
p_6	-8	-8.0000000	-8.0000019
p_7	0	0.0000003	-0.0000014
p_8	-5	-5.0000000	-5.0000002
p_9	3	3.0000000	2.9999992
p_{10}	2	2.0000000	2.0000006

Table S7: Comparison of factors (double precision)

Factors from [52]	-30.0000005908641
	$1 - 0.600000000000000x_1^2 - 0.400000158490569x_1x_2$
	$1 - 0.666666625730982x_1 + 0.166666656432746x_2^2$
Factors from p^*	-30.0000003490653
	$1 - 0.6000000003568289x_1^2 - 0.400000104966183x_1x_2$
	$1 - 0.66666663955509x_1 + 0.166666672330951x_2^2 - 7.87997843076905 \cdot 10^{-8}x_1x_2$

Table S8: Initial (exact), perturbed (4 decimals), and recovered (4 decimals) parameter values

Parameter	Initial ($\tilde{p}$)	Perturbed ($\hat{p}$)	Recovered (p^*)
p_1	1	1.2346	1.0479
p_2	1	1.0089	1.0980

Table S9: Decomposable coupler curve: dimensions and system size as fiber product index a increases

Index a	Dimension	System Size
1	3	53
2	2	102
3	2	151
4	2	200

Table S10: Initial (exact), perturbed (4 decimals), and recovered (4 decimals) parameter values

Parameter	Initial ($\tilde{p}$)	Perturbed ($\hat{p}$)	Recovered (p^*)
p_1	1	1.0025	1.0062
p_2	2	2.0101	2.0057
p_3	1	1.0098	1.0062
p_4	2	2.0014	2.0057

Table S11: Stewart-Gough Platform: dimension and system size as fiber product index a increases

Index a	Dimension	System Size
1	40	72
2	38	102
3	36	132
4	35	162
5	35	192

Table S12: Initial (exact), perturbed (12 decimals), and recovered (12 decimals) parameters

Parameter	Initial ($\tilde{p}$)	Perturbed ($\hat{p}$)	Recovered (p^*)
a_{1x}	0.0000	0.000000000251	0.000000000320
a_{1y}	0.0000	0.000000001013	0.000000000982
a_{1z}	0.0000	0.000000000980	0.000000000477
b_{1x}	0.0000	-0.000000000200	-0.000000000269
b_{1y}	0.0000	-0.000000000637	-0.000000000606
b_{1z}	1.5000	1.499999998979	1.499999999482
d_1	3.2500	3.249999999891	3.249999999724
a_{2x}	1.0000	1.000000000136	1.000000000272
a_{2y}	0.0000	-0.000000000753	-0.000000001236
a_{2z}	0.2500	0.249999998300	0.249999998434
b_{2x}	1.0000	1.000000001806	1.000000001671
b_{2y}	0.0000	-0.000000000886	-0.000000000402
b_{2z}	1.0000	0.999999999658	0.999999999525
d_2	1.5625	1.562499999151	1.562499999240
a_{3x}	1.0000	0.999999999712	0.999999999839
a_{3y}	1.0000	0.999999999733	0.999999999918
a_{3z}	0.0000	0.000000000109	-0.000000000010
b_{3x}	1.0000	0.999999998769	0.999999998641
b_{3y}	1.0000	0.999999999125	0.999999998940
b_{3z}	1.5000	1.499999999413	1.499999999531
d_3	3.2500	3.250000000389	3.250000000350
a_{4x}	-0.5000	-0.500000000115	-0.499999999750
a_{4y}	0.5000	0.500000000098	0.500000000171
a_{4z}	0.0000	0.000000000167	0.000000000893
b_{4x}	-0.5000	-0.499999998762	-0.499999999127
b_{4y}	0.5000	0.499999999424	0.499999999351
b_{4z}	1.0000	1.000000000799	1.000000000073
d_4	2.0000	2.000000000415	2.000000000779
a_{5x}	0.5000	0.500000000717	0.499999999761
a_{5y}	1.5000	1.500000000939	1.500000000405
a_{5z}	0.0000	0.000000000105	0.000000000622
b_{5x}	0.5000	0.499999998819	0.499999999776
b_{5y}	1.5000	1.499999999660	1.500000000194
b_{5z}	1.0000	0.999999999603	0.999999999086
d_5	2.0000	1.999999998435	1.999999998693
a_{6x}	-0.2500	-0.250000000190	-0.249999999931
a_{6y}	1.2500	1.249999999364	1.250000000155
a_{6z}	0.2500	0.250000002270	0.250000001515
b_{6x}	-0.2500	-0.249999999033	-0.249999999292
b_{6y}	1.2500	1.250000001020	1.250000000228
b_{6z}	1.0000	0.999999999682	1.000000000437
d_6	1.5625	1.562500000179	1.562499999676

Table S13: Constant values truncated to single precision

Constant	Single Precision	Constant	Single Precision
q_{08}	$7.4052387 \cdot 10^{-2}$	q_{28}	$1.9594662 \cdot 10^{-1}$
q_{09}	$-8.3050031 \cdot 10^{-2}$	q_{29}	$-1.2280341 \cdot 10^0$
q_{010}	$-3.8615960 \cdot 10^{-1}$	q_{210}	$0.0000000 \cdot 10^0$
q_{011}	$-7.5526603 \cdot 10^{-1}$	q_{211}	$-7.9034219 \cdot 10^{-2}$
q_{012}	$5.0420168 \cdot 10^{-1}$	q_{212}	$2.6387877 \cdot 10^{-2}$
q_{013}	$-1.0916286 \cdot 10^0$	q_{213}	$-5.7131429 \cdot 10^{-2}$
q_{014}	$0.0000000 \cdot 10^0$	q_{214}	$-1.1628081 \cdot 10^0$
q_{015}	$4.0026384 \cdot 10^{-1}$	q_{215}	$1.2587767 \cdot 10^0$
q_{016}	$4.9207289 \cdot 10^{-2}$	q_{216}	$2.1625749 \cdot 10^0$
q_{18}	$-3.7157270 \cdot 10^{-2}$	q_{38}	$-2.0816985 \cdot 10^{-1}$
q_{19}	$3.5436895 \cdot 10^{-2}$	q_{39}	$2.6868319 \cdot 10^0$
q_{110}	$8.5383480 \cdot 10^{-2}$	q_{310}	$-6.9910317 \cdot 10^{-1}$
q_{111}	$0.0000000 \cdot 10^0$	q_{311}	$3.5744412 \cdot 10^{-1}$
q_{112}	$-3.9251967 \cdot 10^{-2}$	q_{312}	$1.2499117 \cdot 10^0$
q_{113}	$0.0000000 \cdot 10^0$	q_{313}	$1.4677360 \cdot 10^0$
q_{114}	$-4.3241927 \cdot 10^{-1}$	q_{314}	$1.1651719 \cdot 10^0$
q_{115}	$0.0000000 \cdot 10^0$	q_{315}	$1.0763397 \cdot 10^0$
q_{116}	$1.3873009 \cdot 10^{-2}$	q_{316}	$-6.9686807 \cdot 10^{-1}$

Table S14: Initial (double precision), perturbed (truncated single precision), and recovered (double precision) parameters

Parameter	Single Double Precision	Recovered
q_{00}	-2.4915068 11232596 $\cdot 10^{-1}$	-2.491506848757833 $\cdot 10^{-1}$
q_{01}	1.6091353 78745045 $\cdot 10^0$	1.609135324728055 $\cdot 10^0$
q_{02}	2.7942342 61384628 $\cdot 10^{-1}$	2.794234123846178 $\cdot 10^{-1}$
q_{03}	1.4348015 88307759 $\cdot 10^0$	1.434801543598025 $\cdot 10^0$
q_{04}	0.0000000 00000000 $\cdot 10^0$	2.329107073061927 $\cdot 10^{-8}$
q_{05}	4.0026384 20852447 $\cdot 10^{-1}$	4.002638399151275 $\cdot 10^{-1}$
q_{06}	-8.0052768 41704895 $\cdot 10^{-1}$	-8.005276506597172 $\cdot 10^{-1}$
q_{07}	0.0000000 00000000 $\cdot 10^0$	1.339330350134300 $\cdot 10^{-8}$
q_{10}	1.2501635 03697273 $\cdot 10^{-1}$	1.250163518996785 $\cdot 10^{-1}$
q_{11}	-6.8660735 90276054 $\cdot 10^{-1}$	-6.866073304900900 $\cdot 10^{-1}$
q_{12}	-1.1922811 66678474 $\cdot 10^{-1}$	-1.192281095708419 $\cdot 10^{-1}$
q_{13}	-7.1994046 84195284 $\cdot 10^{-1}$	-7.199404481832083 $\cdot 10^{-1}$
q_{14}	-4.3241927 30334479 $\cdot 10^{-1}$	-4.324192773933984 $\cdot 10^{-1}$
q_{15}	0.0000000 00000000 $\cdot 10^0$	1.358542627603532 $\cdot 10^{-8}$
q_{16}	0.0000000 00000000 $\cdot 10^0$	-1.039184803095887 $\cdot 10^{-9}$
q_{17}	-8.6483854 60668959 $\cdot 10^{-1}$	-8.648385383114613 $\cdot 10^{-1}$
q_{20}	-6.3555007 06536143 $\cdot 10^{-1}$	-6.355500280163283 $\cdot 10^{-1}$
q_{21}	-1.1571992 24063992 $\cdot 10^{-1}$	-1.157199361445811 $\cdot 10^{-1}$
q_{22}	-6.6640447 34656436 $\cdot 10^{-1}$	-6.664044436579097 $\cdot 10^{-1}$
q_{23}	1.1036211 15850889 $\cdot 10^{-1}$	1.103620867759053 $\cdot 10^{-1}$
q_{24}	2.9070203 22913935 $\cdot 10^{-1}$	2.907020211729024 $\cdot 10^{-1}$
q_{25}	1.2587767 24480555 $\cdot 10^0$	1.258776710166779 $\cdot 10^0$
q_{26}	-6.2938836 22402776 $\cdot 10^{-1}$	-6.293883708977084 $\cdot 10^{-1}$
q_{27}	5.8140406 45827871 $\cdot 10^{-1}$	5.814040462810132 $\cdot 10^{-1}$
q_{30}	1.4894773 41316300 $\cdot 10^0$	1.489477303748473 $\cdot 10^0$
q_{31}	2.3062341 36720304 $\cdot 10^{-1}$	2.306233954795566 $\cdot 10^{-1}$
q_{32}	1.3281073 07376312 $\cdot 10^0$	1.328107268535429 $\cdot 10^0$
q_{33}	-2.5864502 59957599 $\cdot 10^{-1}$	-2.586450384436285 $\cdot 10^{-1}$
q_{34}	1.1651719 51133394 $\cdot 10^0$	1.165171916593329 $\cdot 10^0$
q_{35}	-2.6908493 58556267 $\cdot 10^{-1}$	-2.690849292497942 $\cdot 10^{-1}$
q_{36}	5.3816987 17112534 $\cdot 10^{-1}$	5.381698714725988 $\cdot 10^{-1}$
q_{37}	5.8258597 55666972 $\cdot 10^{-1}$	5.825859575485448 $\cdot 10^{-1}$