Statistics for the Life Sciences

Math 20340 Section 01, Fall 2008

Homework 2 Solutions

- **4.37**:

  The experiment here is to pick one of the possible ways in which the board votes 5-3 in favour. To construct a simple event, we need just selected the five people who voted in favour of the plaintiff; the unchosen three automatically become the three who voted against. There are $C_5^8 = 56$ ways to do this (note that we don't care about the order in which people voted), and so 56 simple events. In only one of those 56 do all 5 women vote in favour of plaintiff; so the probsbility of seeing this split is $1/56$.

- **4.38**:

  There are $C_5^{10} = 252$ ways for the instructor to choose 5 questions (order doesn't matter). Of these 252, only $C_5^6 = 6$ include only questions that the student has prepared. So he probability is $6/252 \approx .0238$.

- **4.42**:

  - **c**: $B \cap C = \{E_4\}$, so $P(B \cap C) = .2$
  - **f**: The simple event that is in A and also in B is E_1 (one simple event) and B consists of 4 events, so $P(A|B) = 1/4$
  - **h**: $A \cap B = \{E_1\}$ so $(A \cap B)^c = \{E_2, E_3, E_4, E_5\}$ and $P((A \cap B)^c) = 4/5$

- **4.43**:

  - **b**: $P((A \cap B)^c) = 1 - P(A \cap B) = 1 - .2 = .8\ (= 4/5)$

- **4.44**:

  - **a**: $P(A|B) = P(A \cap B)/P(B) = .2/.8 = .25\ (= 1/4)$

- **4.45**:

  - **c**: $P(B \cup C) = P(B) + P(C) - P(B \cap C)$ so $P(B \cap C) = P(B) + P(C) - P(B \cup C) = .8 + .4 - 1 = .2$

- **4.46**:

- **a**: $P(A|B) = .25$ and $P(A) = .4$, so the events are NOT independent
- **b**: $A \cap B = \{E_1\}$ which is not the empty set, so the events are NOT mutually exclusive

- **4.50**:
  - **a**: Since the events are mutually exclusive, the probability of the intersection is 0
  - **b**: Since the events are mutually exclusive, the probability of the union is the sum of the probabilities, so $P(A \cup B) = .3 + .5 = .8$

- **4.52**:
  - **a**: $.49$
  - **b**: $.8$
  - **c**: $.34$
  - **d**: $.95$
  - **e**: $.34/.8 = 17/40$
  - **f**: $.34/.49 = 34/49$

- **4.56**:
  - **a**: $.4$
  - **b**: $.37$
  - **c**: $.1$
  - **d**: $.67$
  - **e**: $.6$
  - **f**: $.33$
  - **g**: $.9$
  - **h**: $.1/.37 \approx .27$
  - **i**: $.1/.4 = .25$

- **4.60**:
  - **a**: $S = \{$Starbucks$\}, M = \{$Mocha$\}$. We want $P(S \cap M)$. Since S and M are given to be independent, $P(S \cap M) = P(S)P(M) = .7X.6 = .42$
  - **b**: Yes, it is given in the question that choice of drink is not influenced by choice of coffee shop
  - **c**: $P = \{$Peets$\}$. Want $P(P|M) = P(P \cap M)/P(M) = (.3 * .6)/.6 = .3$. Or, easier: $P(P|M) = P(P) = .3$ since P and M are independent
  - **d**: $P(S \cup M) = P(S) + P(M) - P(S \cap M) = .7 + .6 - .42 = .88$

- **4.62**:

  Let S be the event that a randomly chosen person is a smoker, and L the event that a randomly chosen person dies from lung cancer. We are given:

  - $P(S) = .2$ (so $P(S^c) = .8$)
  - $P(L|S) \approx 10P(L|S^c)$
  - $P(L) = .006$

  A person who dies of lung cancer either smokes or doesn't (but not both), so $P(L) = P(L \cap S) + P(L \cap S^c)$. Since $P(L \cap S) = P(S)P(L|S) = .2P(L|S)$ and $P(L \cap S^c) = P(S^c)P(L|S^c) = .8P(L|S^c)$ we get $.006 = .2P(L|S) + .8P(L|S^c)$. Solving this simultaneously with $P(L|S) \approx 10P(L|S^c)$ we get $.006 \approx 2P(L|S^c) + .8P(L|S^c)$ or $P(L|S^c) \approx .00214$ and so $P(L|S) \approx .0214$.

  Note: If you interpret the question as saying that $P(L \cap S) \approx 10P(L \cap S^c)$ (as I did during Monday's office hours) then from $P(L) = P(L \cap S) + P(L \cap S^c)$ we get $.006 = 11P(L \cap S^c)$ so $P(L \cap S^c) \approx .000545$, and $P(L \cap S) \approx .00545$, so that $P(L|S) = P(L \cap S)/P(S) \approx .00545/.2 \approx .02725$. I think this is a wrong interpretation, but the grader won't mark down for it since I made the mistake in office hours.

- **4.65**:

  - **d**: $88/154$
  - **e**: $44/67$
  - **f**: $23/35$

- **4.67**:

  - **a**: $.8X.8 = .64$ (throws are independent)
  - **b**: Probability that Shaq makes only first of two: $.53X.47 = .2491$. Probability that Shaq makes only second of two: $.47X.53 = .2491$. These are the two mutually exclusive events that combine to the event of Shaq making exactly one; so probability that Shaq makes exactly one: $.2491 + .2491 = .4982$.
  - **c**: $A = \{$Shaq makes both$\}$. $B = \{$Jason makes neither$\}$. $P(A) = .53 * .53 = .2809$. $P(B) = .2 * .2 = .04$. Since A and B are independent, $P(A \cap B) = P(A)P(B) = .011246$