Homework 6
(due Friday, March 6)

Turn in full solutions.

7.3: 2 (Use polar coordinates), 3, 19, 20

From Jones, Chapter 10: 10-41, 10-43

7.6: 4b

Problem 1. (Rewording of Jones 10-24) Let $\Omega \subset \mathbf{R}^2$ be a region. Consider the 'cone'

$$C = \{y = (1-t)(0,0,1) + t(x_1, x_2, 0) \in \mathbf{R}^3 : (x_1, x_2) \in \Omega \text{ and } 0 \leq t \leq 1\}$$

consisting of all line segments joining the apex $(0,0,1)$ to a point in the 'base' Ω. Compute the (3 dimensional) volume of C in terms of the (2 dimensional) volume of Ω using the change of variables $y = (y_1, y_2, y_3) = \Phi(x) := x_3(x_1, x_2, 0) + (1 - x_3)(0,0,1)$.

Problem 2. Given two numbers $R > r > 0$, consider the region $\Omega \subset \mathbf{R}^3$ obtained by revolving the disk $\{(x, z, 0) : (x-R)^2 + z^2 \leq r^2\}$ about the z-axis. The resulting donut shape is known as a torus. Compute the volume of the torus two different ways:

(a) Using cylindrical coordinates.
(b) Using the change of coordinates

$$(x, y, z) = \Phi(t, \phi, \theta) = ((R + t\sin\phi)\cos\theta, (R + t\sin\phi)\sin\theta, t\cos\phi).$$