Statistics for the Life Sciences

Math 20340 Section 01, Fall 2008

Homework 3 Solutions

- **4.75**: Given:
  - $P(L) = .3$, $P(R) = .7$
  - $P(S|R) = .8$, $P(B|R) = .2$
  - $P(S|L) = .1$, $P(B|L) = .9$

  From this we get
  - $P(S \cap R) = P(R)P(S|R) = .7 * .8 = .56$
  - $P(S \cap L) = P(L)P(S|L) = .3 * .1 = .03$
  - $P(B \cap R) = P(R)P(B|R) = .7 * .2 = .14$
  - $P(B \cap L) = P(L)P(B|L) = .3 * .9 = .27$

  and so
  - $P(S) = P(S \cap R) + P(S \cap L) = .56 + .03 = .59$
  - $P(B) = P(B \cap R) + P(B \cap L) = .14 + .27 = .41$

  - **a**: $P(L|B) = P(L \cap B)/P(B) = .27/.41 = .66...$
  - **b**: $P(R|B) = P(R \cap B)/P(B) = .14/.41 = .34...$
  - **c**: On seeing balanced stance, you know that with probability .34 the play will go to the right, and with probability .66 it will go to the left. This suggests it is better to prepare for a play going to the left.

- **4.84**:
  - **a**: Probability distribution: $x = 0, 1, 2, 3, 4, 5, 6$, $p(i) = 1/6$ for each i
  - **b**: $\mu = \sum_{i=0}^{6} i \frac{1}{6} = 3.5$
  - **c**: $\sigma^2 = \sum_{i=0}^{6} (i - 3.5)^2 \frac{1}{6} = 35/12 = 2.916...$ so $\sigma = \sqrt{35/12} = 1.7078...$
  - **d**: The range $\mu \pm 2\sigma$ is from .008... to 6.91...; *all* measurements fall in this range

- **4.86**:

  - **a**: $p(0) = .48^3 = .11$; $p(1) = 3 * .52 * .48^2 = .36$; $p(2) = 3 * .52^2 * .48 = .39$; $p(3) = .52^3 = .14$
  - **b**:
  - **c**: $p(1) = .36$
  - **d**:
    * $\mu = 0 * .11 + 1 * .36 + 2 * .39 + 3 * .14 = 1.56$
    * $\sigma = \sqrt{(0 - 1.56)^2 * .11 + (1 - 1.56)^2 * .36 + (2 - 1.56)^2 * .39 + (3 - 1.56)^2 * .14} = .865...$

- **4.93**:

  - **a**:
    * $p(3) = (.6)^3 + (.4)^3 = .28$ (either A wins all three or B does)
    * $p(4) = 3 * (.6)^3 * .4 + 3 * (.4)^3 * .6 = .3744$ (either A wins three sets to one [three ways for this to happen, $BAAA$, $ABAA$, $AABA$, each with probability $(.6)^3 * .4$] or B wins three sets to one [three ways for this to happen, $ABBB$, $BABB$, $BBAB$, each with probability $(.4)^3 * .6$])
    * $p(5) = 1 - p(3) - p(4) = .3456$

    So $E(x) = 3 * .28 + 4 * .3744 + 5 * .3456 = 4.0656$
  - **b**:
    * $p(3) = (.5)^3 + (.5)^3 = .25$
    * $p(4) = 3 * (.5)^3 * .5 + 3 * (.5)^3 * .5 = .375$
    * $p(5) = 1 - p(3) - p(4) = .375$

    So $E(x) = 3 * .25 + 4 * .375 + 5 * .375 = 4.125$
  - **c**:
    * $p(3) = (.9)^3 + (.1)^3 = .73$
    * $p(4) = 3 * (.9)^3 * .1 + 3 * (.1)^3 * .9 = .2214$
    * $p(5) = 1 - p(3) - p(4) = .0486$

    So $E(x) = 3 * .73 + 4 * .2214 + 5 * .0486 = 3.3186$

- **4.95**:

  Let x be a random variable measuring the insurance company's gain. We have $x = D$ with probability .99, and $x = D - 50000$ with probability .01, so the expectation of x is $E(x) = .99D + .01 * (D + 50000) = D - 500$. Insurance company wants to set D so that gain is 1000 (on average) so want $D - 500 = 1000$, $D = 1500$

- **4.97**:

  - **a**: $p(0) = .28$
  - **b**: Probability of more than two breaks is $p(3) + p(4) + p(5) = .18$

- **c**: $E(x) = 0 * .28 + 1 * .37 + 2 * .17 + 3 * .12 + 4 * .05 + 5 * .01 = 1.32$ and $\sigma^2 = (0 - 1.32)^2 * .28 + (1 - 1.32)^2 * .37 + (2 - 1.32)^2 * .17 + (3 - 1.32)^2 * .12 + (4 - 1.32)^2 * .05 + (5 - 1.32)^2 * .01 = 1.4376$ so $\sigma = 1.199$.
- **d**: The range $\mu \pm 2\sigma$ is -1.06 to 3.7, so covers $x = 0, 1, 2, 3$. Probability of falling in this range is $.94$.

- **5.4**:

  Because the chosen ball is always replaced, all iterations of the experiment are identical (in each case there are 3 red and 2 non-red balls). Also, the trials are independent, so this is a binomial trial with $n = 2$ and $p = 3/5 = .6$.

- **5.7**:

  - **a**: $.0972$
  - **b**: $.3294$
  - **c**: $.6706$
  - **d**: 2.1
  - **e**: 1.21

- **5.15**:

  - **a**: $.7483$
  - **b**: $.6098$
  - **c**: $.3669$
  - **d**: $.9662$
  - **e**: $.6563$

- **5.17** (a, b, c):

  - **a**: $\mu = 100 * .01 = 1$, $\sigma = \sqrt{100 * .01 * .99} = .99498...$
  - **b**: $\mu = 100 * .9 = 90$, $\sigma = \sqrt{100 * .9 * .1} = 3$
  - **c**: $\mu = 100 * .3 = 30$, $\sigma = \sqrt{100 * .3 * .7} = 4.582...$

- **5.20**:

  This is a repeat of the same trial (check to see if it is raining) 30 times in a row. The trials probably don't have the same success probability, though (typically some months, such as April, are rainier than others, such as May; so if the 30 days started at the end of April and went through to May, the success probability might get smaller as the 30 days progress). Also, the trials may not be independent ... rainy spells (and dry spells) tend to occur in clumps, P(dry tomorrow—dry today) is probably greater than P(wet tomorrow—dry today). So it's probably not a binomial trial.

- **5.21**:

  Not a binomial experiment. Instead of repeating the same experiment a *fixed* number of times, and counting number of successes, we are repeating the same experiment a variable number of times and counting how long it takes for the first success ... this is a different process to the binomial trial.

- **5.22**:

  - **a**: Binomial; $n = 100$, $p = .45$
  - **b**: Not binomial
  - **c**: Binomial: $n = 100$,

    $p = $ (#students taking exam *and* scoring above $1518)/($#students taking exam in total).

    Since 45% of students took exam, and of those probably close to 50% scored above average, probably $p = .225$
  - **d**: Not binomial

- **5.25**:

  This is a binomial trial with $n = 25$ and $p = .1$

  - **a**: .0980
  - **b**: .9905
  - **c**: .0980
  - **d**: .1384
  - **e**: .4295
  - **f**: .9020 (More than 20 not black = 4 or fewer are black)

- **5.31**:

  - **a**: $C_8^8 * .6^8 * .4^0 = .0168$
  - **b**: Printout gives .016796
  - **c**: .9832 $(1 - .0168)$

- **5.32**: This is a binomial trial with $n = 25$ and $p = .4$

  - **a**: $\mu = 25 * .4 = 10$ and $\sigma^2 = 25 * .4 * .6 = 6$
  - **b**: $\mu \pm 2\sigma$ is the range from 5.101 to 14.8989.... The values that fall into this interval are $6, 7, 8, 9, 10, 11, 12, 13$ and 14
  - **c**: .9362

- **Bonus Question**:

  I'll just do this for $n = 2$. Possible values of x are 0, 1 and 2. The various probabilities are

- $p(0) = C_0^2 p^0 q^2 = q^2$
- $p(1) = C_1^2 p^1 q^1 = 2pq$
- $p(2) = C_2^2 p^2 q^0 = p^2$

So

$$\begin{aligned}
\mu &= 0 * q^2 + 1 * 2pq + 2 * p^2 \\
&= 2pq + p^2 \\
&= 2p(1-p) + 2p^2 = 2p - 2p^2 + 2p^2 \\
&= 2p
\end{aligned}$$

and

$$\begin{aligned}
\sigma^2 &= (0-2p)^2 * q^2 + (1-2p)^2 * 2pq + (2-2p)^2 * p^2 \\
&= 4p^2q^2 + (1-4p+4p^2)2pq + (4-8p+4p^2)p^2 \\
&= 4p^2(1-p)^2 + (1-4p+4p^2)2p(1-p) + (4-8p+4p^2)p^2 \\
&= 4p^2(1-2p+p^2) + (1-4p+4p^2)(2p-2p^2) + (4-8p+4p^2)p^2 \\
&= 4p^2 - 8p^3 + 4p^4 + 2p - 2p^2 - 8p^2 + 8p^3 + 8p^3 - 8p^4 + 4p^2 - 8p^3 + 4p^4 \\
&= 2p - 2p^2 \\
&= 2p(1-p) \\
&= 2pq.
\end{aligned}$$